

# Artificial Intelligence

## Knowledge, Reasoning and Knowledge Representation Uncertain Knowledge and Reasoning

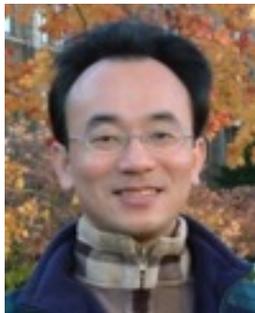
1111AI04

MBA, IM, NTPU (M6132) (Fall 2022)

Wed 2, 3, 4 (9:10-12:00) (B8F40)



[https://meet.google.com/  
miy-fbif-max](https://meet.google.com/miy-fbif-max)



Min-Yuh Day, Ph.D,  
Associate Professor

Institute of Information Management, National Taipei University

<https://web.ntpu.edu.tw/~myday>



# Syllabus

Week	Date	Subject/Topics
1	2022/09/14	Introduction to Artificial Intelligence
2	2022/09/21	Artificial Intelligence and Intelligent Agents
3	2022/09/28	Problem Solving
4	2022/10/05	Knowledge, Reasoning and Knowledge Representation; Uncertain Knowledge and Reasoning
5	2022/10/12	Case Study on Artificial Intelligence I
6	2022/10/19	Machine Learning: Supervised and Unsupervised Learning

# Syllabus

Week	Date	Subject/Topics
7	2022/10/26	The Theory of Learning and Ensemble Learning
8	2022/11/02	Midterm Project Report
9	2022/11/09	Deep Learning and Reinforcement Learning
10	2022/11/16	Deep Learning for Natural Language Processing
11	2022/11/23	Invited Talk: AI for Information Retrieval
12	2022/11/30	Case Study on Artificial Intelligence II

# Syllabus

<b>Week</b>	<b>Date</b>	<b>Subject/Topics</b>
<b>13</b>	<b>2022/12/07</b>	<b>Computer Vision and Robotics</b>
<b>14</b>	<b>2022/12/14</b>	<b>Philosophy and Ethics of AI and the Future of AI</b>
<b>15</b>	<b>2022/12/21</b>	<b>Final Project Report I</b>
<b>16</b>	<b>2022/12/28</b>	<b>Final Project Report II</b>
<b>17</b>	<b>2023/01/04</b>	<b>Self-learning</b>
<b>18</b>	<b>2023/01/11</b>	<b>Self-learning</b>

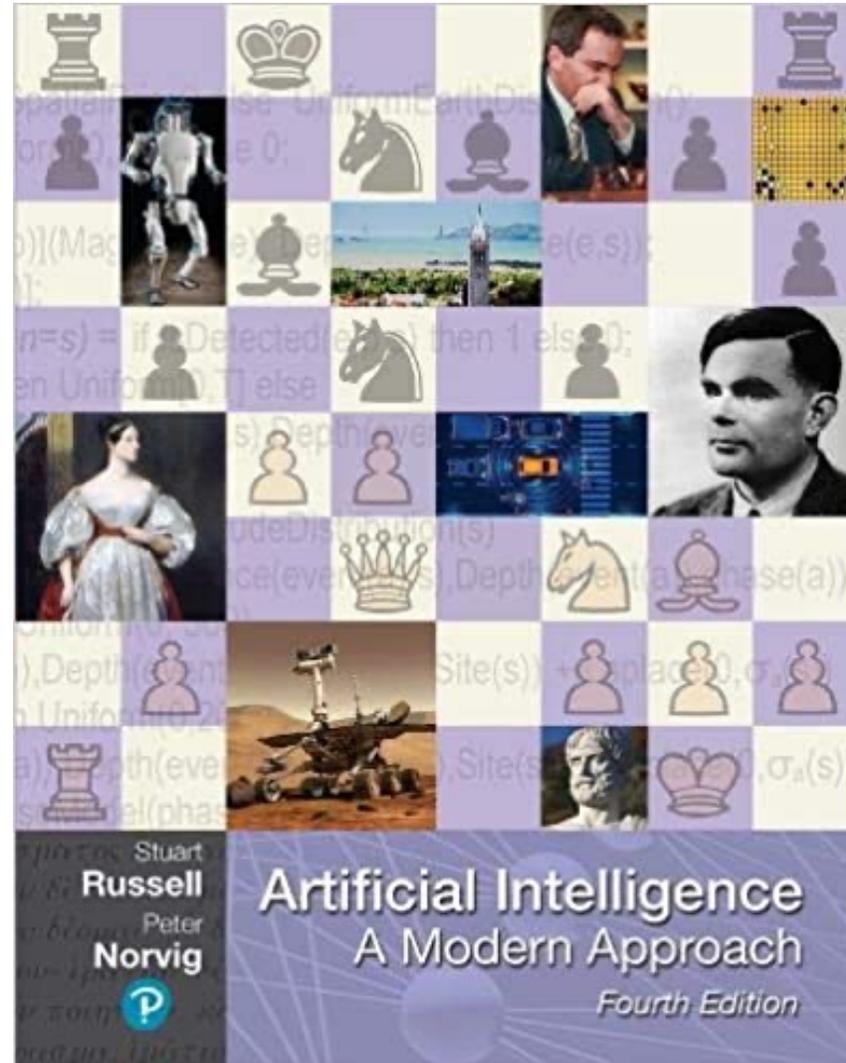
# **Knowledge, Reasoning and Knowledge Representation**

## **Uncertain Knowledge and Reasoning**

# Outline

- **Knowledge and Reasoning**
  - **Logical Agents**
  - **First-Order Logic**
  - **Inference in First-Order Logic**
  - **Knowledge Representation**
  - **Knowledge Graph (KG)**
- **Uncertain Knowledge and Reasoning**
  - **Quantifying Uncertainty**
  - **Probabilistic Reasoning**
  - **Making Complex Decisions**

Stuart Russell and Peter Norvig (2020),  
**Artificial Intelligence: A Modern Approach,**  
4th Edition, Pearson



Source: Stuart Russell and Peter Norvig (2020), Artificial Intelligence: A Modern Approach, 4th Edition, Pearson

<https://www.amazon.com/Artificial-Intelligence-A-Modern-Approach/dp/0134610997/>

# Artificial Intelligence: A Modern Approach

1. Artificial Intelligence
2. Problem Solving
3. Knowledge and Reasoning
4. Uncertain Knowledge and Reasoning
5. Machine Learning
6. Communicating, Perceiving, and Acting
7. Philosophy and Ethics of AI

# Artificial Intelligence: Knowledge and Reasoning

# Artificial Intelligence:

## 3. Knowledge and Reasoning

- **Logical Agents**
- **First-Order Logic**
- **Inference in First-Order Logic**
- **Knowledge Representation**
- **Automated Planning**

# Intelligent Agents

# 4 Approaches of AI

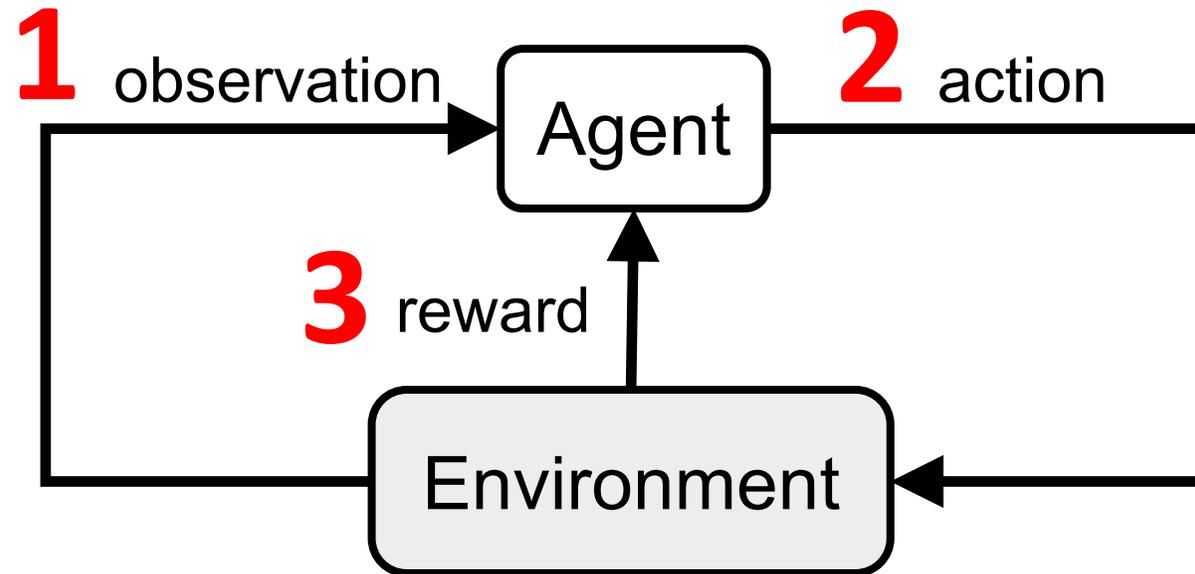
<p><b>2.</b> <b>Thinking Humanly: The Cognitive Modeling Approach</b></p>	<p><b>3.</b> <b>Thinking Rationally: The “Laws of Thought” Approach</b></p>
<p><b>1.</b> <b>Acting Humanly: The Turing Test Approach</b> <small>(1950)</small></p>	<p><b>4.</b> <b>Acting Rationally: The Rational Agent Approach</b></p>

# Reinforcement Learning (DL)

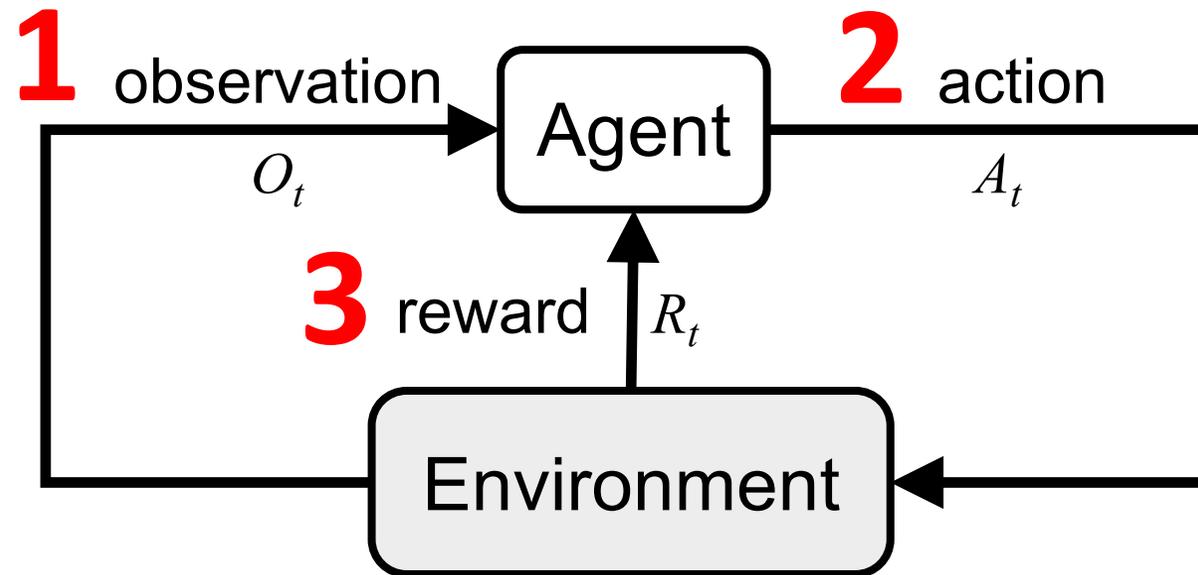
Agent

Environment

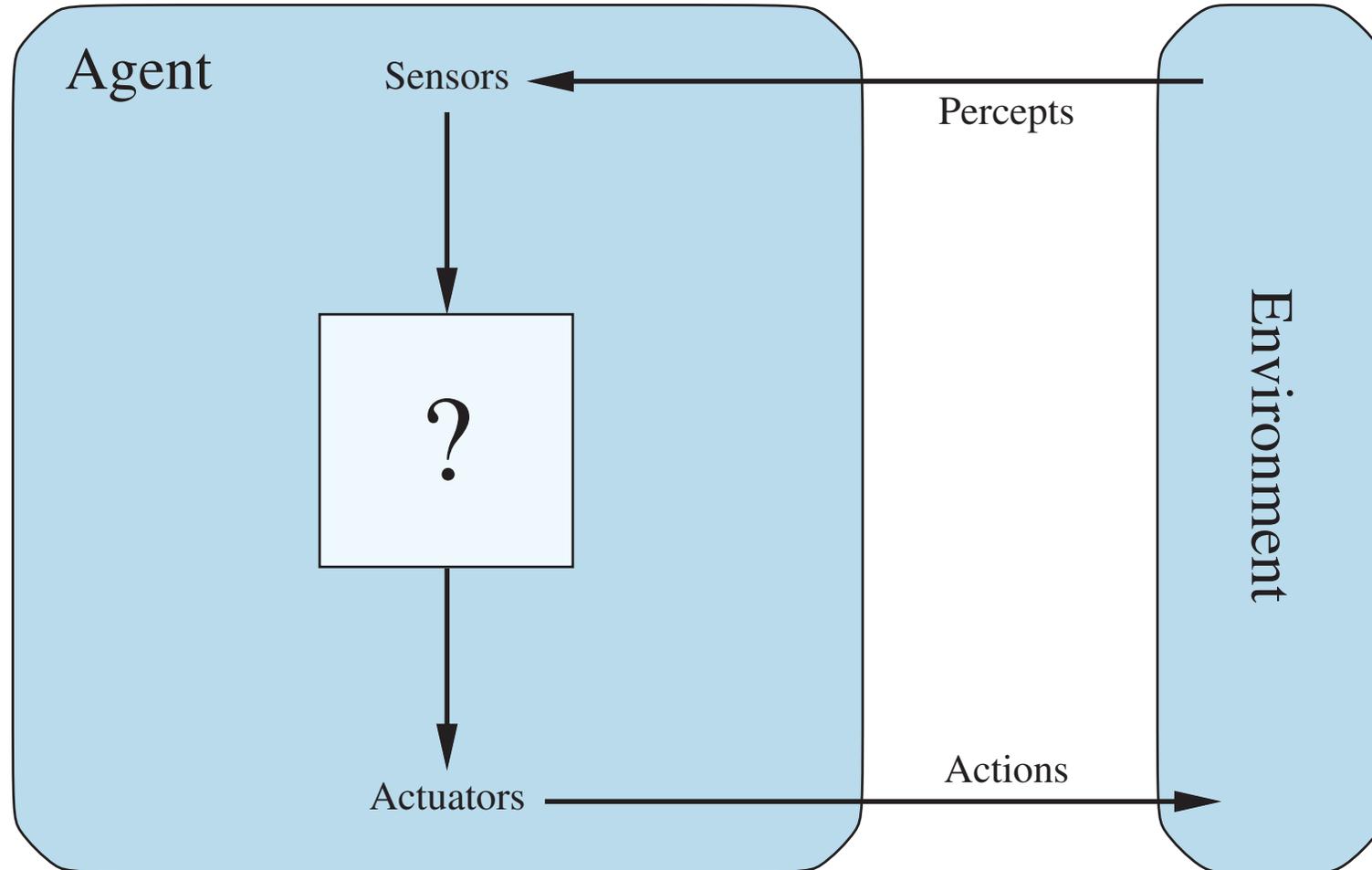
# Reinforcement Learning (DL)



# Reinforcement Learning (DL)



# Agents interact with environments through sensors and actuators



# Logical Agents

# Logical Agents

Knowledge-based Agents

KB Agents

# Knowledge-based Agent (KB Agent)

**function** **KB-AGENT**(*percept*) **returns** an *action*

**persistent:** *KB*, a knowledge base

*t*, a counter, initially 0, indicating time

**TELL**(*KB*, **MAKE-PERCEPT-SENTENCE**(*percept*, *t*))

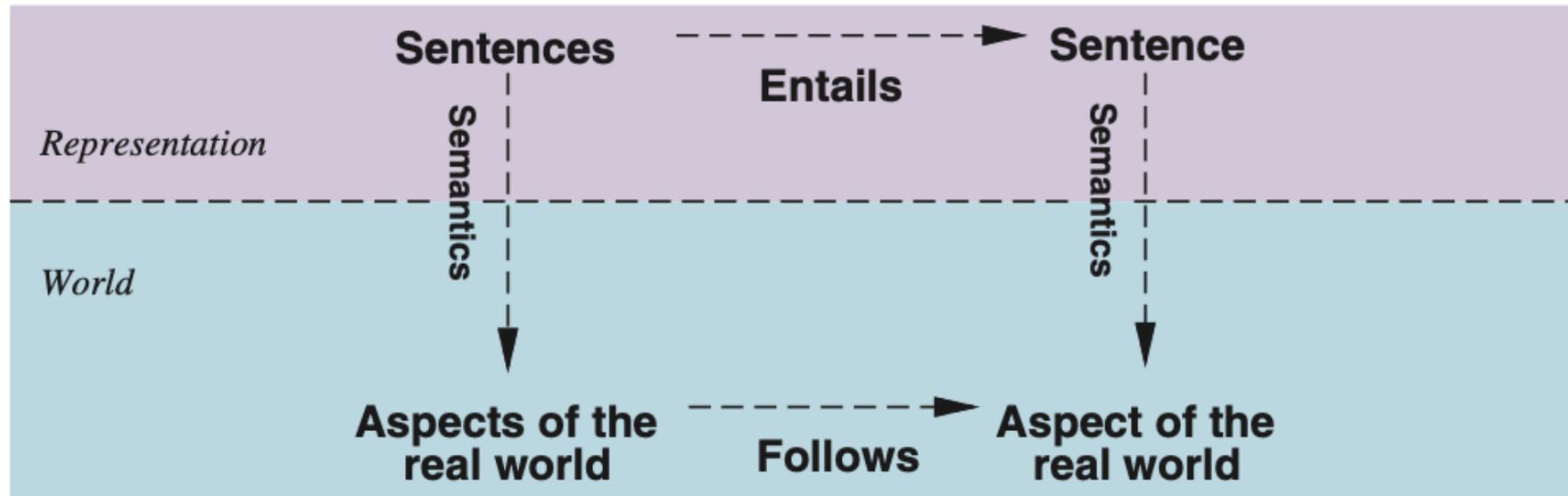
*action*  $\leftarrow$  **ASK**(*KB*, **MAKE-ACTION-QUERY**(*t*))

**TELL**(*KB*, **MAKE-ACTION-SENTENCE**(*action*, *t*))

*t*  $\leftarrow$  *t* + 1

**return** *action*

# Sentences are physical configurations of the agent



**Reasoning** is a process of constructing new physical configurations from old ones

**Logical reasoning** should ensure that the new configurations represent aspects of the world that actually follow from the aspects that the old configurations represent.

# A BNF (Backus–Naur Form) grammar of sentences in propositional logic

*Sentence*  $\rightarrow$  *AtomicSentence* | *ComplexSentence*

*AtomicSentence*  $\rightarrow$  *True* | *False* | *P* | *Q* | *R* | ...

*ComplexSentence*  $\rightarrow$  ( *Sentence* )  
|  $\neg$  *Sentence*  
| *Sentence*  $\wedge$  *Sentence*  
| *Sentence*  $\vee$  *Sentence*  
| *Sentence*  $\Rightarrow$  *Sentence*  
| *Sentence*  $\Leftrightarrow$  *Sentence*

**OPERATOR PRECEDENCE** :  $\neg, \wedge, \vee, \Rightarrow, \Leftrightarrow$

# Truth Tables (TT) for the Five Logical Connectives

$P$	$Q$	$\neg P$	$P \wedge Q$	$P \vee Q$	$P \Rightarrow Q$	$P \Leftrightarrow Q$
<i>false</i>	<i>false</i>	<i>true</i>	<i>false</i>	<i>false</i>	<i>true</i>	<i>true</i>
<i>false</i>	<i>true</i>	<i>true</i>	<i>false</i>	<i>true</i>	<i>true</i>	<i>false</i>
<i>true</i>	<i>false</i>	<i>false</i>	<i>false</i>	<i>true</i>	<i>false</i>	<i>false</i>
<i>true</i>	<i>true</i>	<i>false</i>	<i>true</i>	<i>true</i>	<i>true</i>	<i>true</i>

# A Truth Table constructed for the knowledge base given in the text

$B_{1,1}$	$B_{2,1}$	$P_{1,1}$	$P_{1,2}$	$P_{2,1}$	$P_{2,2}$	$P_{3,1}$	$R_1$	$R_2$	$R_3$	$R_4$	$R_5$	$KB$
<i>false</i>	<i>true</i>	<i>true</i>	<i>true</i>	<i>true</i>	<i>false</i>	<i>false</i>						
<i>false</i>	<i>false</i>	<i>false</i>	<i>false</i>	<i>false</i>	<i>false</i>	<i>true</i>	<i>true</i>	<i>true</i>	<i>false</i>	<i>true</i>	<i>false</i>	<i>false</i>
$\vdots$												
<i>false</i>	<i>true</i>	<i>false</i>	<i>false</i>	<i>false</i>	<i>false</i>	<i>false</i>	<i>true</i>	<i>true</i>	<i>false</i>	<i>true</i>	<i>true</i>	<i>false</i>
<i>false</i>	<i>true</i>	<i>false</i>	<i>false</i>	<i>false</i>	<i>false</i>	<i>true</i>	<i>true</i>	<i>true</i>	<i>true</i>	<i>true</i>	<i>true</i>	<u><i>true</i></u>
<i>false</i>	<i>true</i>	<i>false</i>	<i>false</i>	<i>false</i>	<i>true</i>	<i>false</i>	<i>true</i>	<i>true</i>	<i>true</i>	<i>true</i>	<i>true</i>	<u><i>true</i></u>
<i>false</i>	<i>true</i>	<i>false</i>	<i>false</i>	<i>false</i>	<i>true</i>	<u><i>true</i></u>						
<i>false</i>	<i>true</i>	<i>false</i>	<i>false</i>	<i>true</i>	<i>false</i>	<i>false</i>	<i>true</i>	<i>false</i>	<i>false</i>	<i>true</i>	<i>true</i>	<i>false</i>
$\vdots$												
<i>true</i>	<i>false</i>	<i>true</i>	<i>true</i>	<i>false</i>	<i>true</i>	<i>false</i>						

# A Truth-Table (TT) enumeration algorithm for deciding propositional entailment

```
function TT-ENTAILS?(KB,  $\alpha$ ) returns true or false  
  inputs: KB, the knowledge base, a sentence in propositional logic  
            $\alpha$ , the query, a sentence in propositional logic  
  
  symbols  $\leftarrow$  a list of the proposition symbols in KB and  $\alpha$   
  return TT-CHECK-ALL(KB,  $\alpha$ , symbols, { })  
  
function TT-CHECK-ALL(KB,  $\alpha$ , symbols, model) returns true or false  
  if EMPTY?(symbols) then  
    if PL-TRUE?(KB, model) then return PL-TRUE?( $\alpha$ , model)  
    else return true // when KB is false, always return true  
  else  
    P  $\leftarrow$  FIRST(symbols)  
    rest  $\leftarrow$  REST(symbols)  
    return (TT-CHECK-ALL(KB,  $\alpha$ , rest, model  $\cup$  {P = true})  
            and  
            TT-CHECK-ALL(KB,  $\alpha$ , rest, model  $\cup$  {P = false })))
```

# Standard Logical Equivalences

The symbols  $\alpha$ ,  $\beta$ , and  $\gamma$  stand for arbitrary sentences of propositional logic.

$$\begin{aligned}(\alpha \wedge \beta) &\equiv (\beta \wedge \alpha) && \text{commutativity of } \wedge \\(\alpha \vee \beta) &\equiv (\beta \vee \alpha) && \text{commutativity of } \vee \\((\alpha \wedge \beta) \wedge \gamma) &\equiv (\alpha \wedge (\beta \wedge \gamma)) && \text{associativity of } \wedge \\((\alpha \vee \beta) \vee \gamma) &\equiv (\alpha \vee (\beta \vee \gamma)) && \text{associativity of } \vee \\ \neg(\neg\alpha) &\equiv \alpha && \text{double-negation elimination} \\(\alpha \Rightarrow \beta) &\equiv (\neg\beta \Rightarrow \neg\alpha) && \text{contraposition} \\(\alpha \Rightarrow \beta) &\equiv (\neg\alpha \vee \beta) && \text{implication elimination} \\(\alpha \Leftrightarrow \beta) &\equiv ((\alpha \Rightarrow \beta) \wedge (\beta \Rightarrow \alpha)) && \text{biconditional elimination} \\ \neg(\alpha \wedge \beta) &\equiv (\neg\alpha \vee \neg\beta) && \text{De Morgan} \\ \neg(\alpha \vee \beta) &\equiv (\neg\alpha \wedge \neg\beta) && \text{De Morgan} \\(\alpha \wedge (\beta \vee \gamma)) &\equiv ((\alpha \wedge \beta) \vee (\alpha \wedge \gamma)) && \text{distributivity of } \wedge \text{ over } \vee \\(\alpha \vee (\beta \wedge \gamma)) &\equiv ((\alpha \vee \beta) \wedge (\alpha \vee \gamma)) && \text{distributivity of } \vee \text{ over } \wedge\end{aligned}$$

# A grammar for Conjunctive Normal Form (CNF), Horn clauses, and definite clauses

*CNFSentence*  $\rightarrow$  *Clause*<sub>1</sub>  $\wedge \dots \wedge$  *Clause*<sub>n</sub>

*Clause*  $\rightarrow$  *Literal*<sub>1</sub>  $\vee \dots \vee$  *Literal*<sub>m</sub>

*Fact*  $\rightarrow$  *Symbol*

*Literal*  $\rightarrow$  *Symbol* |  $\neg$ *Symbol*

*Symbol*  $\rightarrow$  *P* | *Q* | *R* | ...

