MECHANISTIC DESIGN MODELS OF LOADING AND CURLING IN CONCRETE PAVEMENTS

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Abstract

Determination of the critical bending stress at the edge of a concrete pavement due to individual and combination effects of wheel loading and thermal curling is very crucial to any mechanistic based design procedure. This paper first presents the recent discovery of two additional dimensionless mechanistic variables such that the problems encountered in previous investigations using dimensional analysis for thermal-related curling problems are resolved. A new regression technique (projection pursuit regression) together with traditional linear and nonlinear regressions were utilized to develop prediction models. Subsequently, closed-form mechanistic design models which have been carefully validated and ready for implementation in a spreadsheet or computer program are presented. These prediction models turned out to be a very accurate representation of the finite element model. They are also simple, easy to comprehend, dimensionally correct, and may be extrapolated to wider ranges of input parameters.

Introduction

Since transverse and longitudinal cracking are the major structural distresses in jointed concrete pavements, the determination of the critical bending stress at the edge (midway between the joints) is very crucial to any

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mechanistic based design procedure. The analysis of edge stress due to the effects of wheel loading and loading plus curling is presented in this paper.

Finite element (FE) models have been successfully utilized to account for the effects of a finite slab size as well as possible loss of support due to a linear temperature differential more realistically than theoretical solutions based on infinite slab and full contact assumptions. However, due to the required run time and complexity of the FE model, FE analysis can not be easily implemented as a part of design procedures.

Therefore, a series of FE factorial runs were performed over a wide range of pavement designs. The resulting edge stresses were compared to theoretical Westergaard solutions and adjustment factors were introduced to account for this discrepancy. Statistical regression techniques were utilized to develop predictive models for each adjustment factor. These predictive models can then be used as an alternative to FE analysis to estimate stresses for pavement design with sufficient accuracy.

Identification of Important Mechanistic Variables

Primary Structural Responses Due to Wheel Loading Only

According to Westergaard (1948), the edge stress of a concrete pavement due to a circular edge loading over a dense liquid foundation (Winkler foundation) can be determined by the following expression:

$$\sigma_w = \frac{3(1+\mu)P}{\pi(3+\mu)h^2}\left[\log_e \frac{Eh^3}{100ka^4} + 1.84 - \frac{4\mu}{3} + \frac{1-\mu}{2} + 1.18(1+2\mu)\frac{a}{l}\right] \tag{1}$$

$$l = \sqrt[4]{\frac{Eh^3}{12(1-\mu^2)k}} \tag{2}$$

Where:

- σ_w = Westergaard's edge stress, [FL^{-2}];
- P = total applied wheel load [F];
- a = radius of the applied circular load [L];
- E = modulus of elasticity of the concrete slab [FL^{-2}];
- h = thickness of the slab [L];
- μ = Poisson's ratio of the concrete;
- k = modulus of subgrade reaction [FL^{-3}]; and
- l = radius of relative stiffness of the slab subgrade system [L].

(Note: Primary dimensions are represented by [F] for force and [L] for length.)

Through the use of the principles of dimensional analysis, Ioannides and Salsilli (1989) have demonstrated that Westergaard solutions for the primary structural responses (i.e., slab bending stress, slab deflection, and subgrade stress) can be reduced to three dimensionless terms, which all depend on the normalized load radius alone for a constant Poisson's ratio (usually $\mu \approx 0.15$). These relationships are given as follows:

$$\frac{\sigma h^2}{P}, \frac{\delta k l^2}{P}, \frac{q l^2}{P} = f\left(\frac{a}{l}\right) \tag{3}$$

Where:

σ, q = slab bending stress and subgrade vertical stress, respectively, [FL^{-2}]; and

δ = slab deflection, [L].

Thus, the relationships between the primary structural responses and the external wheel loading are concisely defined. The dependent variables are $\sigma h^2/P$, $\delta k l^2/P$, and $q l^2/P$, whereas independent variable is the dominating factor a/l, rather than the other input parameters (E, h, μ, k, and a). Note that these relationships are also simple, easy to understand, and dimensionally correct.

