# 第 16 章　數學附錄

一　證明母體兩類別的分配比例是否相同的齊一性檢定，與兩母體比例是否相等的檢定完全一致。

證明下表為兩個母體在兩個類別的次數分配。

**表 18.31　　兩母體兩類別的次數分配**

| 母體\類別 | 第一類 | 第二類 | 總和 |
|---|---|---|---|
| 母體 1 | $X_{11}$ | $X_{12}$ | $n_1$ |
| 母體 2 | $X_{21}$ | $X_{22}$ | $n_2$ |
| 總和 | $X_{11}+X$ | $X_{12}+X$ | |

所以 $\hat{p}=\dfrac{X_{11}+X_{21}}{n_1+n_2}$

**表 18.32　　兩母體兩類別的次數分配**

| | 第一類 | 第二類 | 總和 |
|---|---|---|---|
| 母體 1 | $X_{11}$<br>$\left[\dfrac{n_1(X_{11}+X_{21}}{n_1+n_2}\right.$ | $X_{12}$<br>$\left[\dfrac{n_1(X_{12}+X_{22}}{n_1+n_2}\right.$ | $n_1$ |
| 母體 2 | $X_{21}$<br>$\left[\dfrac{n_2(X_{11}+X_{21}}{n_1+n_2}\right.$ | $X_{22}$<br>$\left[\dfrac{n_2(X_{12}+X_{22}}{n_1+n_2}\right.$ | $n_2$ |
| 總和 | $X_{11}+X_{21}$ | $X_{12}+X_{22}$ | |

註：括弧內為理論次數

$$\chi^2=\frac{\left[X_{11}-\frac{n_1X_{11}+n_1X_{21}}{n_1+n_2}\right]^2}{\frac{n_1X_{11}+n_1X_{21}}{n_1+n_2}}+\frac{\left[X_{12}-\frac{n_1X_{12}+n_1X_{22}}{n_1+n_2}\right]^2}{\frac{n_1X_{12}+n_1X_{22}}{n_1+n_2}}+\frac{\left[X_{21}-\frac{n_2X_{11}+n_2X_{21}}{n_1+n_2}\right]^2}{\frac{n_2X_{11}+n_2X_{21}}{n_1+n_2}}$$

$$+\frac{\left[X_{22}-\frac{n_2X_{12}+n_2X_{22}}{n_1+n_2}\right]^2}{\frac{n_2X_{12}+n_2X_{22}}{n_1+n_2}}$$

$$=\frac{(n_2X_{11}-n_1X_{21})^2}{(n_1+n_2)^2\left(\frac{n_1X_{11}+n_1X_{21}}{n_1+n_2}\right)}+\frac{(n_2X_{12}-n_1X_{22})^2}{(n_1+n_2)^2\left(\frac{n_1X_{12}+n_1X_{22}}{n_1+n_2}\right)}+\frac{(n_1X_{21}-n_2X_{11})^2}{(n_1+n_2)^2\left(\frac{n_2X_{11}+n_2X_{21}}{n_1+n_2}\right)}+\frac{(n_1X_{22}-n_2X_{12})^2}{(n_1+n_2)^2\left(\frac{n_2X_{12}+n_2X_{22}}{n_1+n_2}\right)}$$

$$=\frac{\left(\frac{X_{11}}{n_1}-\frac{X_{21}}{n_2}\right)^2\frac{(n_1+n_2)}{n_1n_2}}{\left(\frac{n_1X_{11}+n_1X_{21}}{n_1+n_2}\right)}+\frac{\left(\frac{X_{11}}{n_1}-\frac{X_{21}}{n_2}\right)^2\frac{(n_1+n_2)}{n_1n_2}}{\left(\frac{n_1X_{12}+n_1X_{22}}{n_1+n_2}\right)}+\frac{\left(\frac{X_{21}}{n_2}-\frac{X_{11}}{n_1}\right)^2\frac{(n_1+n_2)}{n_1n_2}}{\left(\frac{n_2X_{11}+n_2X_{21}}{n_1+n_2}\right)}+\frac{\left(\frac{X_{21}}{n_2}-\frac{X_{11}}{n_1}\right)^2\frac{(n_1+n_2)}{n_1n_2}}{\left(\frac{n_2X_{12}+n_2X_{22}}{n_1+n_2}\right)}$$

$$=\left[\frac{\frac{X_{11}}{n_1}-\frac{X_{21}}{n_2}}{\frac{n_1+n_2}{n_1n_2}}\right]^2\left[\frac{1}{\frac{n_1X_{11}+n_1X_{21}}{n_1+n_2}}+\frac{1}{\frac{n_1X_{12}+n_1X_{22}}{n_1+n_2}}+\frac{1}{\frac{n_2X_{11}+n_2X_{21}}{n_1+n_2}}+\frac{1}{\frac{n_2X_{12}+n_2X_{22}}{n_1+n_2}}\right]$$

$$=\frac{\left(\frac{X_{11}}{n_1}-\frac{X_{21}}{n_2}\right)^2\frac{n_1+n_2}{n_1n_2}}{\left(\frac{X_{11}+X_{21}}{n_1+n_2}\right)\left(\frac{X_{12}+X_{22}}{n_1+n_2}\right)\left(\frac{n_1n_2}{n_1+n_2}\right)}$$

$$=\frac{\left(\bar{p}_1-\bar{p}_2\right)^2}{\left(\sqrt{\frac{X_{11}+X_{21}}{n_1+n_2}}\sqrt{1-\frac{X_{11}+X_{21}}{n_1+n_2}}\sqrt{\frac{n_1+n_2}{n_1n_2}}\right)^2}=\left(\frac{\bar{p}_1-\bar{p}_2}{\sqrt{\bar{p}(1-\bar{p})}\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}}\right)^2=Z^2$$

其中：$\frac{X_{11}}{n_1}=\hat{p}_1$，$\frac{X_{21}}{n_2}=\hat{p}_2$，$\frac{1}{n_1}+\frac{1}{n_2}=\frac{n_1+n_2}{n_1n_2}$，$\bar{p}=\frac{X_{11}+X_{21}}{n_1+n_2}$。

故兩者檢完全一致。