*HornClauseForm*  $\rightarrow$  *DefiniteClauseForm* | *GoalClauseForm*

*DefiniteClauseForm*  $\rightarrow$  *Fact* | (*Symbol*<sub>1</sub>  $\wedge \dots \wedge$  *Symbol*<sub>l</sub>)  $\Rightarrow$  *Symbol*

*GoalClauseForm*  $\rightarrow$  (*Symbol*<sub>1</sub>  $\wedge \dots \wedge$  *Symbol*<sub>l</sub>)  $\Rightarrow$  *False*

# A simple resolution algorithm for propositional logic

**function** PL-RESOLUTION( $KB, \alpha$ ) **returns** *true* or *false*  
**inputs:**  $KB$ , the knowledge base, a sentence in propositional logic  
 $\alpha$ , the query, a sentence in propositional logic

$clauses \leftarrow$  the set of clauses in the CNF representation of  $KB \wedge \neg\alpha$   
 $new \leftarrow \{ \}$   
**while** *true* **do**  
    **for each** pair of clauses  $C_i, C_j$  **in**  $clauses$  **do**  
         $resolvents \leftarrow$  PL-RESOLVE( $C_i, C_j$ )  
        **if**  $resolvents$  contains the empty clause **then return** *true*  
         $new \leftarrow new \cup resolvents$   
    **if**  $new \subseteq clauses$  **then return** *false*  
     $clauses \leftarrow clauses \cup new$

# The forward-chaining algorithm for propositional logic

**function** PL-FC-ENTAILS?( $KB, q$ ) **returns** *true* or *false*

**inputs:**  $KB$ , the knowledge base, a set of propositional definite clauses

$q$ , the query, a proposition symbol

$count \leftarrow$  a table, where  $count[c]$  is initially the number of symbols in clause  $c$ 's premise

$inferred \leftarrow$  a table, where  $inferred[s]$  is initially *false* for all symbols

$queue \leftarrow$  a queue of symbols, initially symbols known to be true in  $KB$

**while**  $queue$  is not empty **do**

$p \leftarrow$  POP( $queue$ )

**if**  $p = q$  **then return** *true*

**if**  $inferred[p] = false$  **then**

$inferred[p] \leftarrow true$

**for each** clause  $c$  in  $KB$  where  $p$  is in  $c$ .PREMISE **do**

            decrement  $count[c]$

**if**  $count[c] = 0$  **then** add  $c$ .CONCLUSION to  $queue$

**return** *false*

# A set of Horn clauses

$$P \Rightarrow Q$$

$$L \wedge M \Rightarrow P$$

$$B \wedge L \Rightarrow M$$

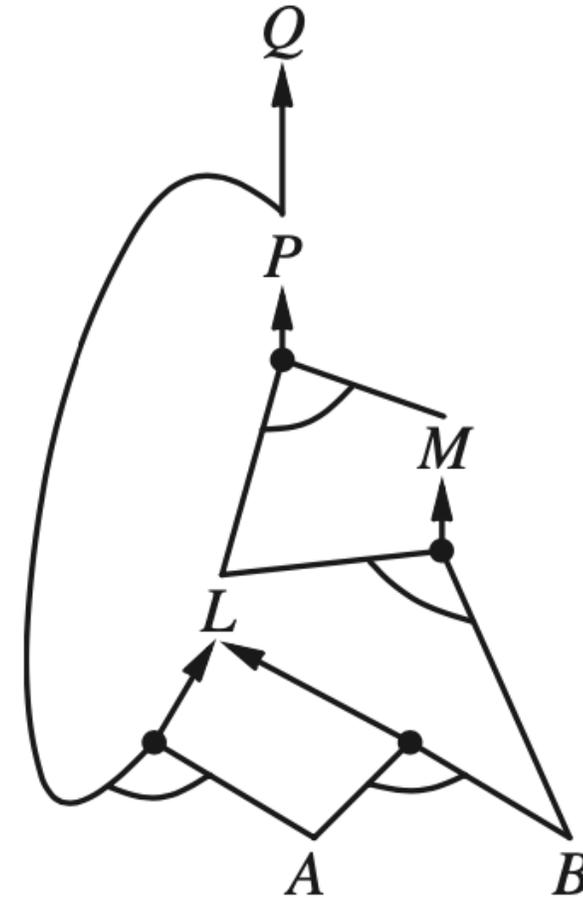
$$A \wedge P \Rightarrow L$$

$$A \wedge B \Rightarrow L$$

$A$

$B$

(a)



(b)

The corresponding AND-OR graph

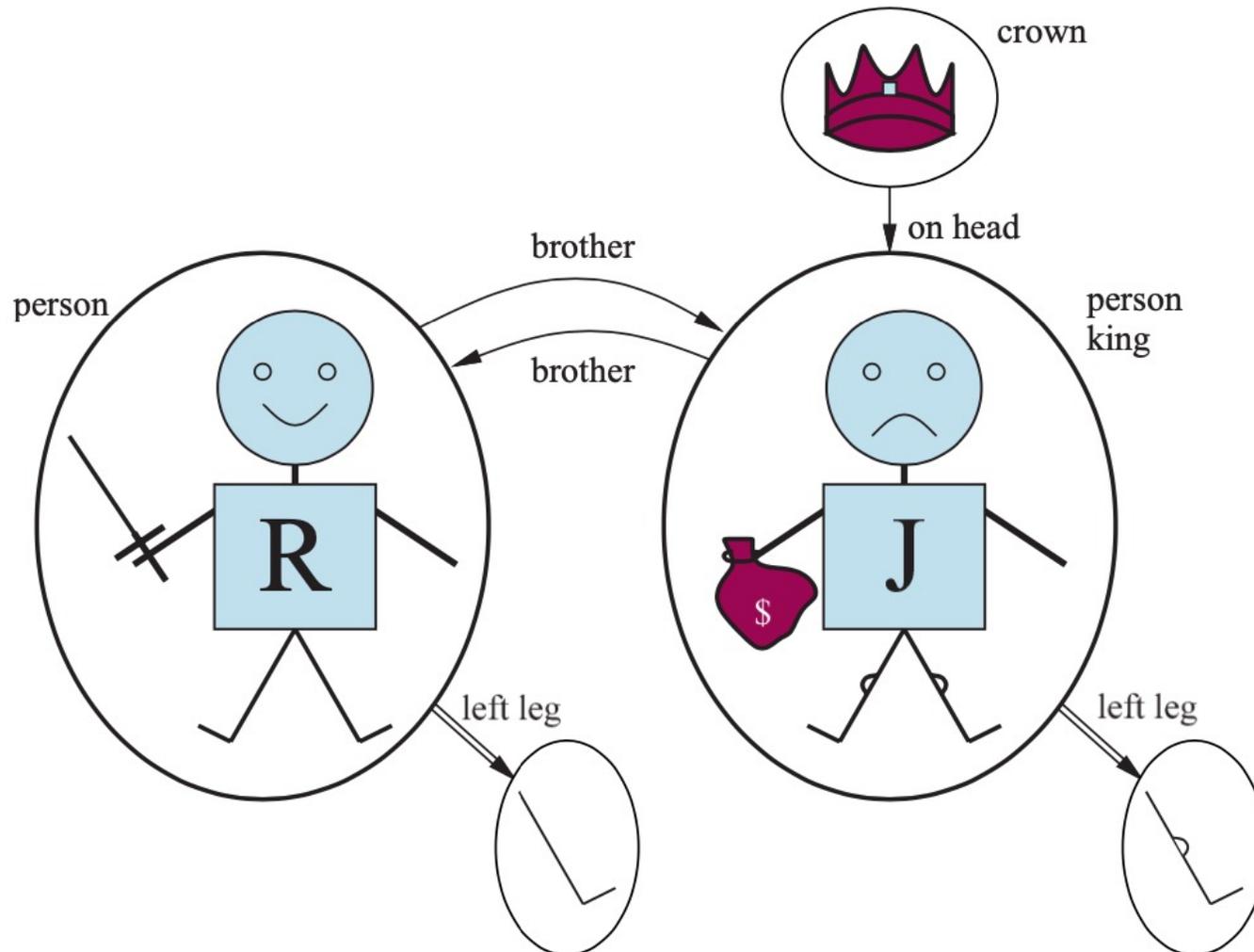
# First-Order Logic

# Formal languages and their ontological and epistemological commitments

Language	Ontological Commitment (What exists in the world)	Epistemological Commitment (What an agent believes about facts)
Propositional logic	facts	true/false/unknown
First-order logic	facts, objects, relations	true/false/unknown
Temporal logic	facts, objects, relations, times	true/false/unknown
Probability theory	facts	degree of belief $\in [0, 1]$
Fuzzy logic	facts with degree of truth $\in [0, 1]$	known interval value

# A model containing five objects

two binary relations (brother and on-head), three unary relations (person, king, and crown), and one unary function (left-leg).



# The syntax of first-order logic with equality

*Sentence* → *AtomicSentence* | *ComplexSentence*

*AtomicSentence* → *Predicate* | *Predicate(Term, ...)* | *Term = Term*

*ComplexSentence* → ( *Sentence* )  
|  $\neg$  *Sentence*  
| *Sentence*  $\wedge$  *Sentence*  
| *Sentence*  $\vee$  *Sentence*  
| *Sentence*  $\Rightarrow$  *Sentence*  
| *Sentence*  $\Leftrightarrow$  *Sentence*  
| *Quantifier Variable, ... Sentence*

*Term* → *Function(Term, ...)*  
| *Constant*  
| *Variable*

*Quantifier* →  $\forall$  |  $\exists$

*Constant* → *A* | *X<sub>1</sub>* | *John* | ...

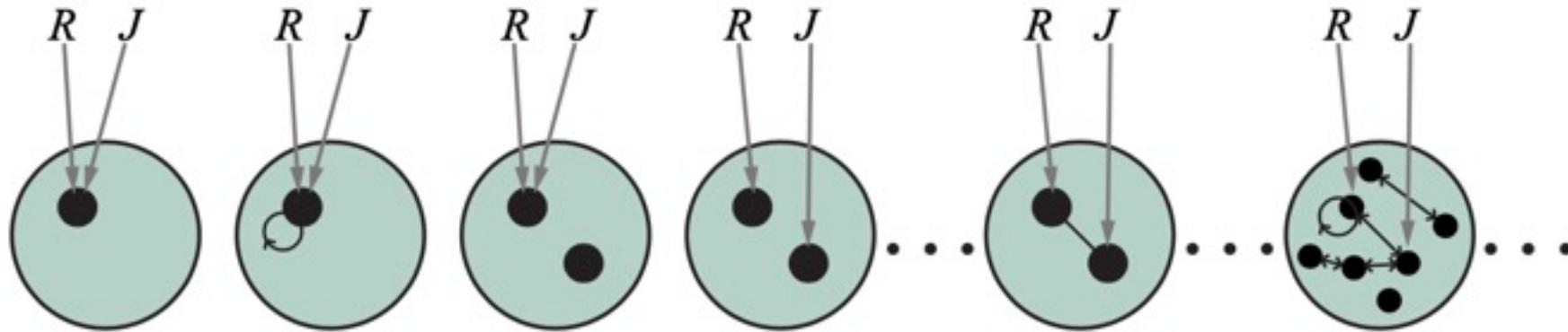
*Variable* → *a* | *x* | *s* | ...

*Predicate* → *True* | *False* | *After* | *Loves* | *Raining* | ...

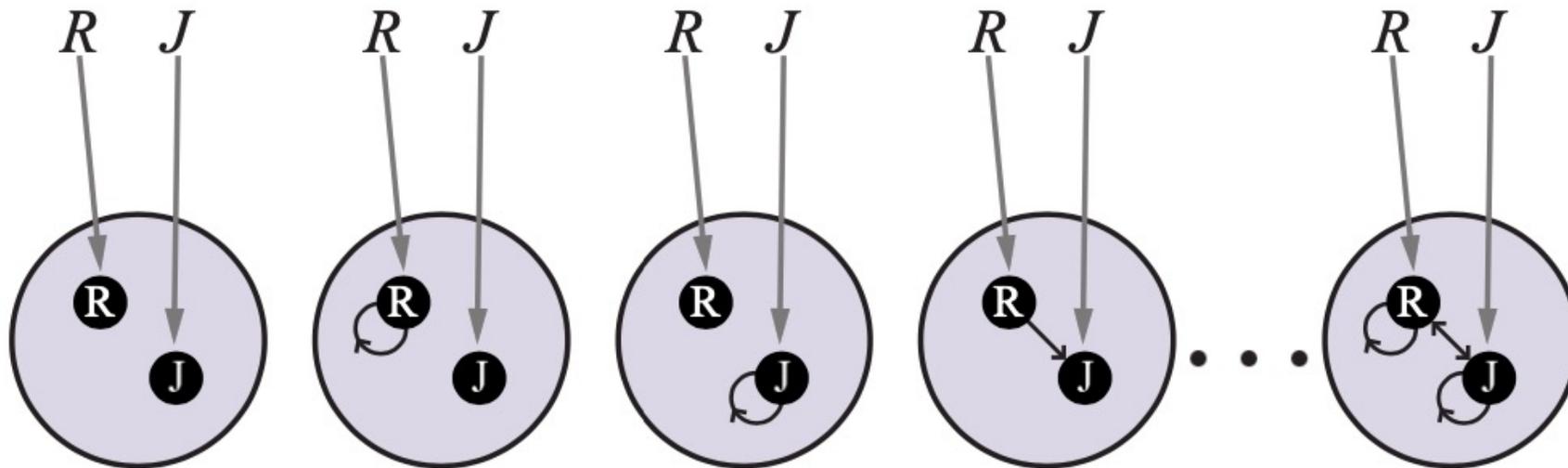
*Function* → *Mother* | *LeftLeg* | ...

**OPERATOR PRECEDENCE** :  $\neg, =, \wedge, \vee, \Rightarrow, \Leftrightarrow$

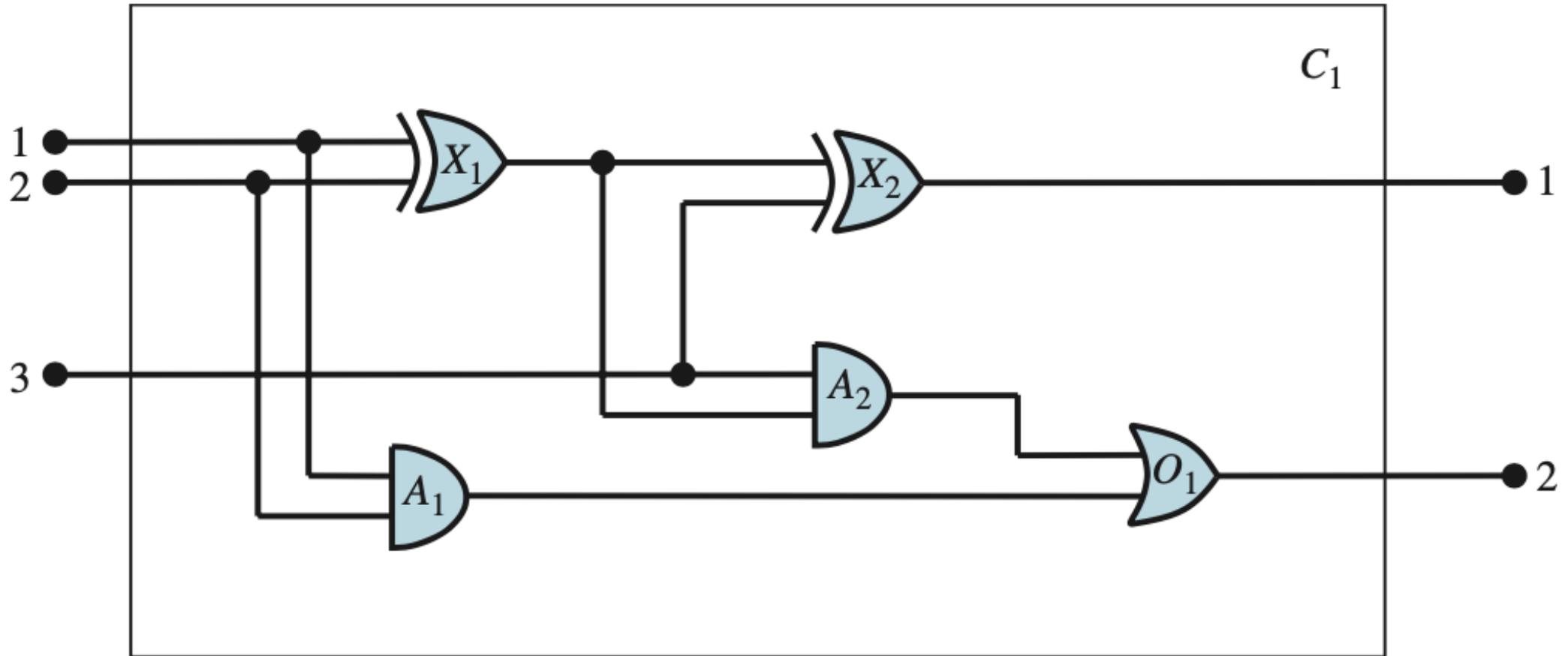
# Some members of the set of all models for a language with two constant symbols, $R$ and $J$ , and one binary relation symbol



# Some members of the set of all models for a language with two constant symbols, $R$ and $J$ , and one binary relation symbol, under database semantics



# A digital circuit C1, purporting to be a one-bit full adder.



# Inference in First-Order Logic

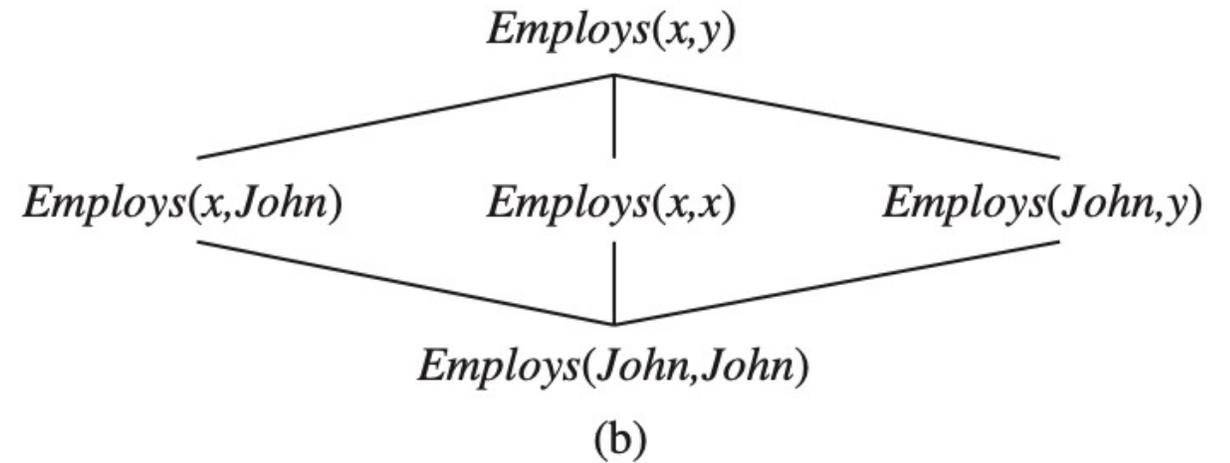
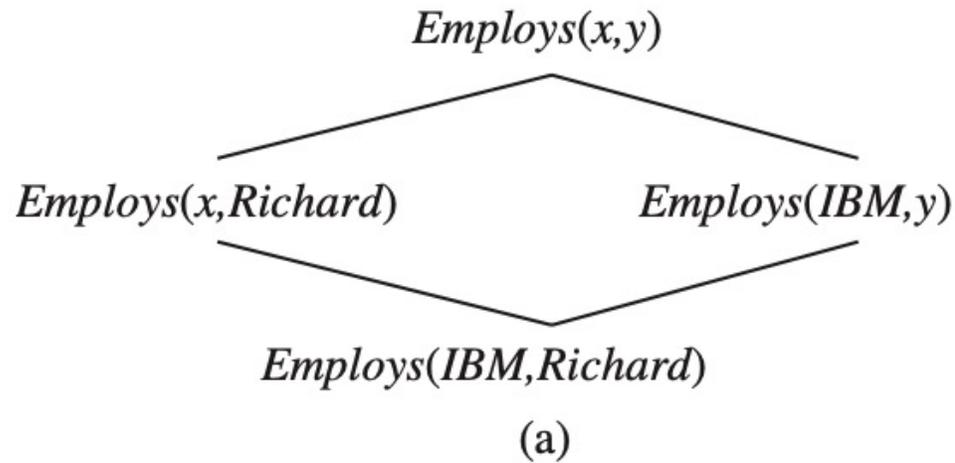
# The unification algorithm