The analysis of finite slab length and width effect due to an external wheel load is not possible until the introduction of FE models. According to previous research (Ioannides et al. 1985, 1989), the normalized slab length and width (L/l and W/l) are the dominating mechanistic variables for the finite extent of slab condition, where L and W are the finite slab length and width, respectively. In fact, theoretical treatments on the effect of thermal curling (rather than wheel loading), which can be traced back as early as Westergaard (1926) and Bradbury (1938), have already shown that the finite extent of slab condition is simply a function of these two dimensionless terms. Thus, the above relationships become:

$$\frac{\sigma h^2}{P}, \frac{\delta k l^2}{P}, \frac{q l^2}{P} = f\left(\frac{a}{l}, \frac{L}{l}, \frac{W}{l}\right) \tag{4}$$

<u>Primary Structural Responses Due to Thermal Curling Only</u>

Considering a linear temperature differential acting on a concrete slab over a dense liquid foundation, Westergaard (1926) first developed solutions for three slab conditions (i.e., infinite, semi-infinite, and an infinite long strip). Bradbury (1938) later expanded Westergaard's stress solution for a slab with finite dimensions in both transverse and longitudinal directions. Thus, the curling stress along the very edge of a very narrow and long strip of slab could be determined by:

$$\sigma_c = \frac{E\alpha\Delta T}{2}\left[1 - \frac{2\cos\lambda\cosh\lambda}{\sin 2\lambda + \sinh 2\lambda}(\tan\lambda + \tanh\lambda)\right] \tag{5}$$

$$\lambda = \frac{B}{l\sqrt{8}} \tag{6}$$

Where:

σ_c = Westergaard's edge curling stress, [FL^{-2}];
α = thermal expansion coefficient of the slab, [T^{-1}];
ΔT = a linear temperature differential through the thickness of the slab, [T]; and
B = finite slab width (W) or length (L).

Similarly, the closed-form solutions derived for the thermal curling problem can be summarized as follows for constant Poisson's ratio:

$$\frac{\sigma}{E}\,,\ \frac{\delta h}{l^2}\,,\ \frac{qh}{kl^2} = f\left(\alpha\Delta T\,,\ \frac{L}{l}\,,\ \frac{W}{l}\right) \tag{7}$$

Two Additional Dimensionless Parameters Identified

It is known that Westergaard/Bradbury's analytical solutions for thermal-related curling problems are based on a full contact assumption between the pavement slab and the subgrade. However, experimental data by Teller and Sutherland (1935) have clearly shown that parts of the pavement slab were found without subgrade support when a temperature (or moisture) differential existed through the slab thickness. Thus, the effect of the loss of subgrade support due to a temperature differential has to be considered in order to more realistically account for the actual pavement condition.

The ILLI-SLAB finite element model (Korovesis 1990), which has been developed over the past 15 years at the University of Illinois and has implemented this effect through an iterative procedure, was selected for this study.

Because of the above theoretical difference, attempts by earlier investigators (e.g., Salsilli 1991) to relate the primary structural responses due to the combination effect of a single wheel load and a temperature influence were not very successful using only the aforementioned dimensionless variables. Thus, it was necessary to search for other possible mechanistic variables to adequately describe this difference.

Particular attention was focused on the effect of the self-weight of the concrete slab on the thermal-induced curling stress. Westergaard (1926) did not explicitly consider the self-weight effect in his deflection equation, since the center of the slab was assumed to be flat and the deflection was zero. On the other hand, the effect of self-weight is included in the ILLI-SLAB model. The deflection at the center is determined by:

$$\delta_\gamma = \frac{\gamma h}{k} \tag{8}$$

where: γ = unit weight of the concrete slab, [FL^{-3}].

In Westergaard's deflection solution due to thermal curling alone, the deflection may be described by the normalized slab length and width (L/l and W/l), the dimensionless product of $\alpha\Delta T$, and the deflection factor (l^2/h). All these parameters are dimensionless except the last one, l^2/h, which has the same dimension as length and can be used as an indicator for the extent of loss of subgrade support. By taking the ratio of self-weight deflection versus l^2/h, the following dimensionless parameter, D_γ, was defined to represent the relative deflection stiffness due to the self-weight of the concrete slab and the possible loss of subgrade support:

$$D_\gamma = \frac{\gamma h^2}{k l^2} \tag{9}$$

It was hypothesized that the resulting ILLI-SLAB edge strain due to curling alone may be well characterized by L/l, W/l, $\alpha\Delta T$, as well as this additional parameter (D_γ). To validate this hypothesis, 16 ILLI-SLAB runs were performed by keeping the above four parameters constant while changing any individual input variables. The resulting edge and interior stresses were numerically confirmed to be directly proportional to the elastic modulus of the concrete slab. In other words, the resulting edge strains were unique for all the runs.