**function** UNIFY( $x, y, \theta = \text{empty}$ ) **returns** a substitution to make  $x$  and  $y$  identical, or *failure*  
**if**  $\theta = \text{failure}$  **then return** *failure*  
**else if**  $x = y$  **then return**  $\theta$   
**else if** VARIABLE?( $x$ ) **then return** UNIFY-VAR( $x, y, \theta$ )  
**else if** VARIABLE?( $y$ ) **then return** UNIFY-VAR( $y, x, \theta$ )  
**else if** COMPOUND?( $x$ ) **and** COMPOUND?( $y$ ) **then**  
    **return** UNIFY(ARGS( $x$ ), ARGS( $y$ ), UNIFY(OP( $x$ ), OP( $y$ ),  $\theta$ ))  
**else if** LIST?( $x$ ) **and** LIST?( $y$ ) **then**  
    **return** UNIFY(REST( $x$ ), REST( $y$ ), UNIFY(FIRST( $x$ ), FIRST( $y$ ),  $\theta$ ))  
**else return** *failure*

**function** UNIFY-VAR( $var, x, \theta$ ) **returns** a substitution  
**if**  $\{var/val\} \in \theta$  for some  $val$  **then return** UNIFY( $val, x, \theta$ )  
**else if**  $\{x/val\} \in \theta$  for some  $val$  **then return** UNIFY( $var, val, \theta$ )  
**else if** OCCUR-CHECK?( $var, x$ ) **then return** *failure*  
**else return** add  $\{var/x\}$  to  $\theta$

# The subsumption lattice whose lowest node is **Employs** **(IBM , Richard )**

The subsumption lattice for the sentence **Employs**  
**(John, John )**



# A conceptually straightforward, but inefficient, forward-chaining algorithm

**function** FOL-FC-ASK( $KB, \alpha$ ) **returns** a substitution or *false*

**inputs:**  $KB$ , the knowledge base, a set of first-order definite clauses

$\alpha$ , the query, an atomic sentence

**while** *true* **do**

$new \leftarrow \{ \}$  // *The set of new sentences inferred on each iteration*

**for each** *rule* **in**  $KB$  **do**

$(p_1 \wedge \dots \wedge p_n \Rightarrow q) \leftarrow \text{STANDARDIZE-VARIABLES}(\text{rule})$

**for each**  $\theta$  such that  $\text{SUBST}(\theta, p_1 \wedge \dots \wedge p_n) = \text{SUBST}(\theta, p'_1 \wedge \dots \wedge p'_n)$

for some  $p'_1, \dots, p'_n$  in  $KB$

$q' \leftarrow \text{SUBST}(\theta, q)$

**if**  $q'$  does not unify with some sentence already in  $KB$  or *new* **then**

add  $q'$  to *new*

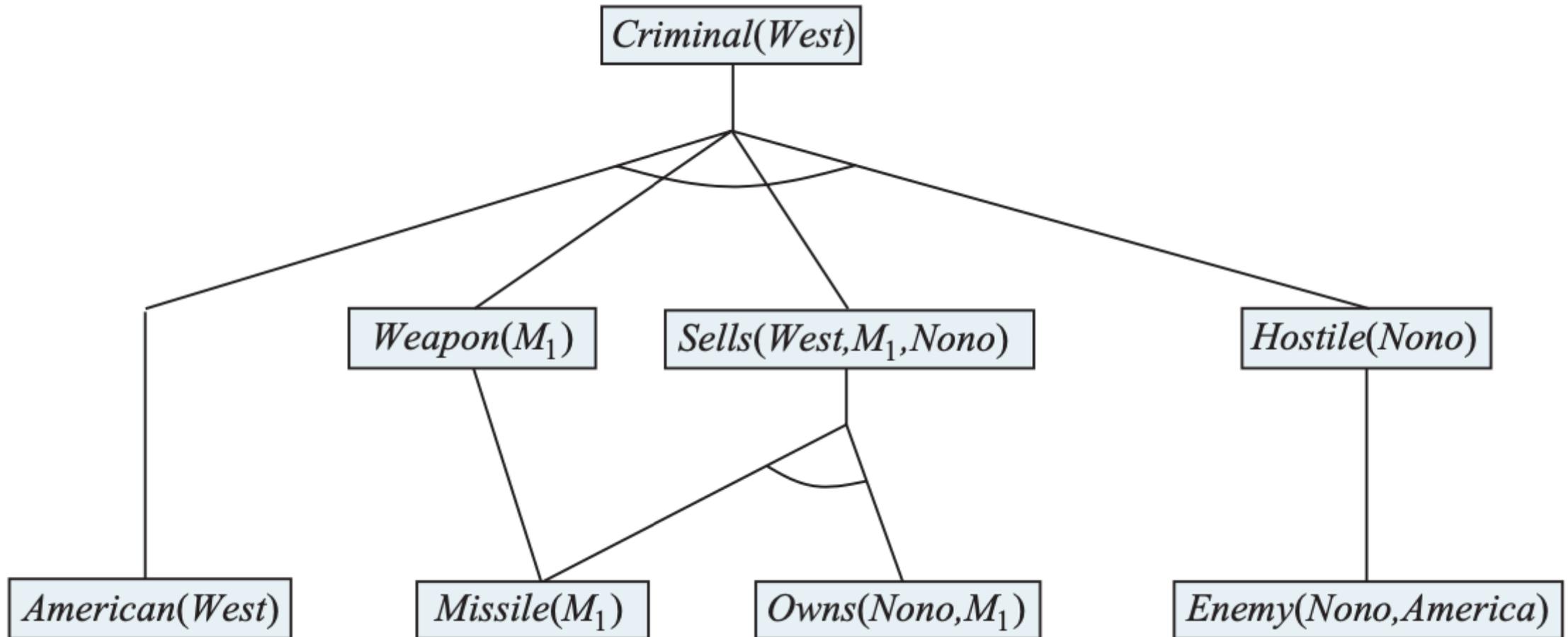
$\phi \leftarrow \text{UNIFY}(q', \alpha)$

**if**  $\phi$  is not *failure* **then return**  $\phi$

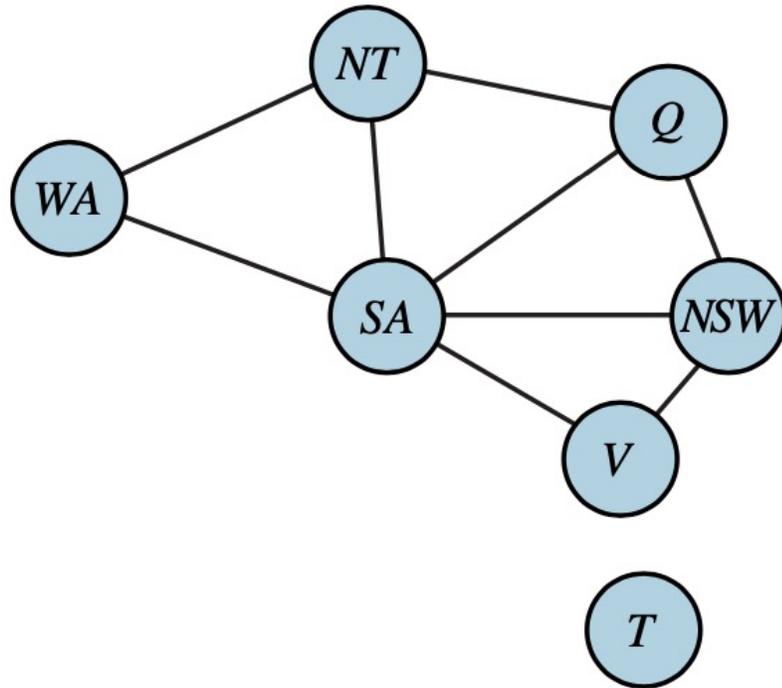
**if**  $new = \{ \}$  **then return** *false*

add *new* to  $KB$

# The proof tree generated by forward chaining on the crime example



# Constraint graph for coloring the map of Australia



(a)

$$\begin{aligned} & Diff(wa, nt) \wedge Diff(wa, sa) \wedge \\ & Diff(nt, q) \wedge Diff(nt, sa) \wedge \\ & Diff(q, nsw) \wedge Diff(q, sa) \wedge \\ & Diff(nsw, v) \wedge Diff(nsw, sa) \wedge \\ & Diff(v, sa) \Rightarrow Colorable() \end{aligned}$$

$$\begin{aligned} & Diff(Red, Blue) \quad Diff(Red, Green) \\ & Diff(Green, Red) \quad Diff(Green, Blue) \\ & Diff(Blue, Red) \quad Diff(Blue, Green) \end{aligned}$$

(b)

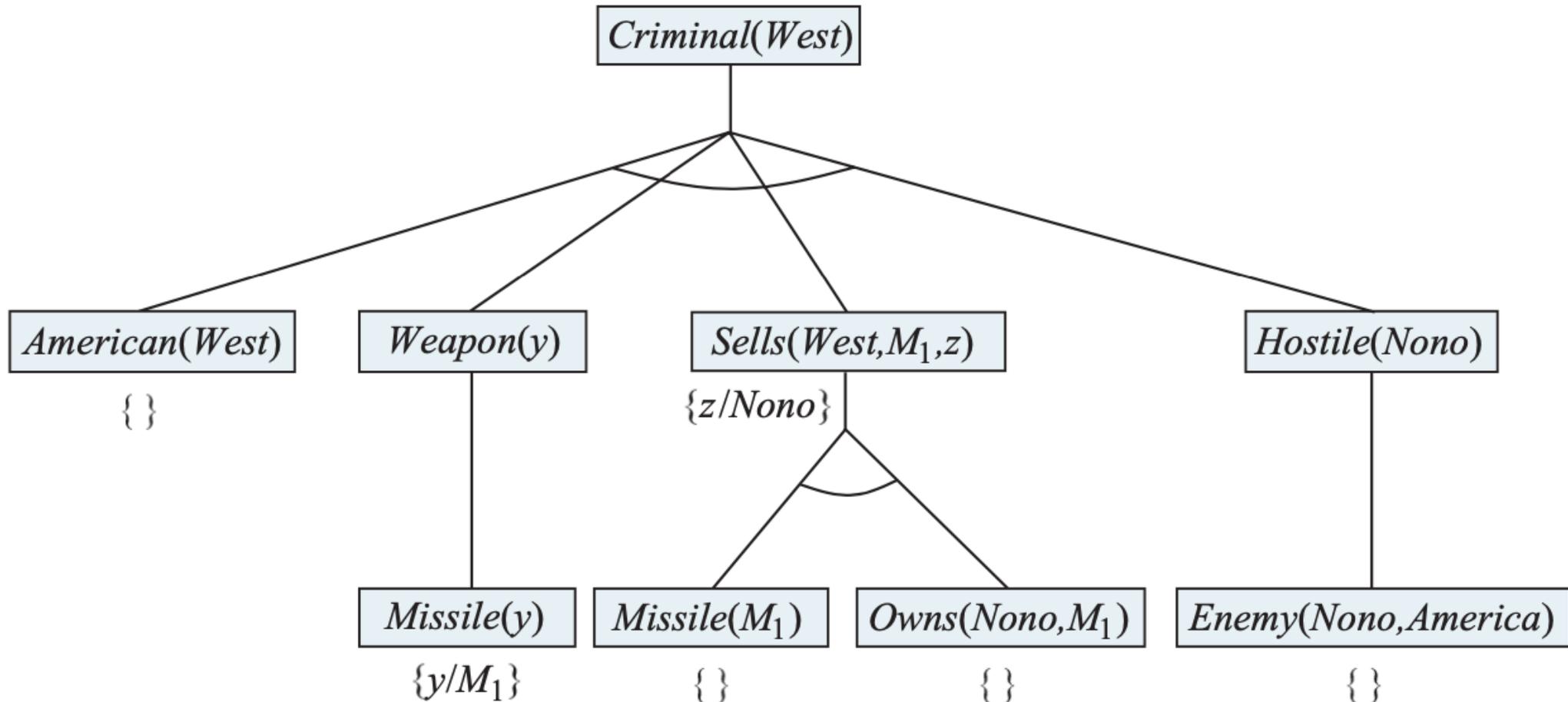
# A simple backward-chaining algorithm for first-order knowledge bases

**function** FOL-BC-ASK( $KB, query$ ) **returns** a generator of substitutions  
**return** FOL-BC-OR( $KB, query, \{ \}$ )

**function** FOL-BC-OR( $KB, goal, \theta$ ) **returns** a substitution  
**for each**  $rule$  in FETCH-RULES-FOR-GOAL( $KB, goal$ ) **do**  
     $(lhs \Rightarrow rhs) \leftarrow$  STANDARDIZE-VARIABLES( $rule$ )  
    **for each**  $\theta'$  in FOL-BC-AND( $KB, lhs, UNIFY(rhs, goal, \theta)$ ) **do**  
        **yield**  $\theta'$

**function** FOL-BC-AND( $KB, goals, \theta$ ) **returns** a substitution  
**if**  $\theta = failure$  **then return**  
**else if** LENGTH( $goals$ ) = 0 **then yield**  $\theta$   
**else**  
     $first, rest \leftarrow$  FIRST( $goals$ ), REST( $goals$ )  
    **for each**  $\theta'$  in FOL-BC-OR( $KB, SUBST(\theta, first), \theta$ ) **do**  
        **for each**  $\theta''$  in FOL-BC-AND( $KB, rest, \theta'$ ) **do**  
            **yield**  $\theta''$

# Proof tree constructed by backward chaining to prove that West is a criminal



# Pseudocode representing the result of compiling the Append predicate

**procedure** APPEND( $ax, y, az, continuation$ )

$trail \leftarrow$  GLOBAL-TRAIL-POINTER()

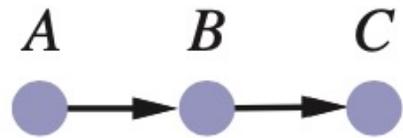
**if**  $ax = []$  and UNIFY( $y, az$ ) **then** CALL( $continuation$ )

RESET-TRAIL( $trail$ )

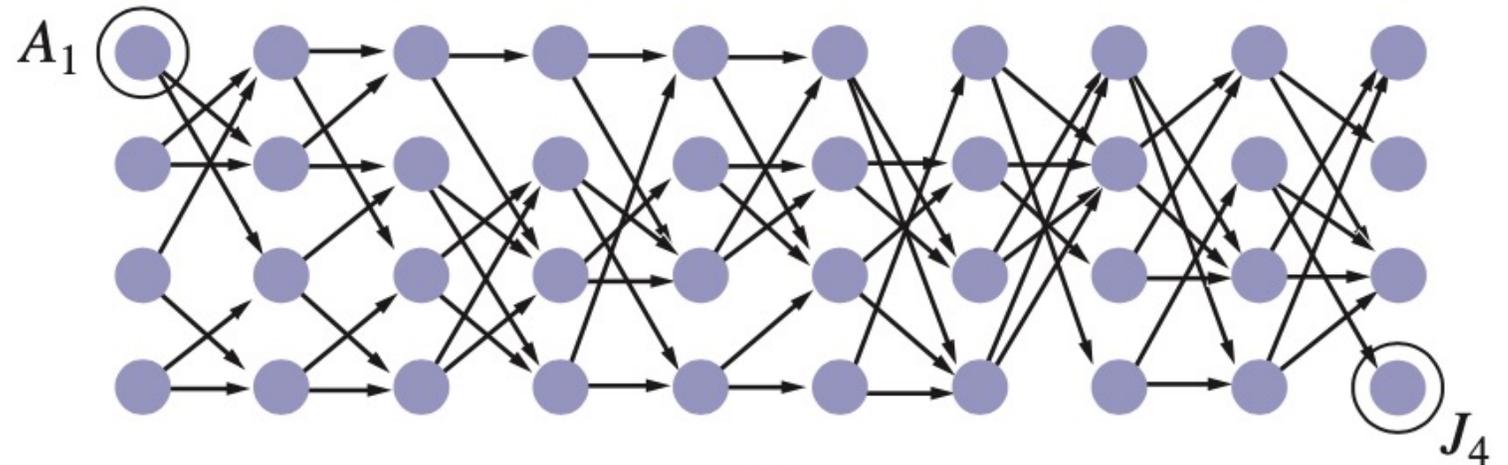
$a, x, z \leftarrow$  NEW-VARIABLE(), NEW-VARIABLE(), NEW-VARIABLE()

**if** UNIFY( $ax, [a] + x$ ) and UNIFY( $az, [a | z]$ ) **then** APPEND( $x, y, z, continuation$ )

# Finding a path from A to C can lead Prolog into an infinite loop.

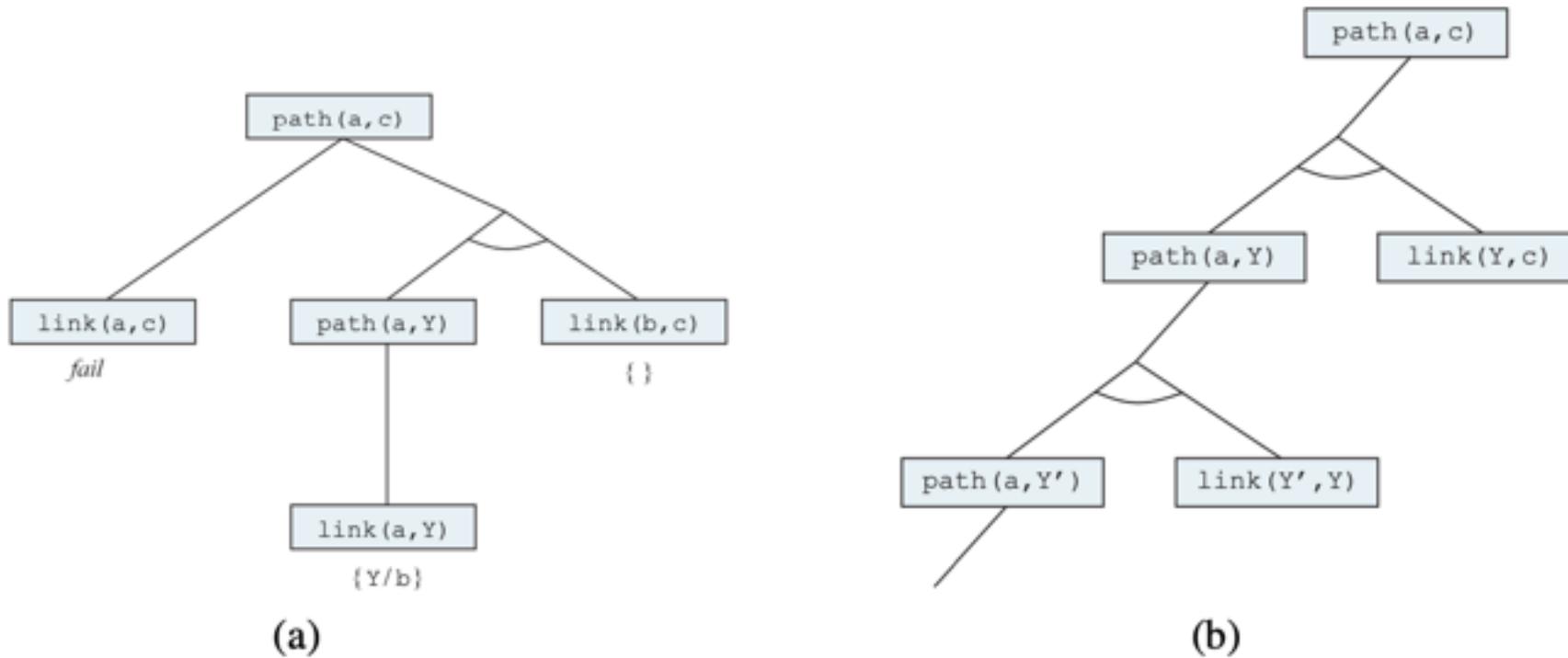


(a)



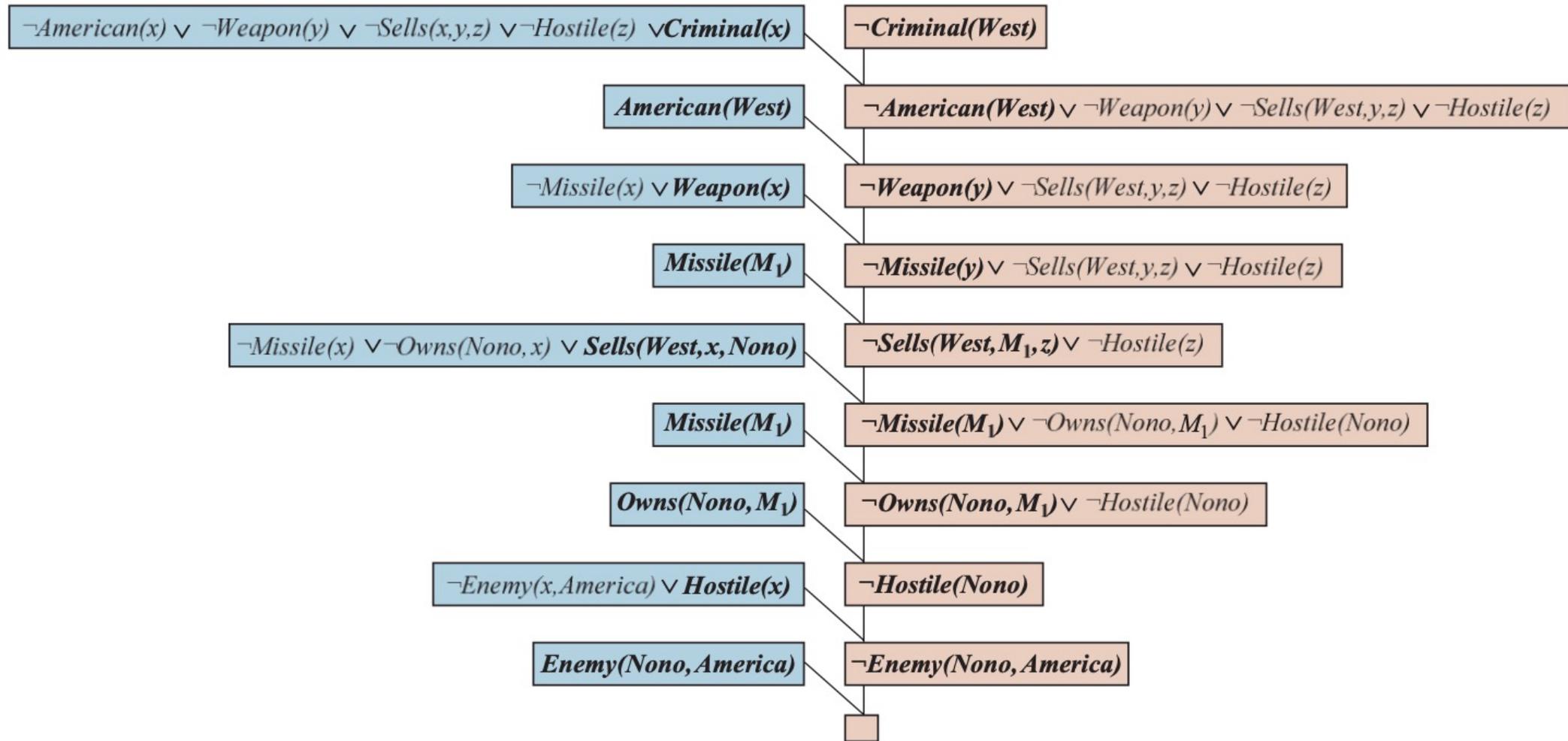
(b)

# Proof that a path exists from A to C.

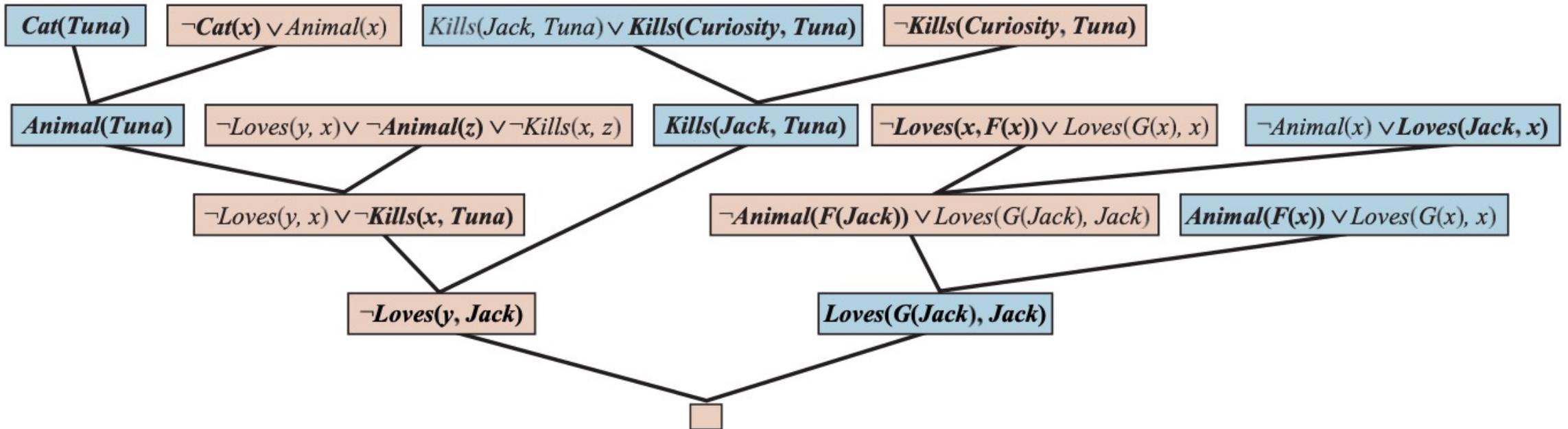


Infinite proof tree generated when the clauses are in the “wrong” order

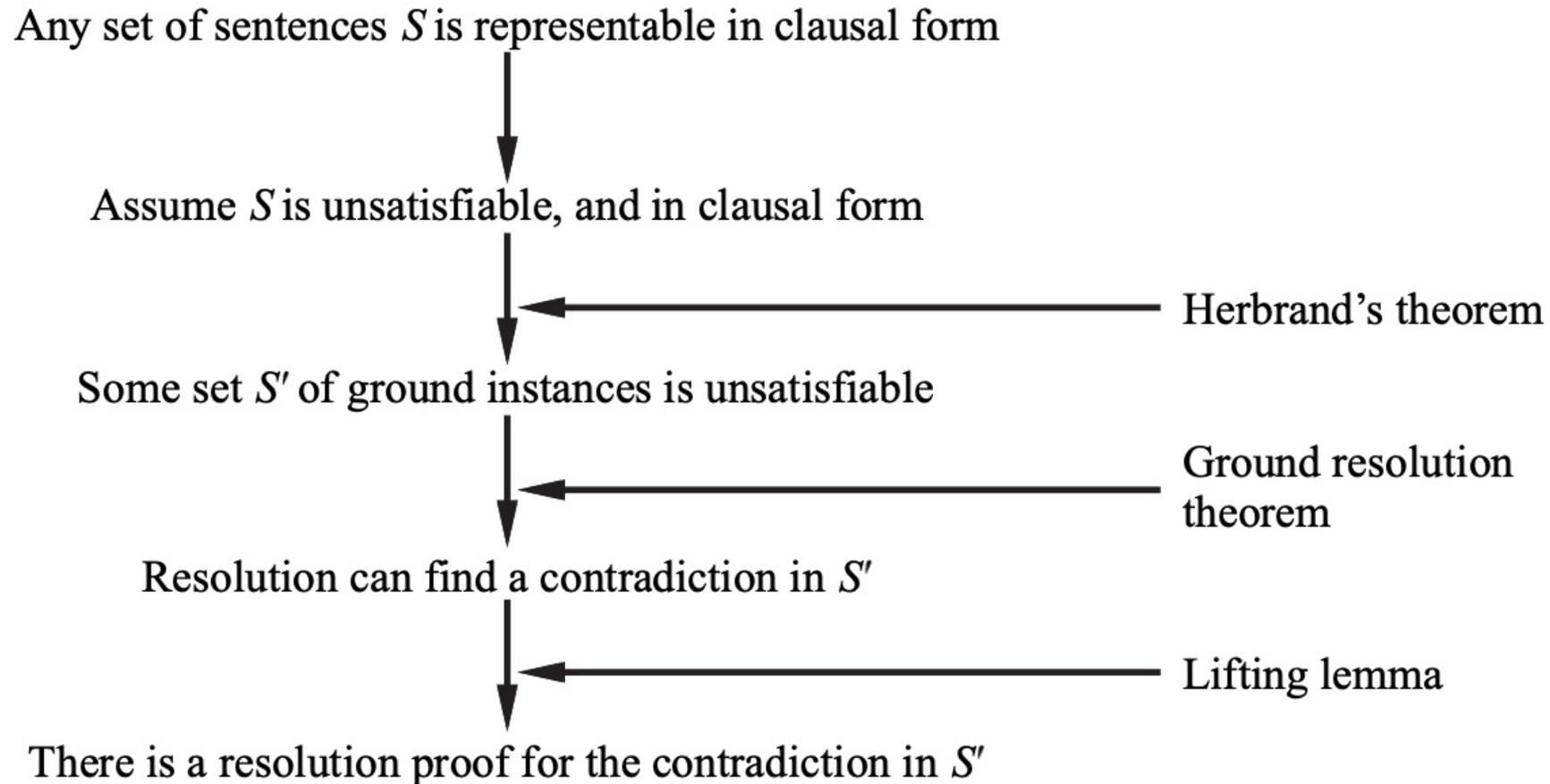
# A resolution proof that West is a criminal



# A resolution proof that Curiosity killed the cat

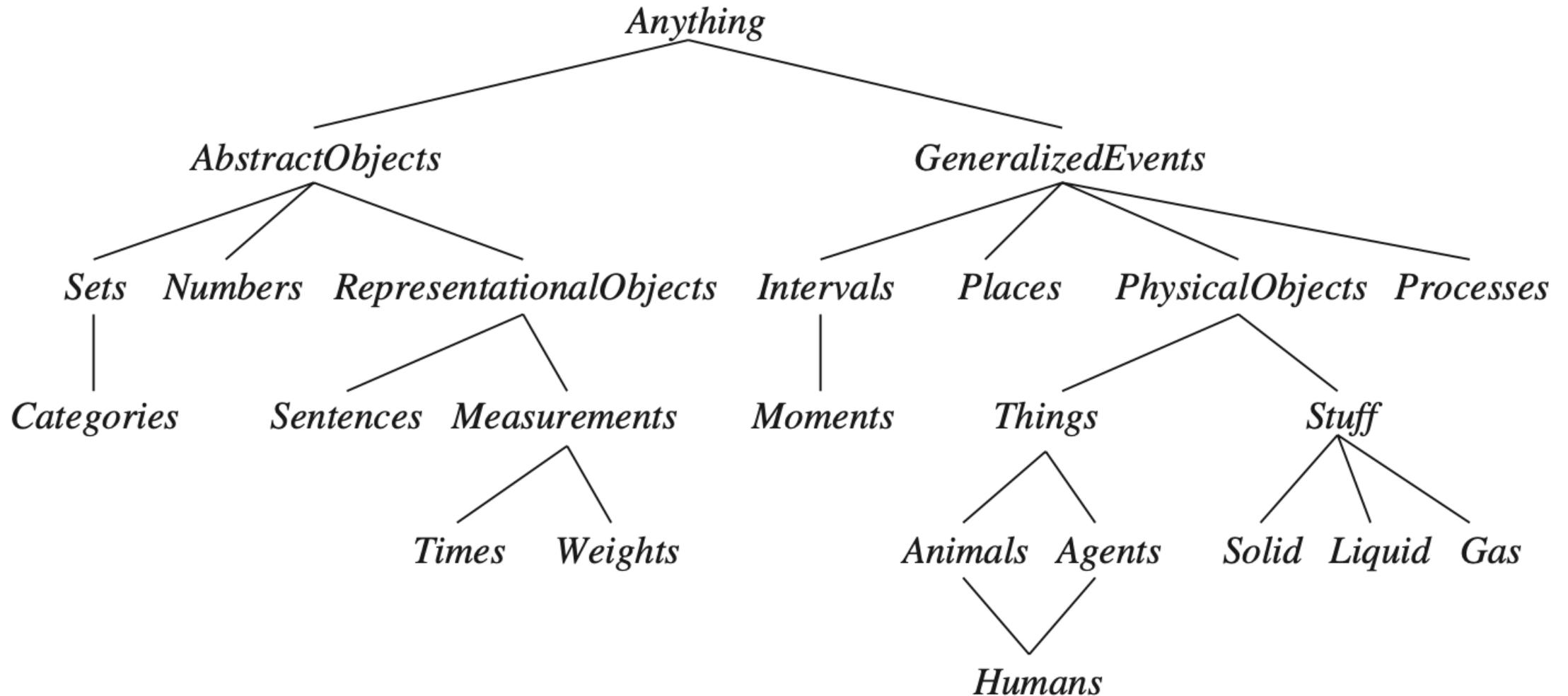


# Structure of a completeness proof for resolution

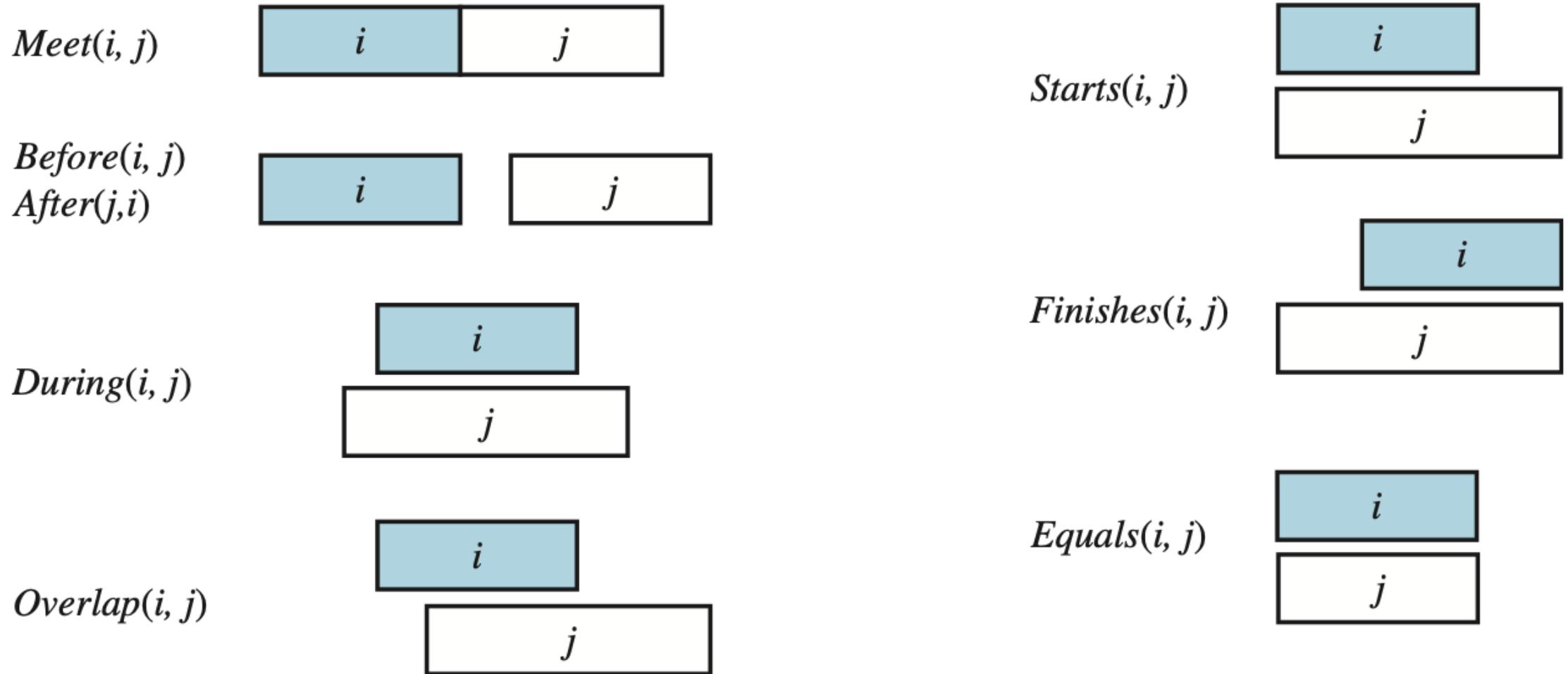


# Knowledge Representation

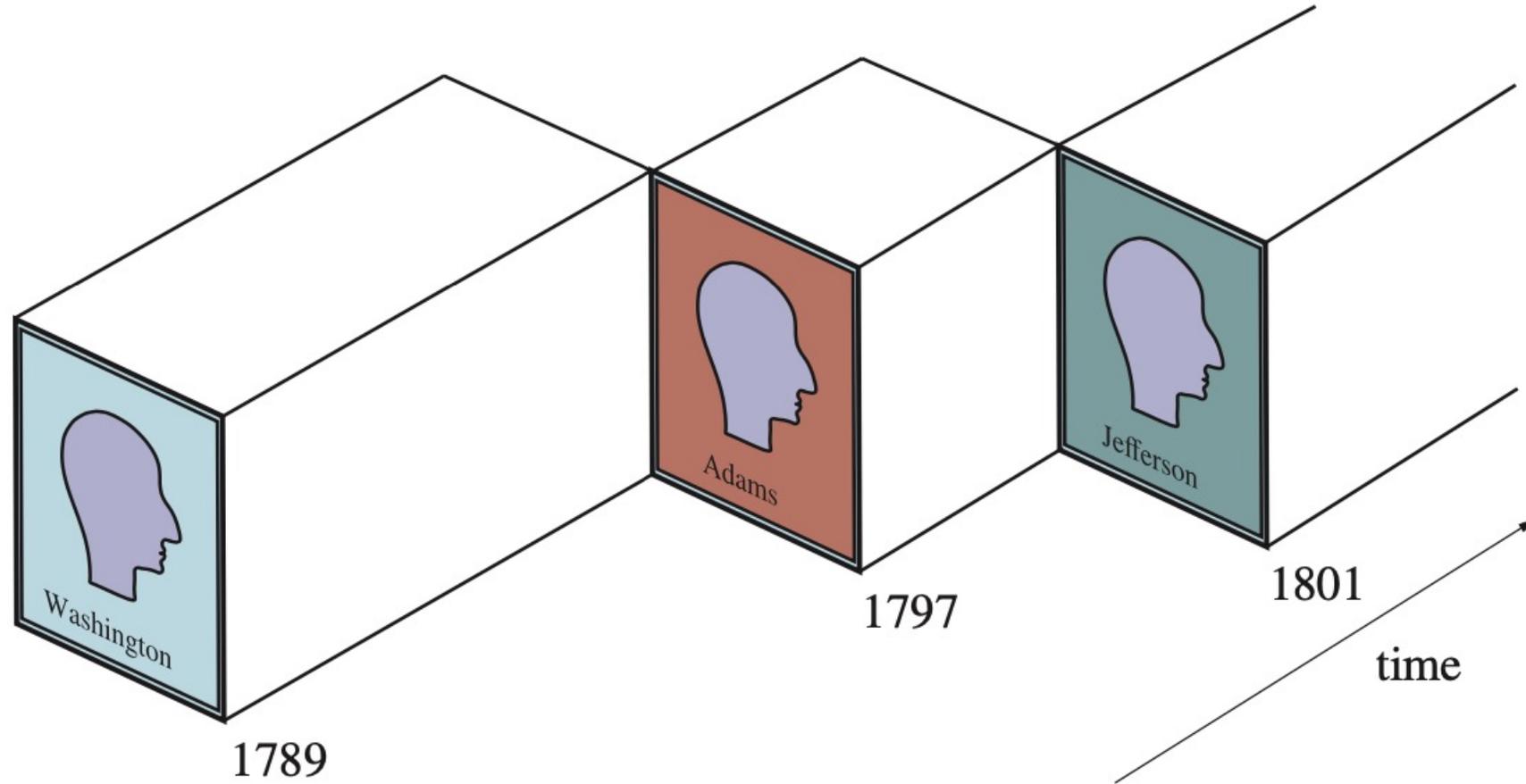
# The Upper Ontology of the World



# Predicates on time intervals

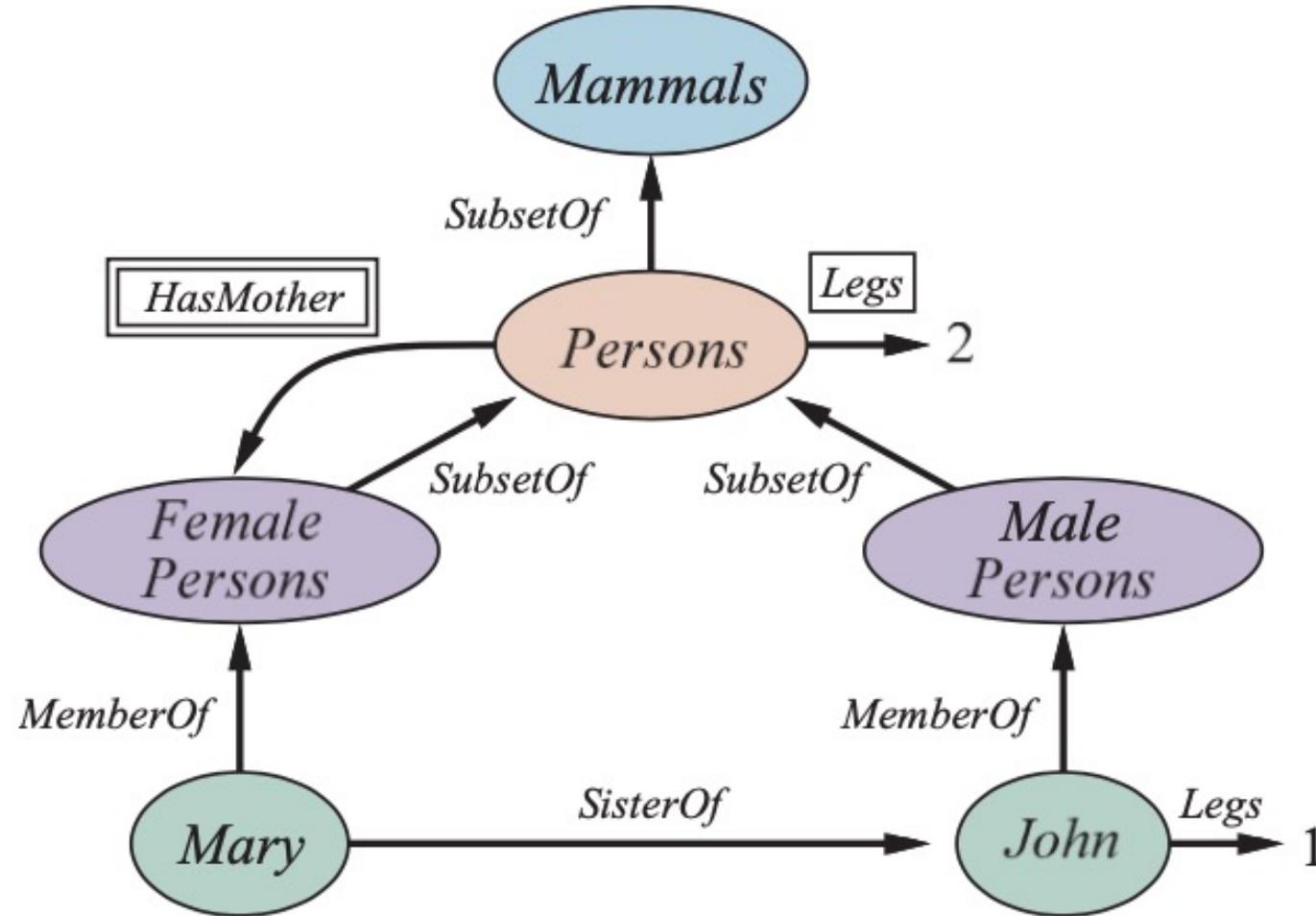


# A schematic view of the object President (USA) for the early years



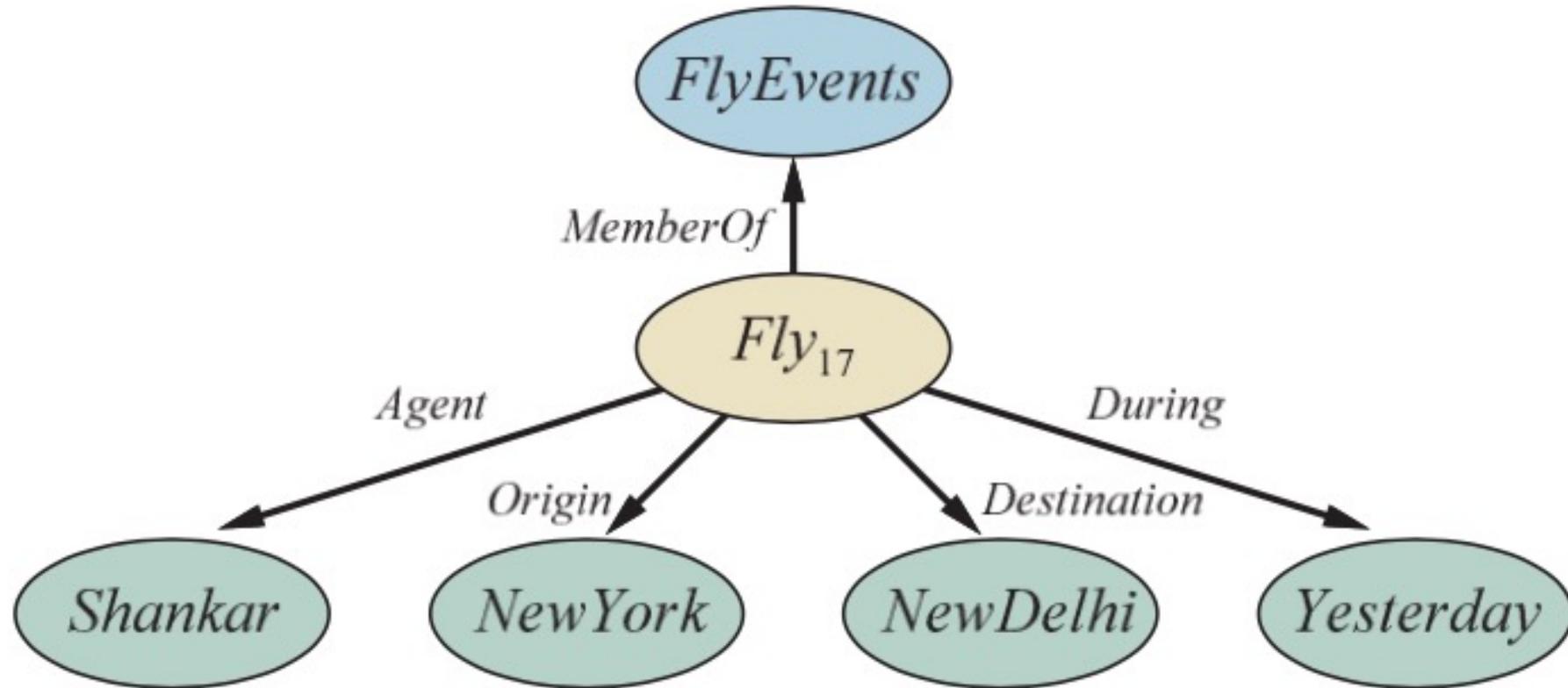
# A semantic network

with four objects (John, Mary, 1, and 2) and four categories Relations are denoted by labeled links



# Semantic network

Representation of the logical assertion  
Fly (Shankar, NewYork, NewDelhi, Yesterday)



# The syntax of descriptions in a subset of the CLASSIC language.

*Concept* → **Thing** | *ConceptName*  
| **And**(*Concept*, ...)  
| **All**(*RoleName*, *Concept*)  
| **AtLeast**(*Integer*, *RoleName*)  
| **AtMost**(*Integer*, *RoleName*)  
| **Fills**(*RoleName*, *IndividualName*, ...)  
| **SameAs**(*Path*, *Path*)  
| **OneOf**(*IndividualName*, ...)