By the same token, this "deflection-ratio" concept was applied again in searching for another dimensionless parameter for the effect of an applied wheel loading on a curled slab due to a linear temperature differential. Based on Westergaard's deflection solution due to loading only, the edge deflection can be represented by two parameters, a/l and $P/(Eh^3k)^{1/2}$. The second one is a deflection factor which is analogous to P/kl^2 and has the same dimension as length. By taking the ratio of this deflection factor versus l^2/h, the following dimensionless parameter, D_P, which may be used as an indicator to represent the relative deflection stiffness due to the external wheel load and the loss of subgrade support:

$$D_P = \frac{Ph}{kl^4} = 12\left(1 - \mu^2\right)\frac{P}{Eh^2} \tag{10}$$

Thus, it was also hypothesized that the following six dimensionless parameters: a/l, L/l, W/l, $\alpha\Delta T$, D_γ, and D_P, would adequately describe the primary structural responses for the combination effect of loading plus thermal curling. To numerically validate this hypothesis, 16 ILLI-SLAB runs were performed while keeping these six parameters constant and changing any individual input variables. The resulting ILLI-SLAB edge and interior stresses were again directly proportional to the elastic modulus of the concrete slab. Thus, unique edge strain was obtained for all the runs as well.

New Predictive Modeling Procedures

The proper selection of regression techniques is one of the most important factors to the success of prediction modeling. Traditional "parametric" regression techniques such as linear and nonlinear regressions require imposing a parametric form on the functions and then obtaining the parameter estimates afterwards. With the multi-dimensional pavement engineering problems in mind, several untangled deficiencies are frequently identified in the use of traditional stepwise regression and nonlinear regression. These include problems in the selection of correct functional form, violations of the embedded statistical assumptions, and failure to satisfy some engineering boundary conditions.

In situations where little knowledge about the shape and the form of a function exists, several new "nonparametric" regression techniques developed over the past ten years have gradually gained popularity. Without imposing an unjustified parametric assumption, nonparametric regression techniques strive to estimate the actual functional form which best fits the data through the use of scatter plot smoothers.

The projection pursuit regression (PPR or "projection") algorithm introduced by Friedman and Stuetzle (1981) appears to have the most favorable features due to its capability of handling variable interactions when suggesting transformations to improve the fit. The "projection" algorithm strives to model a multi-dimensional response surface as a sum of several projected curves through the use of local smoothing technique. The projected curves are essentially two-dimensional curves, which can be graphically displayed, easily visualized, and properly formulated. The relative importance of each projected curve can be measured by the absolute value of the regression coefficients as well.

A new statistical package named S-PLUS which has been widely used by statisticians for data analysis (Statistical Science Inc. 1991[a], 1991[b]) was selected due to the availability of this new regression technique. As a result, the following two-step modeling procedure proposed by Lee (1993) was adopted:

1. Use the "projection" algorithm to break down the multi-dimensional response surface into a sum of several smooth projected curves; and
2. Use traditional linear and nonlinear regression techniques to obtain the parameter estimates of each individual projected curves and the overall regression statistics.

Predictive Models for Loading Only

The effects of an external wheel loading subjected to a finite slab length and a finite slab width were analyzed separately. Adjustment (multiplication) factors for each individual effect were developed. Their combination effects can be approximately represented by multiplying both adjustment factors together.

Effect of Finite Slab Length

An adjustment factor (R_L) for the finite slab length effect was developed based on the following relationship:

$$R_L = \frac{\sigma_i}{\sigma_w} = f\left(\frac{a}{l}\ ,\ \frac{L}{l}\right) \tag{11}$$

Where: σ_i = edge stress determined by the FE model, [FL^{-2}].

Based on previous investigation (Ioannides et al. 1985), the maximum edge stress condition or Westergaard's infinite slab assumption may be achieved by selecting a value of 5.0 or more for the normalized slab length term (L/l). Thus, a more conservative value of 7.0 for both the normalized slab length and the normalized slab width (W/l) was selected to ensure an infinite slab condition. The following factorial FE runs were performed:

a/l: 0.05, 0.10, 0.20, 0.30

L/l: 2.0, 2.5, 3.0, 3.5, 4.0, 4.5, 5.0, 6.0, 7.0

Note that W/l was remained a constant value of 7.0 for all runs. The resulting ILLI-SLAB edge stresses when L/l = 7.0 and W/l = 7.0 were used to approximate Westergaard's solution in calculating adjustment factors. A three-dimensional perspective plot providing a very clear picture of the relationship among R_L, a/l, and L/l is shown in Figure 1.