*Path* → [*RoleName*, ...]

*ConceptName* → *Adult* | *Female* | *Male* | ...

*RoleName* → *Spouse* | *Daughter* | *Son* | ...

# Knowledge Graph (KG)

# Knowledge Graph (KG)

- **Knowledge Graph (KG)**

- A knowledge graph is a multi-relational graph composed of **entities** and **relations**, which are regarded as **nodes** and different types of **edges**, respectively (Ji et al., 2021).

- Represents knowledge as **concepts (entities)** and their **relationships (Facts)**

- **Triple** of facts

- *SPO: (subject, predicate, object)*

- *HRT: (head, relation, tail)*

- **Common Knowledge Graph: DBpedia, YAGO, Wikidata**

# Knowledge Graph, Facts, Triple, Embedding

- $G$ 
  - Knowledge graph
- $F$ 
  - Set of facts
- $(h, r, t)$ 
  - Triple of head, relation, and tail
- $(\mathbf{h}, \mathbf{r}, \mathbf{t})$ 
  - Embedding of head, relation, and tail

# Knowledge Representation

## Factual Triple and Knowledge Graph

- Albert Einstein, **winner of** the 1921 Nobel prize in physics
- The Nobel Prize in Physics 1921 was **awarded to** Albert Einstein "for his services to Theoretical Physics, and especially for his discovery of the law of the photoelectric effect."

**Triple** (Albert Einstein, **WinnerOf**, Nobel Prize in Physics)

**Knowledge  
Graph**



# Factual Triples in Knowledge Base

$(h, r, t)$

(Albert Einstein, **BornIn**, German Empire)

(Albert Einstein, **SonOf**, Hermann Einstein)

(Albert Einstein, **GraduateFrom**, University of Zurich)

(Albert Einstein, **WinnerOf**, Nobel Prize in Physics)

(Albert Einstein, **ExpertIn**, Physics)

(Nobel Prize in Physics, **AwardIn**, Physics)

(The theory of relativity, **TheoryOf**, Physics)

(Albert Einstein, **SupervisedBy**, Alfred Kleiner)

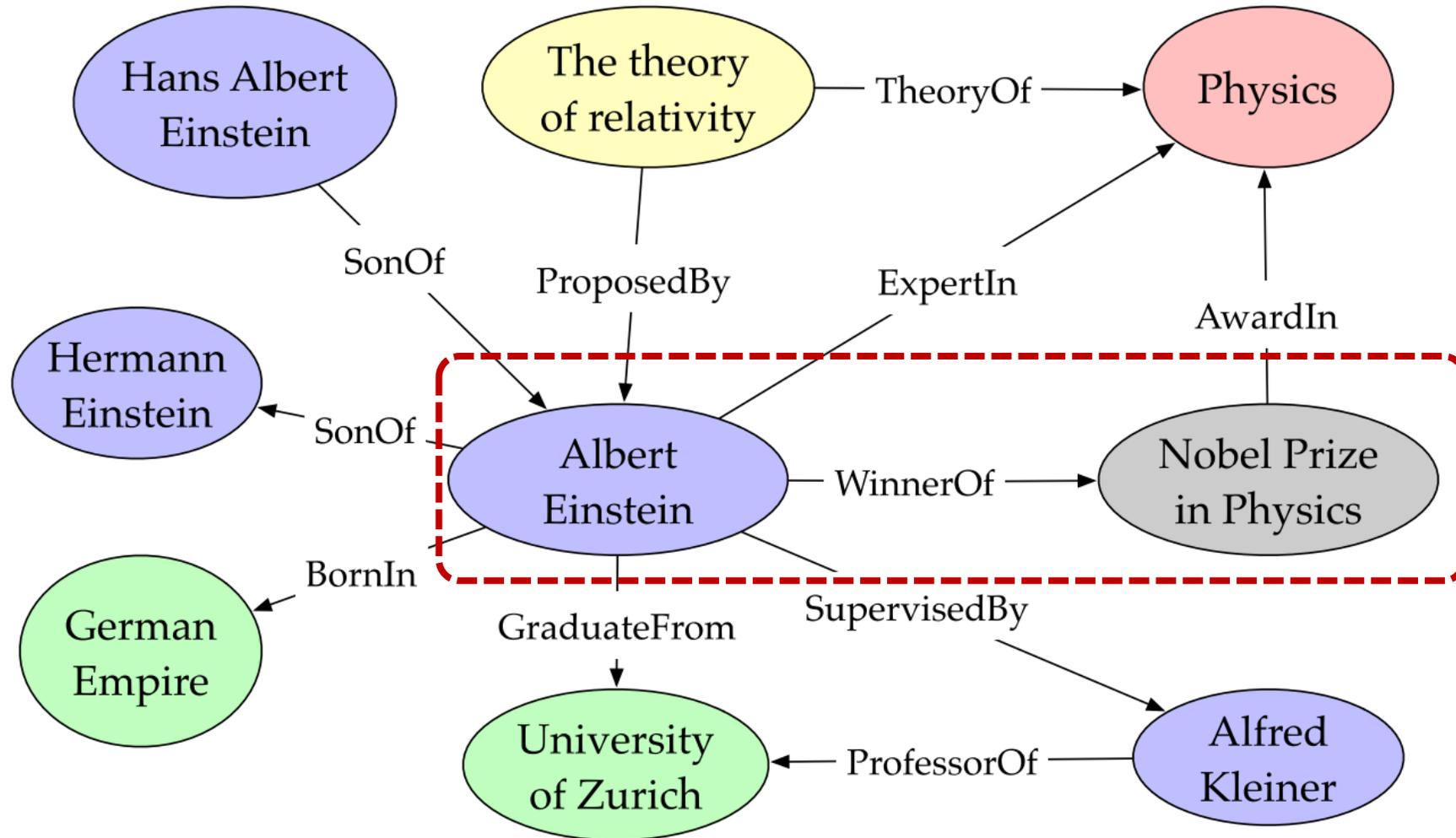
(Alfred Kleiner, **ProfessorOf**, University of Zurich)

(The theory of relativity, **ProposedBy**, Albert Einstein)

(Hans Albert Einstein, **SonOf**, Albert Einstein)

# Entities and Relations in Knowledge Graph

(Albert Einstein, **WinnerOf**, Nobel Prize in Physics)



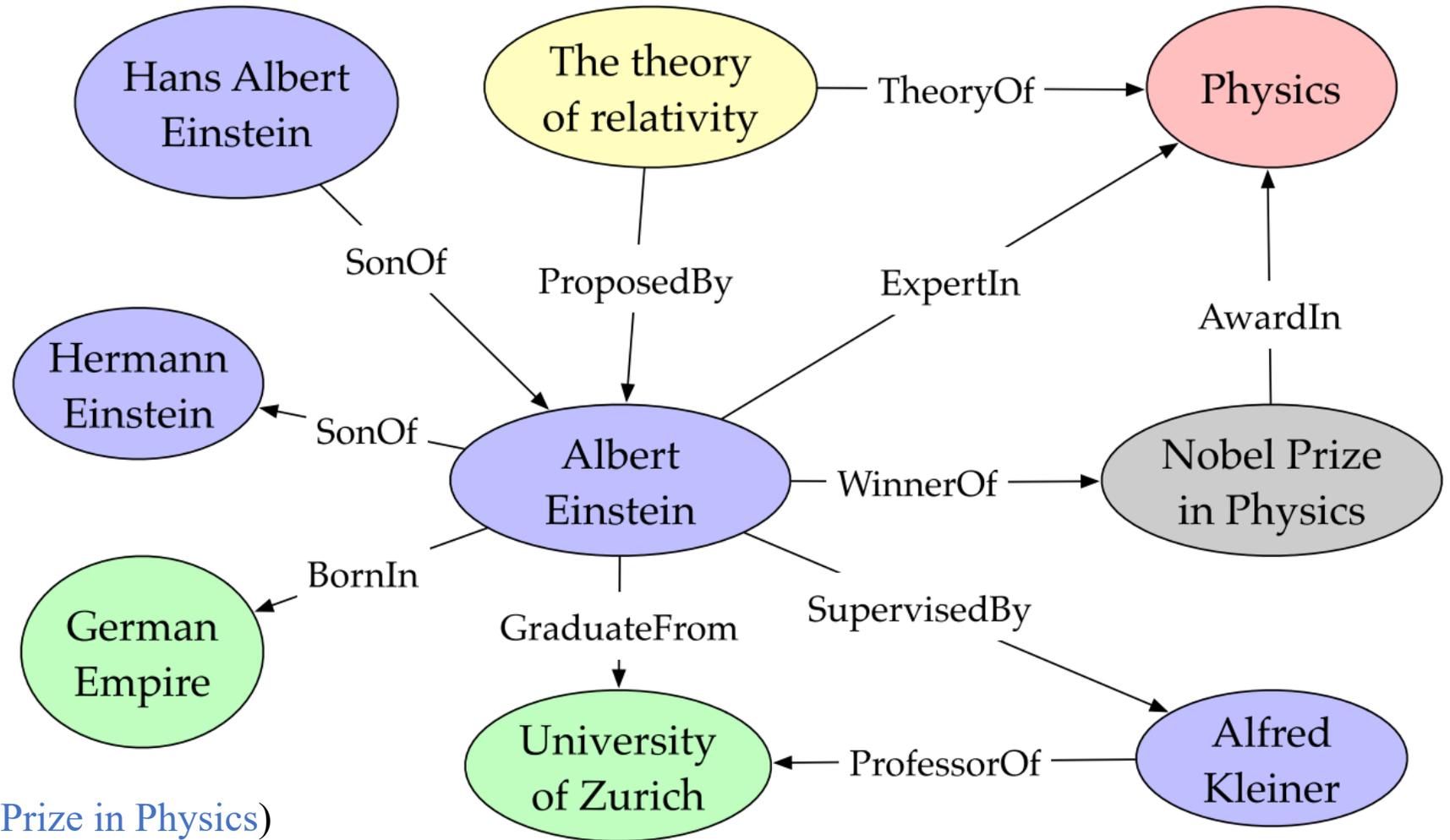
# knowledge base and knowledge graph

## Factual triples in knowledge base

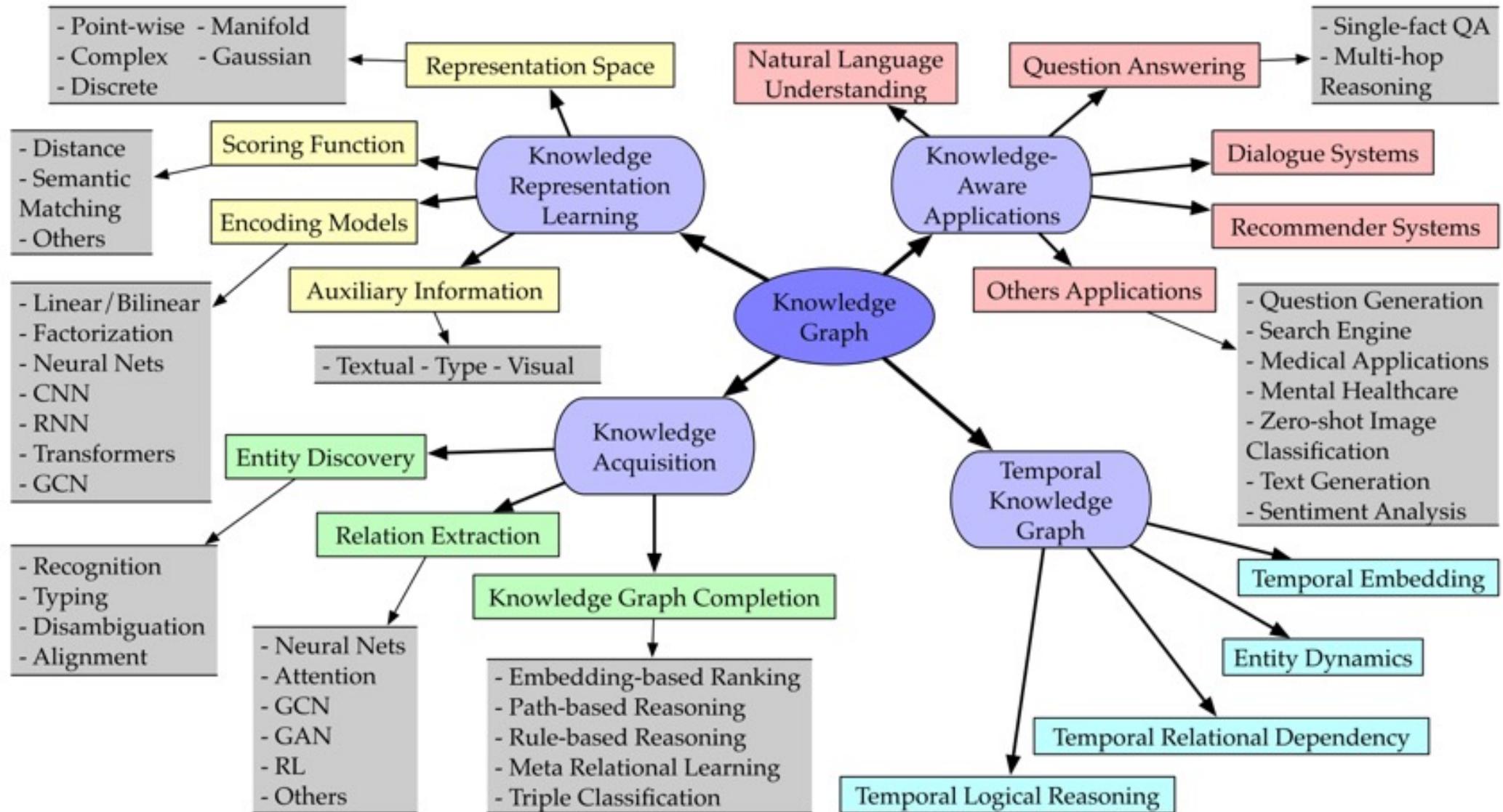
(Albert Einstein, **BornIn**, German Empire)  
(Albert Einstein, **SonOf**, Hermann Einstein)  
(Albert Einstein, **GraduateFrom**, University of Zurich)  
(Albert Einstein, **WinnerOf**, Nobel Prize in Physics)  
(Albert Einstein, **ExpertIn**, Physics)  
(Nobel Prize in Physics, **AwardIn**, Physics)  
(The theory of relativity, **TheoryOf**, Physics)  
(Albert Einstein, **SupervisedBy**, Alfred Kleiner)  
(Alfred Kleiner, **ProfessorOf**, University of Zurich)  
(The theory of relativity, **ProposedBy**, Albert Einstein)  
(Hans Albert Einstein, **SonOf**, Albert Einstein)

(Albert Einstein, **WinnerOf**, Nobel Prize in Physics)

## Entities and relations in knowledge graph



# Categorization of Research on Knowledge Graphs



Source: Ji, S., Pan, S., Cambria, E., Marttinen, P., & Philip, S. Y. (2021). A survey on knowledge graphs: Representation, acquisition, and applications.

IEEE Transactions on Neural Networks and Learning Systems.

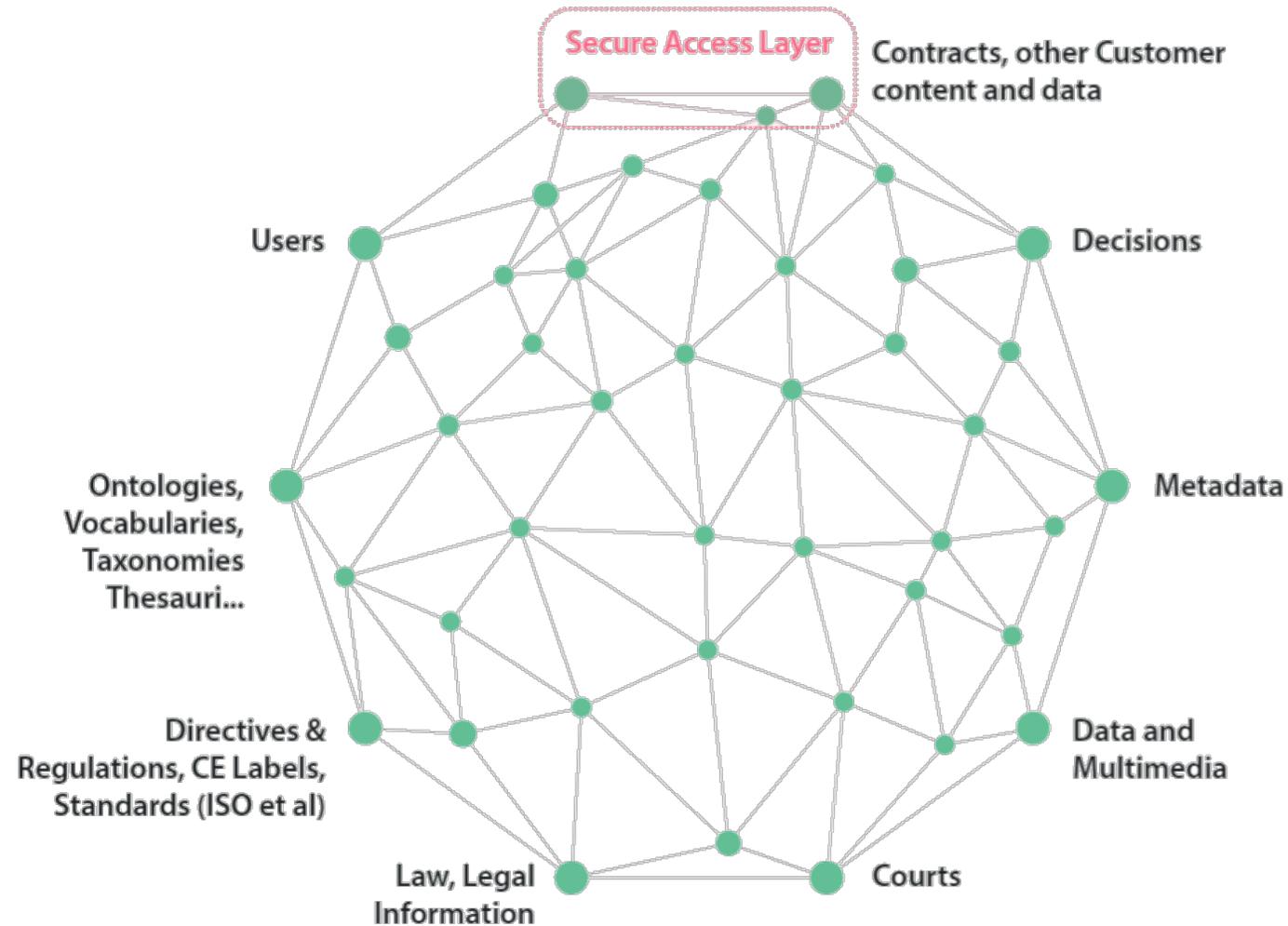
# Knowledge Graph Completion (KGC) Datasets

Knowledge Graph Completion (KGC) Dataset	#Entity	#Relation	#Train	#Valid	#Test	Reference
WN18RR	40,943	11	86,835	3,034	3,134	Toutanova & Chen (2015); Zhang et al. (2020)
FB15k-237	14,541	237	272,115	17,535	20,466	Dettmers et al. (2018); Zhang et al. (2020)
YAGO3-10	123,182	37	1,079,040	5,000	5,000	Mahdisoltani et al. (2015); Zhang et al. (2020)

# Domain-Specific Knowledge Graph

- **Domain-Specific Knowledge Graph**
  - **PubMed Knowledge Graph (PKG)**
    - **Extracting biological entities from 29 million PubMed abstracts**
  - **Lynx: Legal Knowledge Graph for Multilingual Compliance Services**
    - **Legal Knowledge Graph (LKG) integrates and links heterogeneous compliance data sources including legislation, case law, standards and other private contracts.**

# Lynx: Legal Knowledge Graph for Multilingual Compliance Services



# Automated Planning

# A PDDL description of an air cargo transportation planning problem

*Init*( $At(C_1, SFO) \wedge At(C_2, JFK) \wedge At(P_1, SFO) \wedge At(P_2, JFK)$   
 $\wedge Cargo(C_1) \wedge Cargo(C_2) \wedge Plane(P_1) \wedge Plane(P_2)$   
 $\wedge Airport(JFK) \wedge Airport(SFO)$ )

*Goal*( $At(C_1, JFK) \wedge At(C_2, SFO)$ )

*Action*(*Load*( $c, p, a$ ),

PRECOND:  $At(c, a) \wedge At(p, a) \wedge Cargo(c) \wedge Plane(p) \wedge Airport(a)$

EFFECT:  $\neg At(c, a) \wedge In(c, p)$ )

*Action*(*Unload*( $c, p, a$ ),

PRECOND:  $In(c, p) \wedge At(p, a) \wedge Cargo(c) \wedge Plane(p) \wedge Airport(a)$

EFFECT:  $At(c, a) \wedge \neg In(c, p)$ )

*Action*(*Fly*( $p, from, to$ ),

PRECOND:  $At(p, from) \wedge Plane(p) \wedge Airport(from) \wedge Airport(to)$

EFFECT:  $\neg At(p, from) \wedge At(p, to)$ )

# The simple spare tire problem

*Init*(*Tire*(*Flat*)  $\wedge$  *Tire*(*Spare*)  $\wedge$  *At*(*Flat*, *Axle*)  $\wedge$  *At*(*Spare*, *Trunk*))

*Goal*(*At*(*Spare*, *Axle*))

*Action*(*Remove*(*obj*, *loc*),

PRECOND: *At*(*obj*, *loc*)

EFFECT:  $\neg$  *At*(*obj*, *loc*)  $\wedge$  *At*(*obj*, *Ground*))

*Action*(*PutOn*(*t*, *Axle*),

PRECOND: *Tire*(*t*)  $\wedge$  *At*(*t*, *Ground*)  $\wedge$   $\neg$  *At*(*Flat*, *Axle*)  $\wedge$   $\neg$  *At*(*Spare*, *Axle*)

EFFECT:  $\neg$  *At*(*t*, *Ground*)  $\wedge$  *At*(*t*, *Axle*))

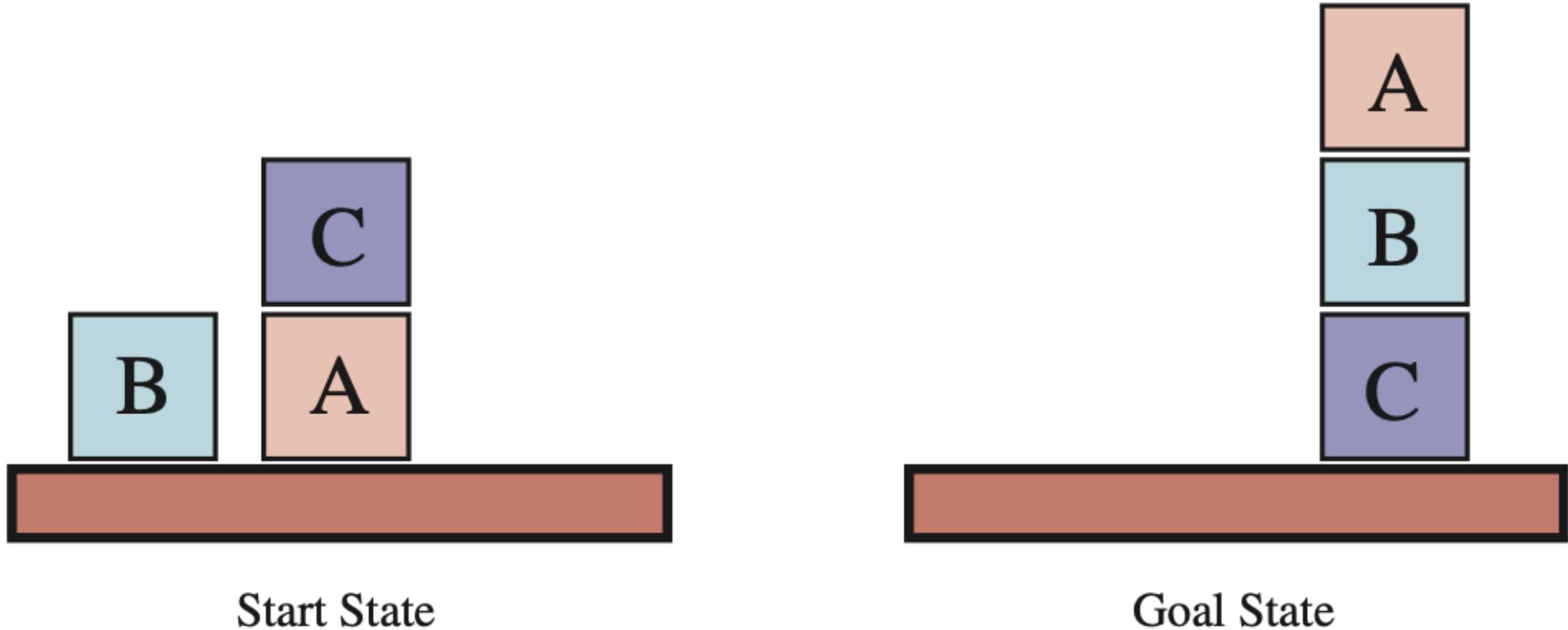
*Action*(*LeaveOvernight*,

PRECOND:

EFFECT:  $\neg$  *At*(*Spare*, *Ground*)  $\wedge$   $\neg$  *At*(*Spare*, *Axle*)  $\wedge$   $\neg$  *At*(*Spare*, *Trunk*)

$\wedge$   $\neg$  *At*(*Flat*, *Ground*)  $\wedge$   $\neg$  *At*(*Flat*, *Axle*)  $\wedge$   $\neg$  *At*(*Flat*, *Trunk*))

# Diagram of the blocks-world problem



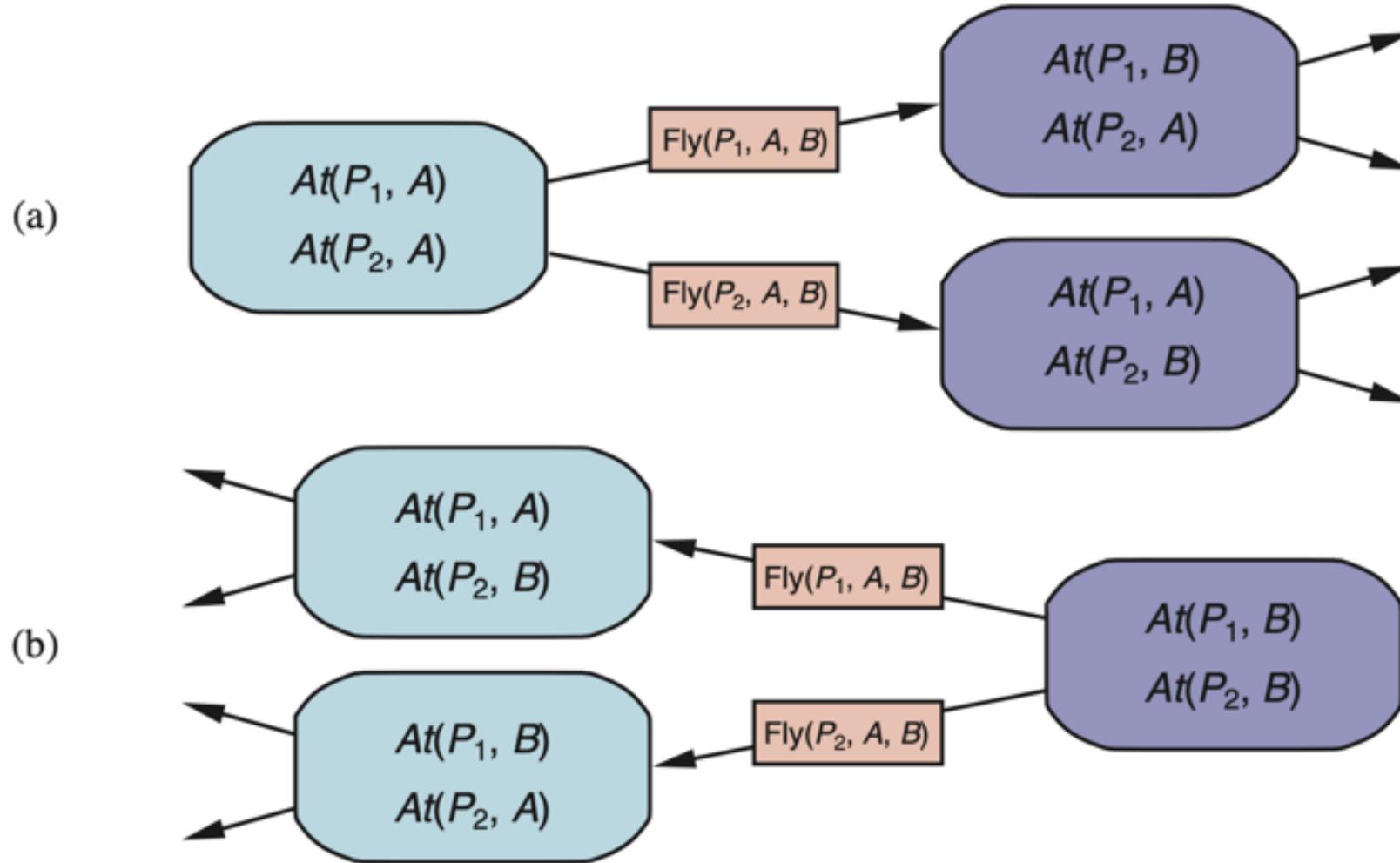
# A planning problem in the blocks world: building a three-block tower

*Init*( $On(A, Table) \wedge On(B, Table) \wedge On(C, A)$   
 $\wedge Block(A) \wedge Block(B) \wedge Block(C) \wedge Clear(B) \wedge Clear(C) \wedge Clear(Table)$ )  
*Goal*( $On(A, B) \wedge On(B, C)$ )  
*Action*(*Move*( $b, x, y$ ),  
PRECOND:  $On(b, x) \wedge Clear(b) \wedge Clear(y) \wedge Block(b) \wedge Block(y) \wedge$   
 $(b \neq x) \wedge (b \neq y) \wedge (x \neq y)$ ,  
EFFECT:  $On(b, y) \wedge Clear(x) \wedge \neg On(b, x) \wedge \neg Clear(y)$ )  
*Action*(*MoveToTable*( $b, x$ ),  
PRECOND:  $On(b, x) \wedge Clear(b) \wedge Block(b) \wedge Block(x)$ ,  
EFFECT:  $On(b, Table) \wedge Clear(x) \wedge \neg On(b, x)$ )

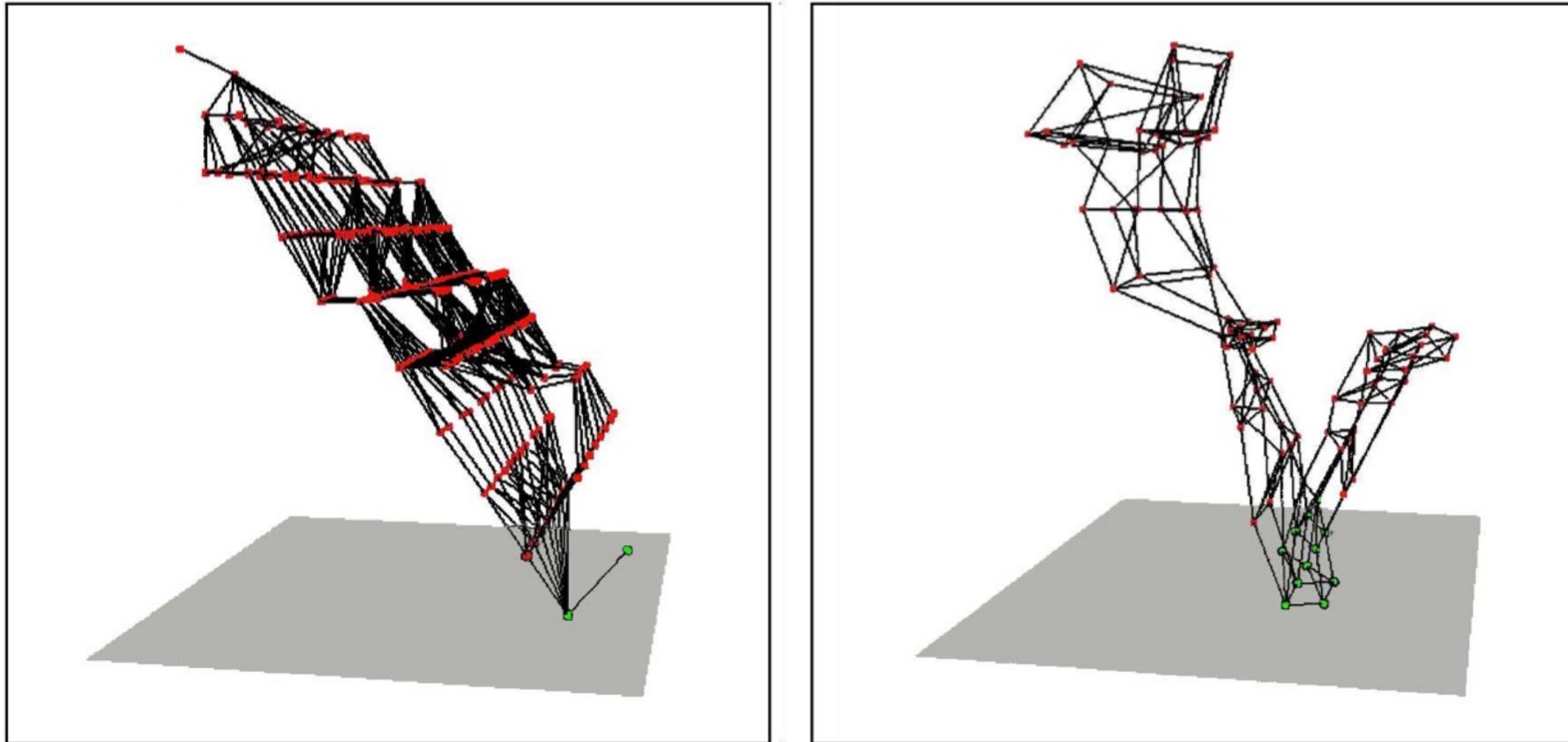
# Two approaches to searching for a plan (a)

Forward (progression) search

(b) Backward (regression) search



# Two state spaces from planning problems with the ignore-delete-lists heuristic



# Definitions of possible refinements for two high-level actions

*Refinement*( *Go*(*Home*, *SFO*),  
    *STEPS*: [*Drive*(*Home*, *SFO**LongTermParking*),  
            *Shuttle*(*SFO**LongTermParking*, *SFO*)] )

*Refinement*( *Go*(*Home*, *SFO*),  
    *STEPS*: [*Taxi*(*Home*, *SFO*)] )

*Refinement*(*Navigate*([*a*, *b*], [*x*, *y*]),  
    *PRECOND*:  $a = x \wedge b = y$   
    *STEPS*: [] )

*Refinement*(*Navigate*([*a*, *b*], [*x*, *y*]),  
    *PRECOND*: *Connected*([*a*, *b*], [*a* - 1, *b*])  
    *STEPS*: [*Left*, *Navigate*([*a* - 1, *b*], [*x*, *y*])] )

*Refinement*(*Navigate*([*a*, *b*], [*x*, *y*]),  
    *PRECOND*: *Connected*([*a*, *b*], [*a* + 1, *b*])  
    *STEPS*: [*Right*, *Navigate*([*a* + 1, *b*], [*x*, *y*])] )

# A breadth-first implementation of hierarchical forward planning search

**function** HIERARCHICAL-SEARCH(*problem*, *hierarchy*) **returns** a solution or *failure*

*frontier*  $\leftarrow$  a FIFO queue with [*Act*] as the only element

**while** *true* **do**

**if** IS-EMPTY(*frontier*) **then return** *failure*

*plan*  $\leftarrow$  POP(*frontier*)      // chooses the shallowest plan in frontier

*hla*  $\leftarrow$  the first HLA in *plan*, or *null* if none

*prefix*, *suffix*  $\leftarrow$  the action subsequences before and after *hla* in *plan*

*outcome*  $\leftarrow$  RESULT(*problem*.INITIAL, *prefix*)

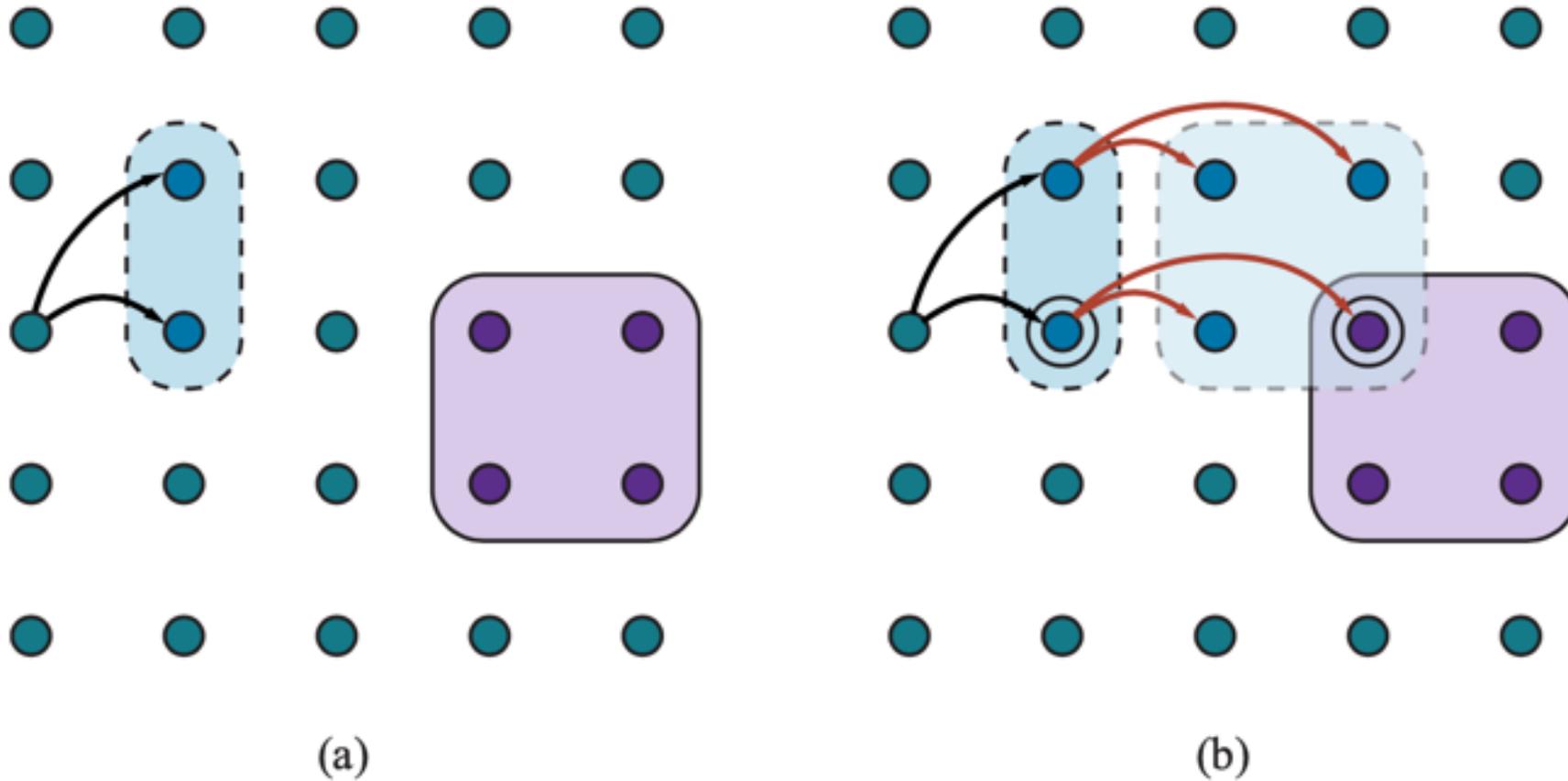
**if** *hla* is *null* **then**      // so *plan* is primitive and *outcome* is its result

**if** *problem*.IS-GOAL(*outcome*) **then return** *plan*

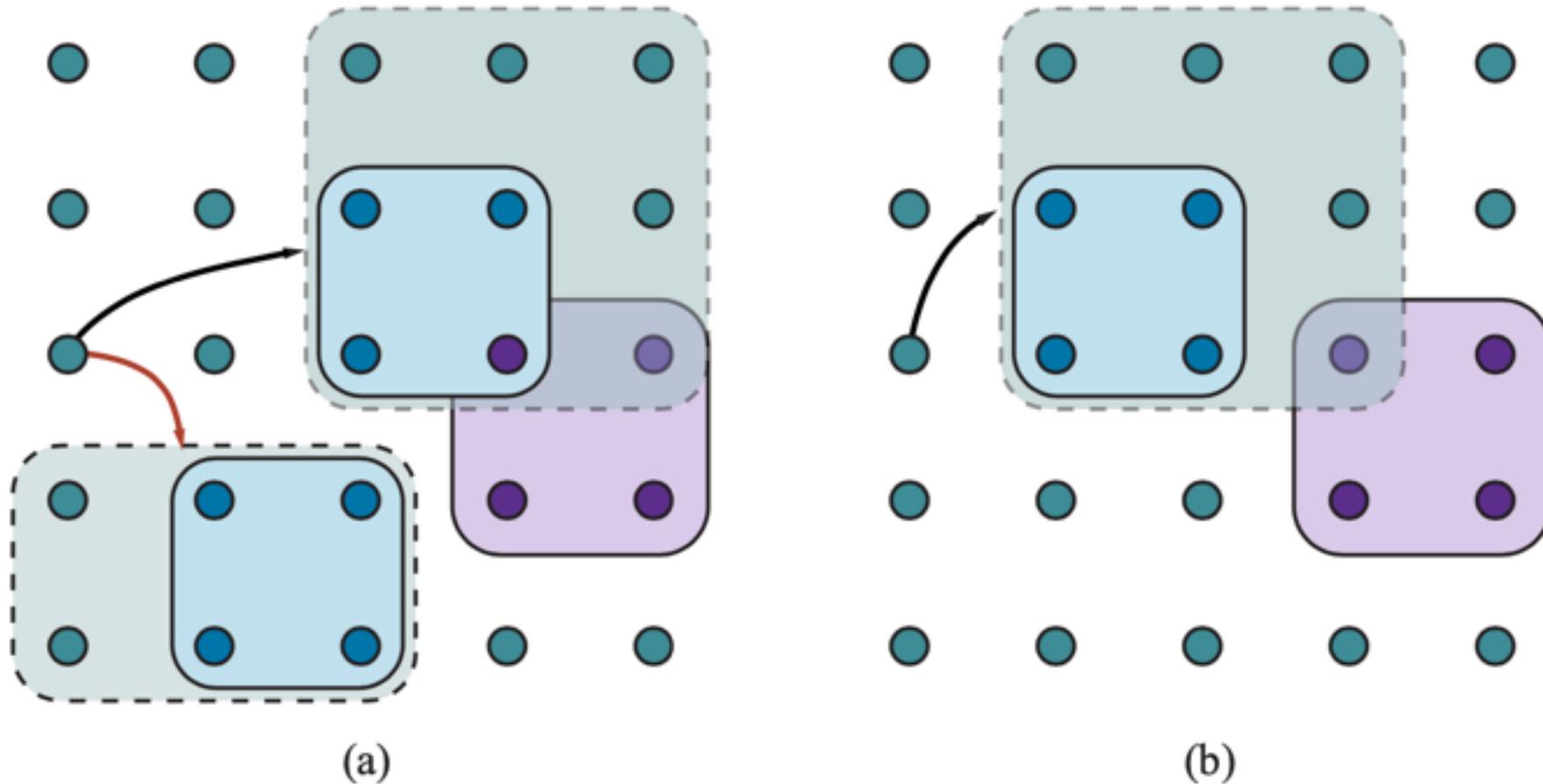
**else for each** *sequence* **in** REFINEMENTS(*hla*, *outcome*, *hierarchy*) **do**

        add APPEND(*prefix*, *sequence*, *suffix*) to *frontier*

# Schematic examples of reachable sets



# Goal achievement for high-level plans with approximate descriptions



# A hierarchical planning algorithm

**function** ANGELIC-SEARCH(*problem*, *hierarchy*, *initialPlan*) **returns** solution or *fail*

*frontier*  $\leftarrow$  a FIFO queue with *initialPlan* as the only element

**while** *true* **do**

**if** EMPTY?(*frontier*) **then return** *fail*

*plan*  $\leftarrow$  POP(*frontier*)      // chooses the shallowest node in *frontier*