By using the aforementioned modeling approach, the following one-term "projection" model was developed:

$$R_L = 0.940 + 0.0799\Phi_1(A1) \tag{12}$$

$$\Phi_1(A1) = -4.031 + \frac{1}{0.203 + 0.0345A1^{-3.304}} \tag{13}$$

$$A1 = -0.944\frac{a}{l} + 0.331\frac{L}{l} \tag{14}$$

Statistics: N = 36, R^2 = 0.994, SEE = 0.0063, CV = 0.67%

Limits: $2 \le L/l \le 7$, $0.05 \le a/l \le 0.3$

(Note: N = number of data points, R^2 = coefficient of determination, SEE = standard error of estimates, CV = coefficient of variation)

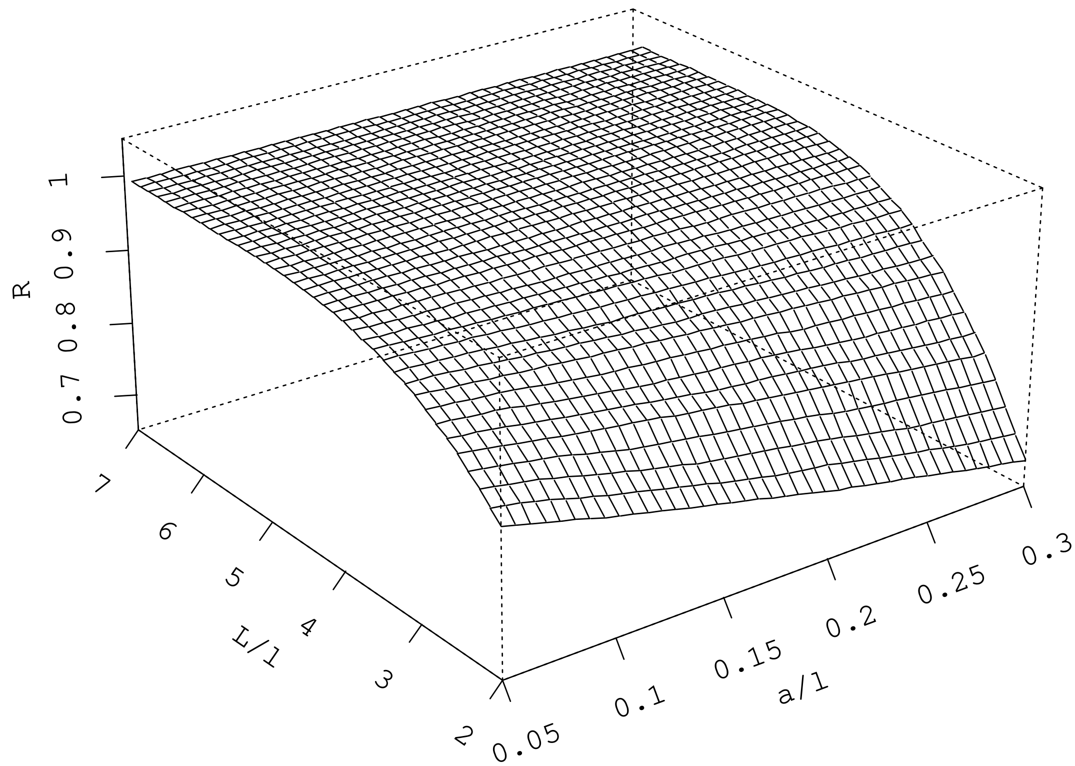

Figure 1 – Finite Slab Length Effect: 3-D Perspective Plot

Effect of Finite Slab Width

The effect of finite slab width was often neglected in practice, since it is not as significant as the slab length effect. However, this effect as represented by the following adjustment factor (R_W) was considered in this study to have a more complete coverage on this topic:

$$R_W = \frac{\sigma_i}{\sigma_w} = f\left(\frac{a}{l}, \frac{W}{l}\right) \tag{15}$$

Thus, the following factorial ILLI-SLAB runs were performed:

a/l: 0.05, 0.10, 0.20, 0.30

W/*l*: 2.0, 2.5, 3.0, 3.5, 4.0, 4.5, 5.0, 6.0, 7.0

Note that L/*l* was kept a constant value of 7.0 for all runs. As expected, the slab width effect is not very significant and it is negligible when W/*l* is greater than 3 or 4. Besides, this effect is more pronounced for a pavement slab with a smaller W/*l* but a larger *a/l*.