**if** REACH<sup>+</sup>(*problem*.INITIAL, *plan*) intersects *problem*.GOAL **then**

**if** *plan* is primitive **then return** *plan*      // REACH<sup>+</sup> is exact for primitive plans

*guaranteed*  $\leftarrow$  REACH<sup>-</sup>(*problem*.INITIAL, *plan*)  $\cap$  *problem*.GOAL

**if** *guaranteed*  $\neq$  { } and MAKING-PROGRESS(*plan*, *initialPlan*) **then**

*finalState*  $\leftarrow$  any element of *guaranteed*

**return** DECOMPOSE(*hierarchy*, *problem*.INITIAL, *plan*, *finalState*)

*hla*  $\leftarrow$  some HLA in *plan*

*prefix*, *suffix*  $\leftarrow$  the action subsequences before and after *hla* in *plan*

*outcome*  $\leftarrow$  RESULT(*problem*.INITIAL, *prefix*)

**for each** *sequence* **in** REFINEMENTS(*hla*, *outcome*, *hierarchy*) **do**

*frontier*  $\leftarrow$  Insert(APPEND(*prefix*, *sequence*, *suffix*), *frontier*)

# A hierarchical planning algorithm

## Decompose solution

**function** DECOMPOSE(*hierarchy*,  $s_0$ , *plan*,  $s_f$ ) **returns** a solution

*solution*  $\leftarrow$  an empty plan

**while** *plan* is not empty **do**

*action*  $\leftarrow$  REMOVE-LAST(*plan*)

$s_i$   $\leftarrow$  a state in  $\text{REACH}^-(s_0, \text{plan})$  such that  $s_f \in \text{REACH}^-(s_i, \text{action})$

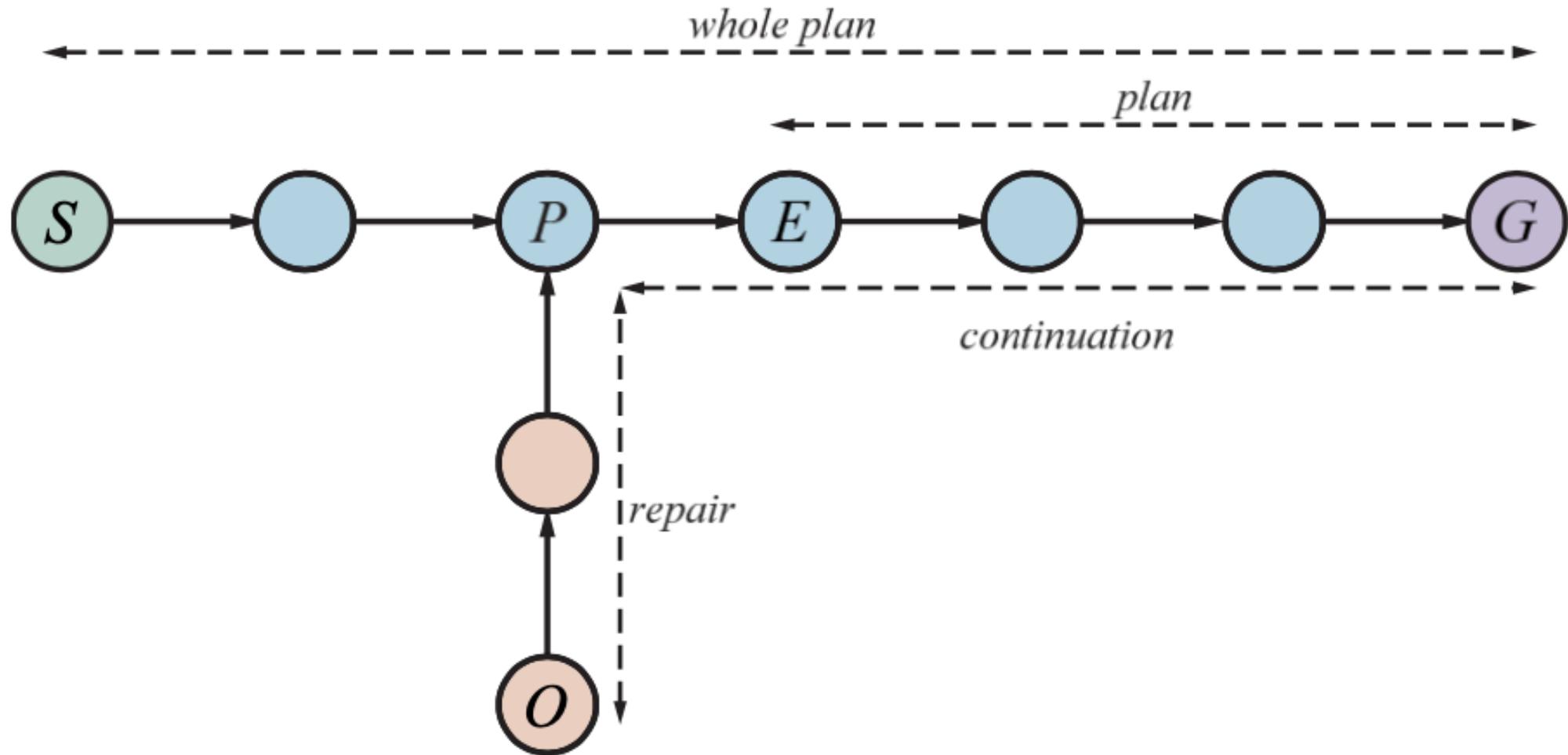
*problem*  $\leftarrow$  a problem with INITIAL =  $s_i$  and GOAL =  $s_f$

*solution*  $\leftarrow$  APPEND(ANGELIC-SEARCH(*problem*, *hierarchy*, *action*), *solution*)

$s_f$   $\leftarrow$   $s_i$

**return** *solution*

At first, the sequence “whole plan” is expected to get the agent from S to G



# A job-shop scheduling problem for assembling two cars, with resource constraints

*Jobs*({*AddEngine1*  $\prec$  *AddWheels1*  $\prec$  *Inspect1* },  
{*AddEngine2*  $\prec$  *AddWheels2*  $\prec$  *Inspect2* })

*Resources*(*EngineHoists*(1), *WheelStations*(1), *Inspectors*(e2), *LugNuts*(500))

*Action*(*AddEngine1*, DURATION:30,  
USE:*EngineHoists*(1))

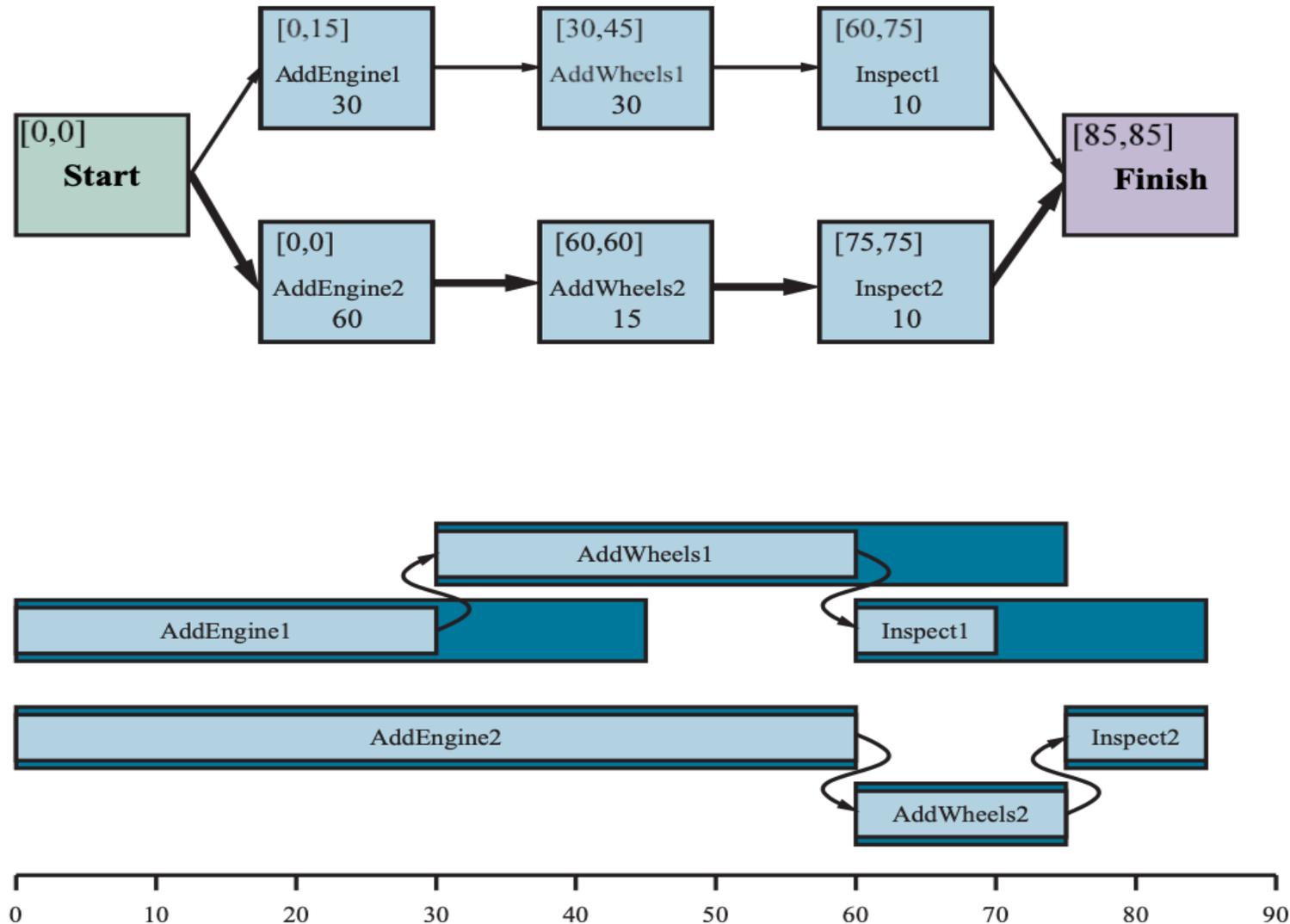
*Action*(*AddEngine2*, DURATION:60,  
USE:*EngineHoists*(1))

*Action*(*AddWheels1*, DURATION:30,  
CONSUME:*LugNuts*(20), USE:*WheelStations*(1))

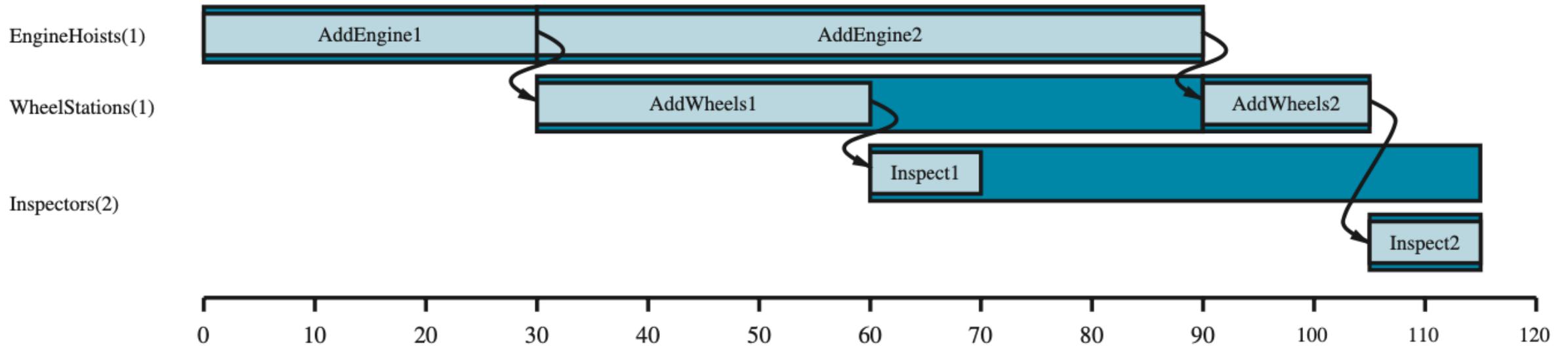
*Action*(*AddWheels2*, DURATION:15,  
CONSUME:*LugNuts*(20), USE:*WheelStations*(1))

*Action*(*Inspect<sub>i</sub>*, DURATION:10,  
USE:*Inspectors*(1))

# A representation of the temporal constraints for the job-shop scheduling problem



# A solution to the job-shop scheduling problem



# **Artificial Intelligence: Uncertain Knowledge and Reasoning**

# Artificial Intelligence:

## 4. Uncertain Knowledge and Reasoning

- **Quantifying Uncertainty**
- **Probabilistic Reasoning**
- **Probabilistic Reasoning over Time**
- **Probabilistic Programming**
- **Making Simple Decisions**
- **Making Complex Decisions**
- **Multiagent Decision Making**

# Quantifying Uncertainty

# DT-Agent

## A Decision-Theoretic Agent that Selects Rational Actions

**function** DT-AGENT(*percept*) **returns** an *action*

**persistent:** *belief\_state*, probabilistic beliefs about the current state of the world  
*action*, the agent's action

update *belief\_state* based on *action* and *percept*

calculate outcome probabilities for actions,

    given action descriptions and current *belief\_state*

select *action* with highest expected utility

    given probabilities of outcomes and utility information

**return** *action*

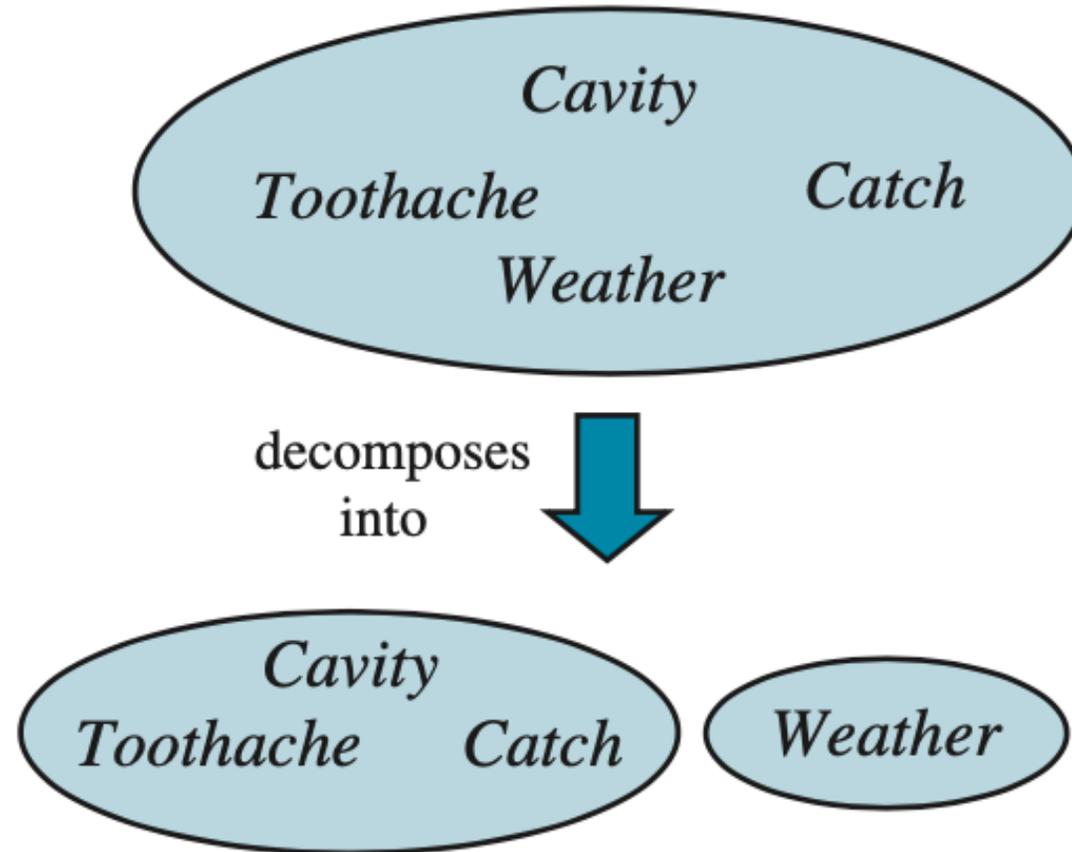
# Agent 1 has inconsistent beliefs

Proposition	Agent 1's belief	Agent 2 bets	Agent 1 bets	Agent 1 payoffs for each outcome			
				$a, b$	$a, \neg b$	$\neg a, b$	$\neg a, \neg b$
$a$	0.4	\$4 on $a$	\$6 on $\neg a$	-\$6	-\$6	\$4	\$4
$b$	0.3	\$3 on $b$	\$7 on $\neg b$	-\$7	\$3	-\$7	\$3
$a \vee b$	0.8	\$2 on $\neg(a \vee b)$	\$8 on $a \vee b$	\$2	\$2	\$2	-\$8
				-\$11	-\$1	-\$1	-\$1

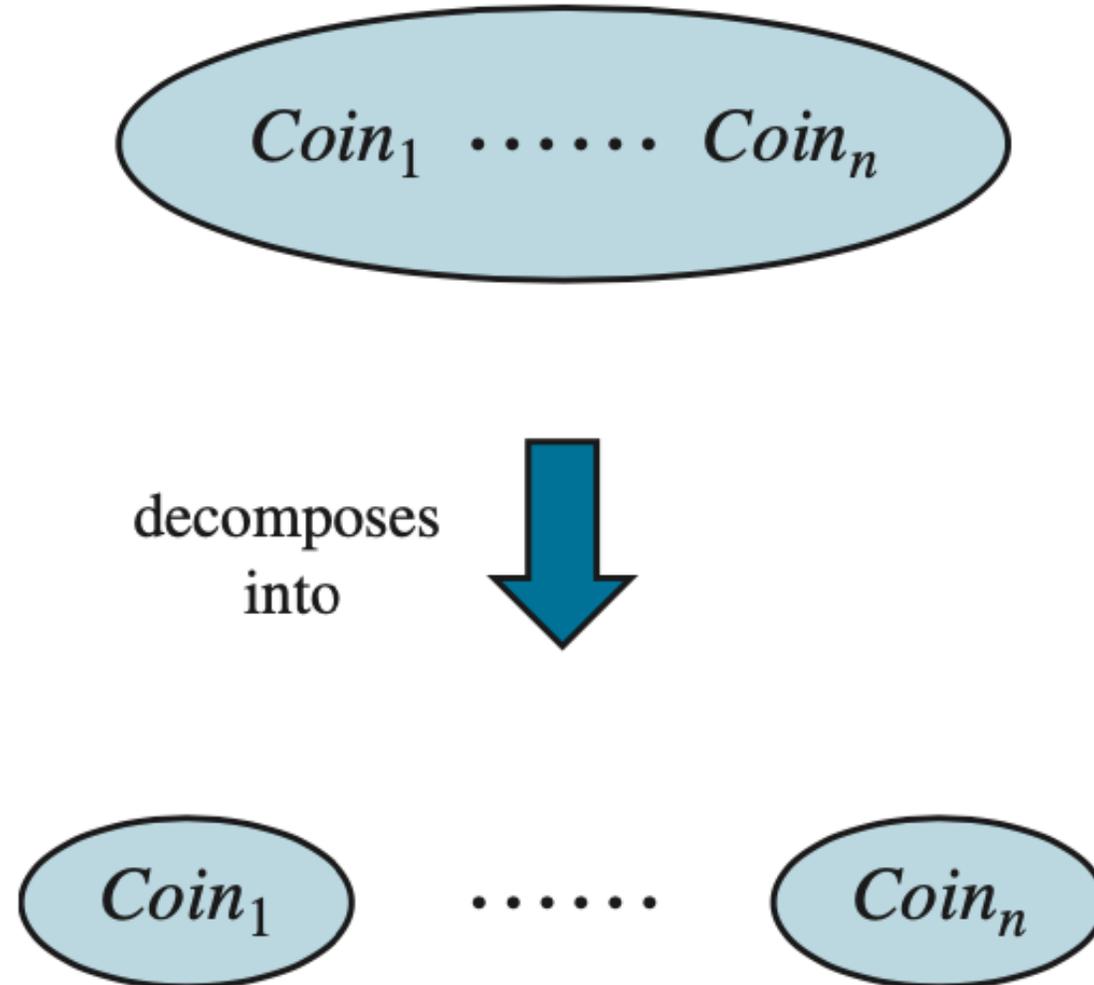
# A full joint distribution for the Toothache, Cavity, Catch world

	<i>toothache</i>		$\neg$ <i>toothache</i>	
	<i>catch</i>	$\neg$ <i>catch</i>	<i>catch</i>	$\neg$ <i>catch</i>
<i>cavity</i>	0.108	0.012	0.072	0.008
$\neg$ <i>cavity</i>	0.016	0.064	0.144	0.576