Similarly, the following one-term "projection" model was developed:

$$R_W = 1.0048 + 0.0121\Phi_1(A1) \tag{16}$$

$$\Phi_1(A1) = -0.534 + 1.654(1 - A1)^{-10.741} \quad (17)$$

$$A1 = 0.995\frac{a}{l} - 0.0986\frac{W}{l} \quad \cancel{(18)}$$

Statistics: N = 36, R^2 = 0.947, SEE = 0.00299, CV = 0.30%
Limits: 2.0 ≤ W/*l* ≤ 7.0, 0.05 ≤ *a/l* ≤ 0.3

Note that very narrow range of the adjustment factors was obtained due to limited number of runs performed. This "projection" model was considered relatively adequate for practical pavement design. With finer grids and more FE runs performed, the above equations can be easily improved using the same modeling approach to achieve higher accuracy if necessary.

Effect of Loading Plus Thermal Curling

The combination effects of loading and thermal curling can not be adequately described by simply superimposing the individual effects of loading and thermal curling alone, since the assumption of full contact condition between slab and subgrade is often violated. Thus, the following adjustment factor (R_T) was introduced to quantify this difference:

$$\sigma_i = \sigma_L + R_T\sigma_c \quad (19)$$

Together with knowledge of the dominating mechanistic variables involved, it may be determined by:

$$R_T = \frac{\sigma_i - \sigma_L}{\sigma_c} = f\left(\frac{a}{l}\ ,\ \alpha\Delta T\ ,\ \frac{L}{l}\ ,\ \frac{W}{l}\ ,\ \frac{\gamma h^2}{k l^2}\ ,\ \frac{Ph}{k l^4}\right) \quad (20)$$

Where: σ_L = edge stress determined by the FE model due to loading alone, which may be estimated by (R_L * R_W * σ_w), [FL^{-2}].

Predictive Models for Loading Plus Thermal Curling

To access the effect of all these six parameters on edge stress, a complete full factorial of ILLI-SLAB runs which requires a tremendous amount of computer time is not feasible. Thus, it was decided to generate a partial factorial based on the following parameters while randomizing the relationships among *a/l*, D_γ, and D_P:

a/l: 0.05, 0.1, 0.2, 0.3
L/*l*: 3, 5, 7, 9, 11, 13, 15
W/*l*: 3, 5, 7, 9, 11
ΔT: 0, ±10, ±20, ±30, ±40 °F

This was done by changing unit weight of the slab and wheel loads (or tire pressure) while keeping any other input variables same as before for different *a/l* values.

For the case of day-time curling plus loading, the following three-term "projection" model was developed:

$$R_T = 0.948 + 0.151\Phi_1(A1) + 0.0372\Phi_2(A2) + 0.0340\Phi_3(A3) \qquad (21)$$

$$\Phi_1(A1) = \begin{cases} -2.558 + 0.800A1 & , if\ A1 \le 3 \\ -2.634 + 1.104A1 - 0.0914A1^2 & , if\ 3 < A1 \le 7 \\ 0.756 - 0.0155A1 & , if\ A1 > 7 \end{cases} \qquad \cancel{(22)}$$

$$\Phi_2(A2) = \begin{cases} -0.679 + 0.0107A2 & , if\ A2 \le 3 \\ 3.767 - 2.297A2 + 0.296A2^2 & , if\ 3 < A2 \le 7 \\ -7.034 + 1.295A2 & , if\ A2 > 7 \end{cases} \qquad (23)$$

$$\Phi_3(A3) = \begin{cases} 4.084 + 4.824A3 & , if\ A3 \le -1 \\ 0.182 + 0.0541A3 - 1.090A3^2 & , if\ -1 < A3 \le 0.5 \\ 0.0453 + 0.0383A3 & , if\ A3 > 0.5 \end{cases} \qquad (24)$$

$$\begin{aligned} A1 = &- 0.0472\frac{W}{l} + 0.570\frac{L}{l} - 0.0841ADT + 0.200\frac{a}{l} \\ &- 0.266DG + 0.00375DP + 0.739\frac{a}{l}\frac{L}{l} - 0.0114ADT\frac{L}{l} \qquad (25) \\ &+ 0.0953DG\frac{L}{l} + 0.0112DG\frac{W}{l} \end{aligned}$$

$$\begin{aligned} A2 = &\ 0.0387\frac{W}{l} + 0.358\frac{L}{l} + 0.0908ADT - 0.0405\frac{a}{l} \\ &+ 0.864DG + 0.0164DP - 0.312\frac{a}{l}\frac{L}{l} + 0.00552ADT\frac{L}{l} \qquad (26) \\ &- 0.127DG\frac{L}{l} - 0.0177DG\frac{W}{l} \end{aligned}$$

$$\begin{aligned} A3 = &\ 0.586\frac{W}{l} + 0.258\frac{L}{l} + 0.148ADT + 0.150\frac{a}{l} \\ &+ 0.127DG - 0.0501DP + 0.723\frac{a}{l}\frac{L}{l} - 0.0131ADT\frac{L}{l} \qquad (27) \\ &- 0.0130DG\frac{L}{l} - 0.0659DG\frac{W}{l} \end{aligned}$$

Statistics: N = 432, R^2 = 0.970, SEE = 0.0280, CV = 2.95%

Similarly, the following three-term "projection" model was developed for night-time curling plus loading:

$$R_T = 0.761 + 0.285\Phi_1(A1) + 0.107\Phi_2(A2) + 0.100\Phi_3(A3) \qquad (28)$$

$$\Phi_1(A1) = \begin{cases} -2.530 + 0.387A1 & , if\ A1 \le 6 \\ -4.094 + 0.880A1 - 0.0395A1^2 & , if\ 6 < A1 \le 12 \\ 0.318 + 0.0435A1 & , if\ A1 > 12 \end{cases} \qquad (29)$$

$$\Phi_2(A2) = \begin{cases} -1.498 - 1.136A2 & , if\ A2 \le -1 \\ -0.496 - 0.0892A2 + 0.0174A2^2 & , if\ -1 < A2 \le 5 \\ -1.128 + 0.108A2 & , if\ A2 > 5 \end{cases} \qquad (30)$$

$$\Phi_3(A3) = \begin{cases} 3.634 + 0.651A3 & , if\ A3 \le -7 \\ -0.268 - 0.681A3 - 0.112A3^2 & , if\ -7 < A3 \le -1 \\ 0.197 - 0.206A3 & , if\ A3 > -1 \end{cases} \quad (31)$$

$$\begin{aligned} A1 = &-0.140\frac{W}{l} + 0.858\frac{L}{l} + 0.0700ADT + 0.196\frac{a}{l} \\ &+ 0.366DG + 0.0595DP - 0.242\frac{a}{l}\frac{L}{l} + 0.0724DG\frac{L}{l} \\ &- 0.00482DP\frac{L}{l} + 0.00797ADT\frac{L}{l} + 0.0122ADT\frac{W}{l} \end{aligned} \quad (32)$$

$$\begin{aligned} A2 = &-0.151\frac{W}{l} + 0.0844\frac{L}{l} + 0.282ADT - 0.488\frac{a}{l} \\ &+ 0.175DG - 0.0319DP - 0.784\frac{a}{l}\frac{L}{l} - 0.0200DG\frac{L}{l} \\ &+ 0.00206DP\frac{L}{l} - 0.0695ADT\frac{L}{l} - 0.00419ADT\frac{W}{l} \end{aligned} \quad (33)$$

$$\begin{aligned} A3 = &\ 0.110\frac{W}{l} - 0.603\frac{L}{l} - 0.268ADT + 0.0697\frac{a}{l} \\ &- 0.715DG - 0.127DP + 0.0285\frac{a}{l}\frac{L}{l} + 0.134DG\frac{L}{l} \\ &+ 0.0109DP\frac{L}{l} + 0.0225ADT\frac{L}{l} + 0.00260ADT\frac{W}{l} \end{aligned} \quad (34)$$

Statistics: N = 432, R^2 = 0.961, SEE = 0.0546, CV = 7.17%
Where: ADT = $\alpha\Delta T \times 10^5$, DG = $D_\gamma \times 10^5$, DP = $D_P \times 10^5$.
Limits: $0.05 \le a/l \le 0.30$, $3 \le L/l \le 15$, $3 \le W/l \le 11$,
$5.5 \le ADT \le 22$, $1.06 \le DG \le 9.93$, $2.61 \le DP \le 140.74$

Validation of the Proposed Predictive Models

To further validate the applicability of the proposed predictive models, a series of ILLI-SLAB factorial runs were performed based on a wide range of input parameters. These factorial runs were totally independent of the previous modeling process. The following input parameters covering most of the practical cases found in the field were selected:

Concrete modulus of the slab, E = 3.0, 5.5, 8.0 Million psi (Mpsi);
Modulus of subgrade reaction, k = 50, 250, 500 pci;
Slab length, L = 10, 20, 30 ft;
Slab thickness, h = 8, 12, 16 in.; and
Temperature differential, $\Delta T = \pm 20, \pm 40$ °F.

The width of the slab was 12 feet, the slab Poisson's ratio was 0.15, the coefficient of thermal expansion was 5.5×10^{-6} /°F, and the unit weight of concrete slab was set to 0.087 pci.

A wheel load of 9,000 pounds was applied to the 10-ft slabs on a loaded rectangle of 10 x 10 in^2, whereas an 18,000-pound load was applied to the 20-ft and 30-ft slabs on a 20 x 10 in^2 rectangle. The tire pressure was set to 90 psi.

Thus, the above factorials essentially result in a group of pavement slabs whose data ranges are a/l = 0.07 to 0.35, L/l = 1.4 to 15.9, W/l = 1.7 to 6.4, $\alpha\Delta T$ = -0.00022 to 0.00022, D_γ = 1.3E-05 to 9.7E-05, and D_P = 5.2E-05 to 110.0E-05. To examine the applicability of the predictive models to pavement slabs with any other input parameters, some of the data points which are outside of the specified limits of the proposed predictive models were not considered.

The adequacy of the proposed predictive models was validated using the above factorial data. The predicted edge stresses were plotted against the actual values as shown in Figure 2 and Figure 3. Clearly, these predictive models were able to make fairly good prediction even when extrapolated to wider ranges of other input parameters.

Numerical Examples

Consider a pavement slab with the following characteristics: E = 5.5 Mpsi, k = 250 pci, L = 10 ft, W = 12 ft, h = 12 in., γ = 0.087 pci, μ = 0.15, and α = 5.5 x 10^{-6} /°F. A single wheel load of 9,000 lbs with a loaded rectangle of the size of 10 x 10 in^2 is applied. A linear temperature differential of +20 °F (day-time condition) exists through the slab thickness. Determine the critical edge stresses due to loading and loading plus curling.

The equivalent radius of the loaded area is a = 5.64 in. and the radius of relative stiffness of the slab-subgrade system is l = 31.20 in. Therefore, the actual dominating mechanistic variables are a/l = 0.18, L/l = 3.83, W/l = 4.60, $\alpha\Delta T$ = +11.0E-05, D_γ = 2.27E-05, and D_P = 29.99E-05. The theoretical Westergaard's solutions based on equations (1) and (5) are σ_w = 345.75 psi and σ_c = 118.25 psi for loading only and curling only, respectively.

For the case of loading only, the adjustment factors for finite slab length and width are R_L = 0.968 and R_W = 1.000 using equations (12) and (16), respectively. Thus, the edge stress determined by the proposed models is 0.968 x 1.000 x 345.75 = 334.7 psi. (Note that the actual ILLI-SLAB edge stress was 344.36 psi.)

For the case of loading plus curling, the adjustment factor is R_T = 0.732 based on equation (21). Thus, the predicted total edge stress determined by the proposed model is 334.7 + 0.732 x 118.25 = 421.3 psi using equation (19). (Note that the actual edge stress was 436.42 psi.)

Conclusions

The edge stress of a concrete pavement due to the effects of wheel loading and loading plus curling was conducted under this study. The subgrade was assumed to act as a dense liquid foundation and a linear temperature differential was considered for the curling problem.

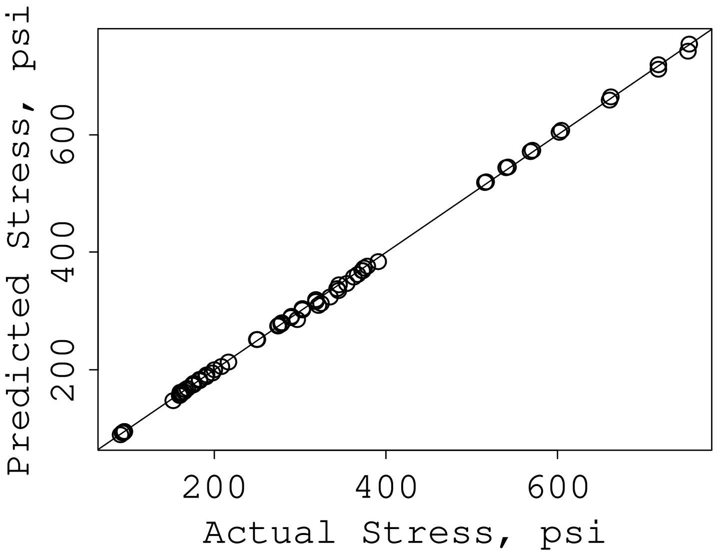

Figure 2 – Validation: Loading Only

Based on previous research using dimensional analysis, the major independent variables were: normalized load radius (*a/l*), normalized slab length (L/*l*), normalized slab width (W/*l*), and a dimensionless product ($\alpha\Delta T$) of a temperature differential and thermal expansion coefficient. However, the actual structural response to a temperature influence could not be adequately described using only these four parameters. Fortunately, two additional dimensionless parameters (D_γ and D_P) representing the relative deflection stiffness due to the

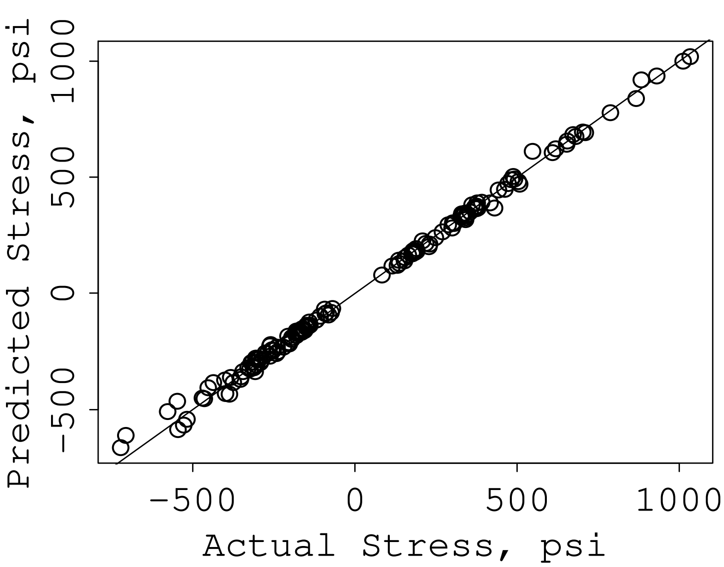

Figure 3 – Validation: Loading + Curling

self-weight of the concrete slab, the applied load, and the possible loss of support were recently identified. With this discovery, the problems encountered in previous investigations using dimensional analysis for thermal-related curling problems are now resolved.

A new modeling approach proposed by Lee, which makes use of projection pursuit regression algorithm, linear and nonlinear regression techniques, was utilized to develop the proposed predictive models. The new models use only the dominating mechanistic variables as opposed to earlier attempts using arbitrary linear combination of input parameters with very few insights to the actual relationships among the variables.

Consequently, closed-form mechanistic design models which have been carefully validated and ready for implementation on a spreadsheet or computer program were presented. These stress models turned out to be very accurate representations of the finite element model. The new models were also properly formulated to satisfy applicable engineering boundary conditions. They are also simple, easy to comprehend, dimensionally correct, and may be extrapolated to wider ranges of other input parameters. Practical numerical examples showing the use of the new models are also provided.

The predictive models cover almost all practical ranges of pavement designs. Since they are dimensionally correct, they can be used in any other unit systems as well. However, extrapolation outside the specified ranges of these dominating parameters is not recommended. Practical pavement design guidelines for edge stress analysis may be easily developed based on these predictive models together with the use of some reliability concepts. Besides, the predictive modeling procedures and the "deflection-ratio" concept may be utilized in other pavement related modeling problems as well.

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Appendix II. Conversion to SI Units

To convert	To	Multiply by
inch (in.)	millimeter (mm)	25.4
feet (ft)	centimeter (cm)	30.48
pound force (lbf)	newtons (N)	4.45
pound/cubic inch (pci)	MN/m^3	0.27
pound/square inch (psi)	kilopascals (kPa)	6.89
Fahrenheit temperature (°F)	Celcius temperature (°C)	(F - 32) / 1.8

Key Words

Concrete pavements, stress analysis, wheel loads, curling, dimensional analysis, statistical analysis, projection pursuit regression (PPR), predictive models.

(18)	(22)	L
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