# Weather and Dental problems are independent



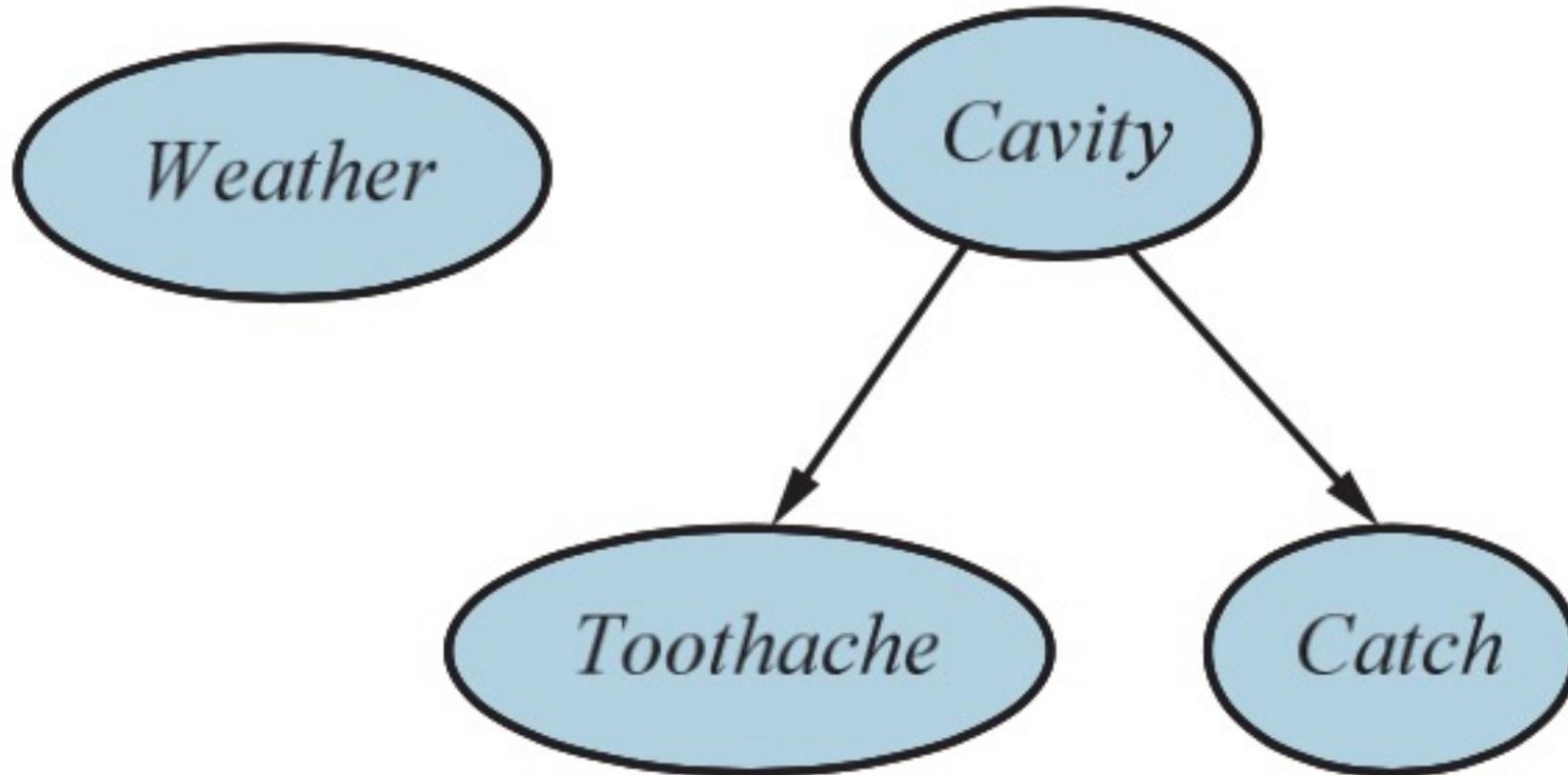
# Coin flips are independent



# Probabilistic Reasoning

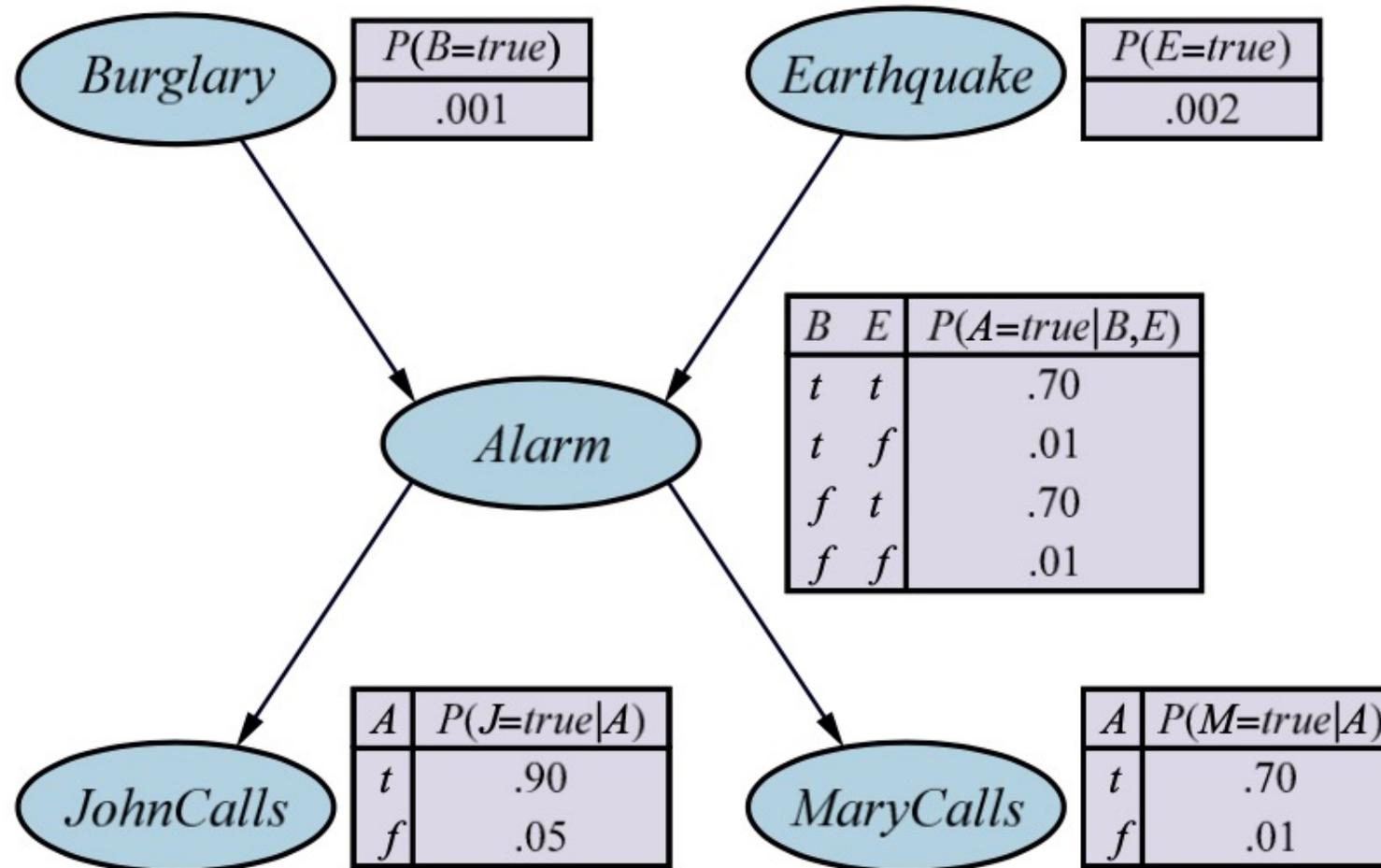
# A Simple Bayesian Network

Weather is independent to the other three variables.  
Toothache and Catch are conditionally independent, given Cavity.



# A Typical Bayesian Network

Topology and the Conditional Probability Tables (CPTs)



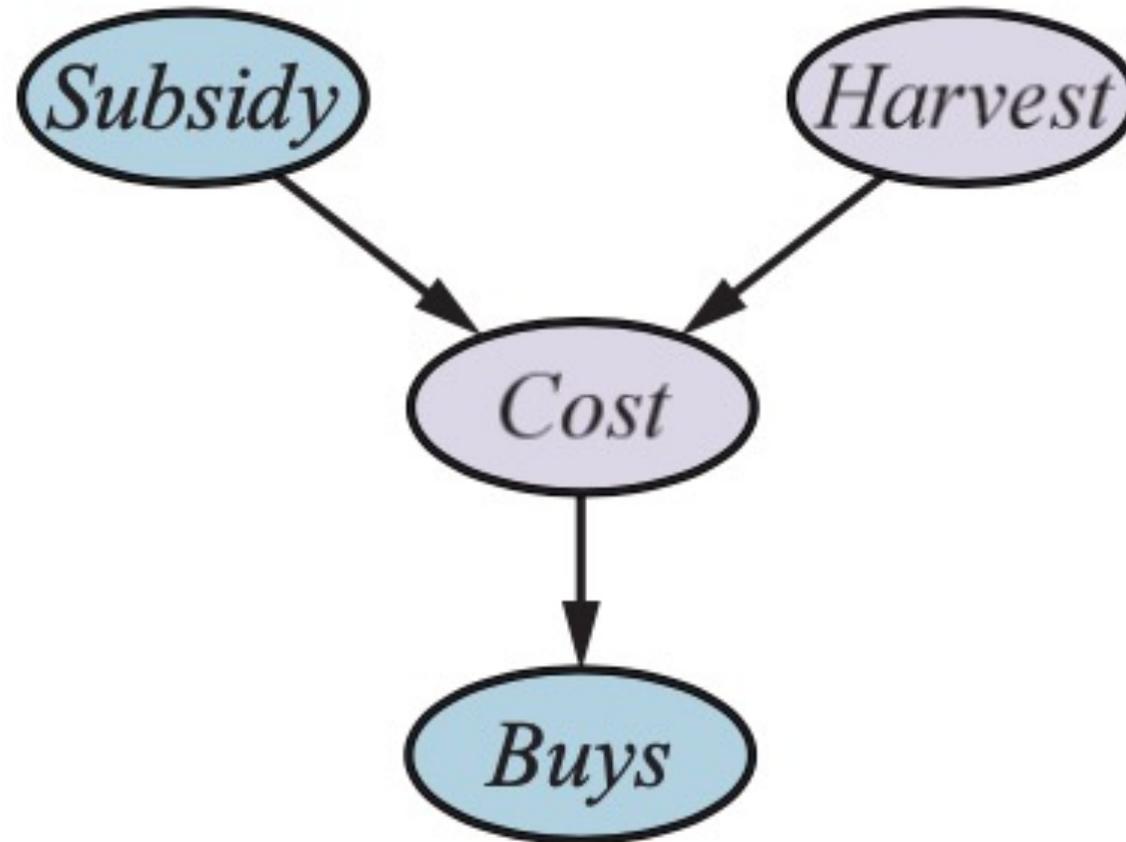
# Conditional Probability Table

for  $P(\text{Fever} \mid \text{Cold, Flu, Malaria})$

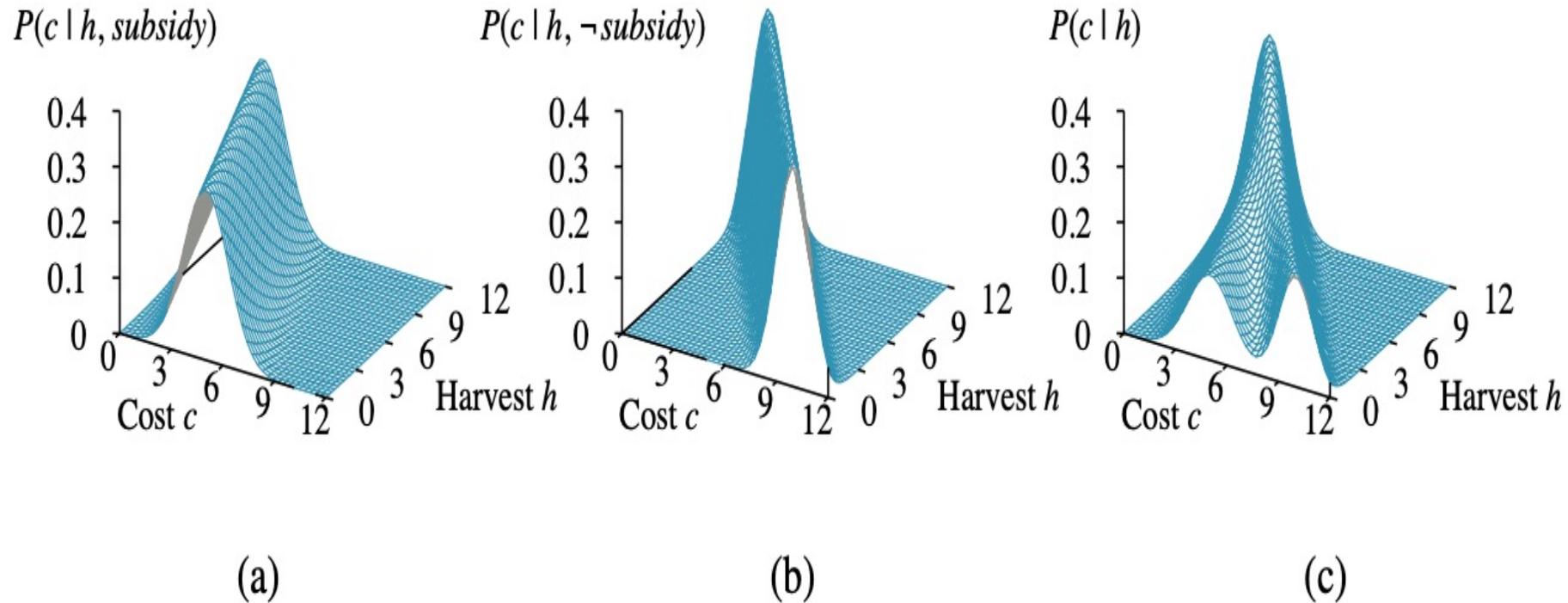
<i>Cold</i>	<i>Flu</i>	<i>Malaria</i>	$P(\text{fever} \mid \cdot)$	$P(\neg\text{fever} \mid \cdot)$
<i>f</i>	<i>f</i>	<i>f</i>	0.0	1.0
<i>f</i>	<i>f</i>	<i>t</i>	0.9	<b>0.1</b>
<i>f</i>	<i>t</i>	<i>f</i>	0.8	<b>0.2</b>
<i>f</i>	<i>t</i>	<i>t</i>	0.98	$0.02 = 0.2 \times 0.1$
<i>t</i>	<i>f</i>	<i>f</i>	0.4	<b>0.6</b>
<i>t</i>	<i>f</i>	<i>t</i>	0.94	$0.06 = 0.6 \times 0.1$
<i>t</i>	<i>t</i>	<i>f</i>	0.88	$0.12 = 0.6 \times 0.2$
<i>t</i>	<i>t</i>	<i>t</i>	0.988	$0.012 = 0.6 \times 0.2 \times 0.1$

# A Simple Network

with discrete variables (Subsidy and Buys)  
and continuous variables (Harvest and Cost )

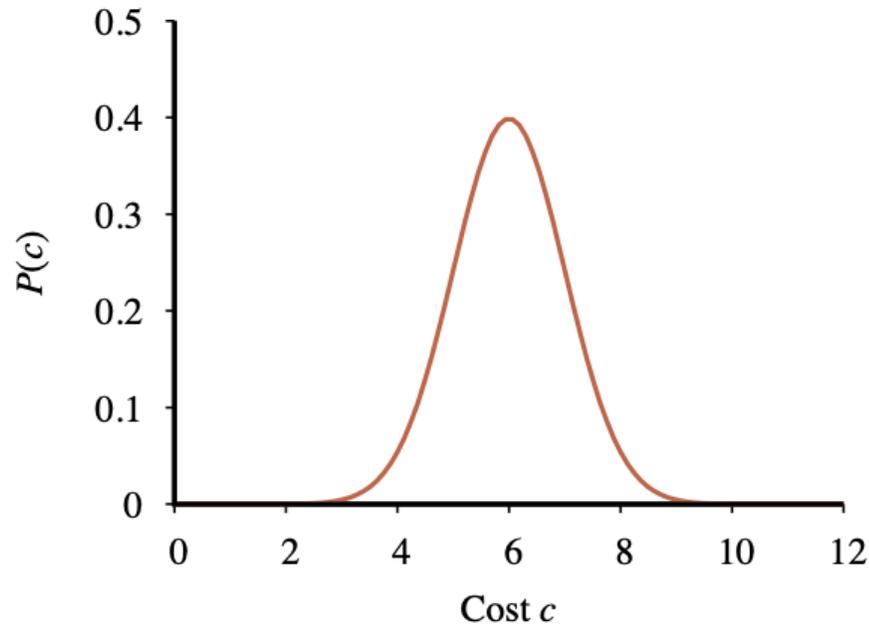


# Probability distribution over Cost as a function of Harvest size

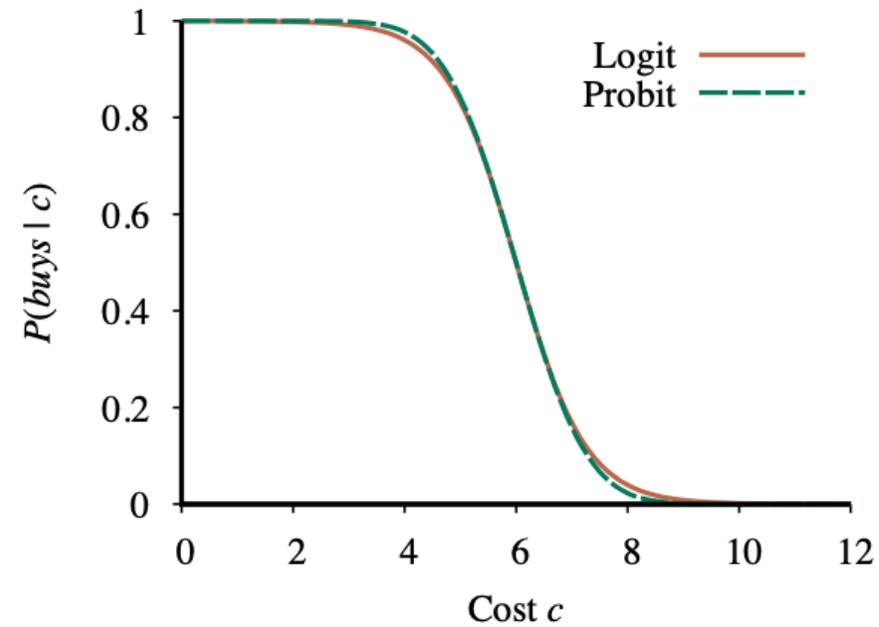


distribution  $P(\text{Cost} | \text{Harvest})$ ,  
obtained by summing over the  
two subsidy cases.

# A normal (Gaussian) distribution for the cost threshold



(a)

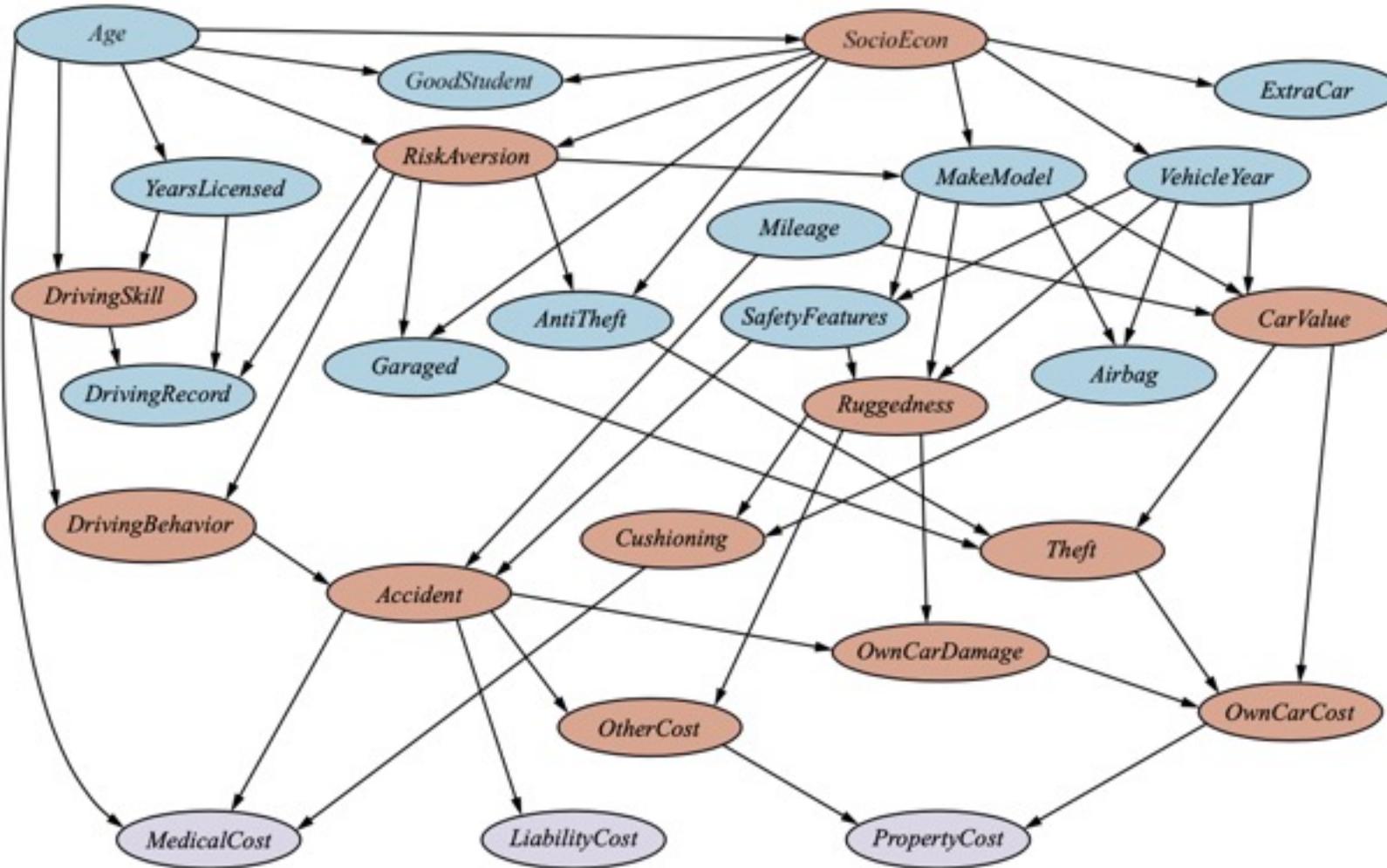


(b)

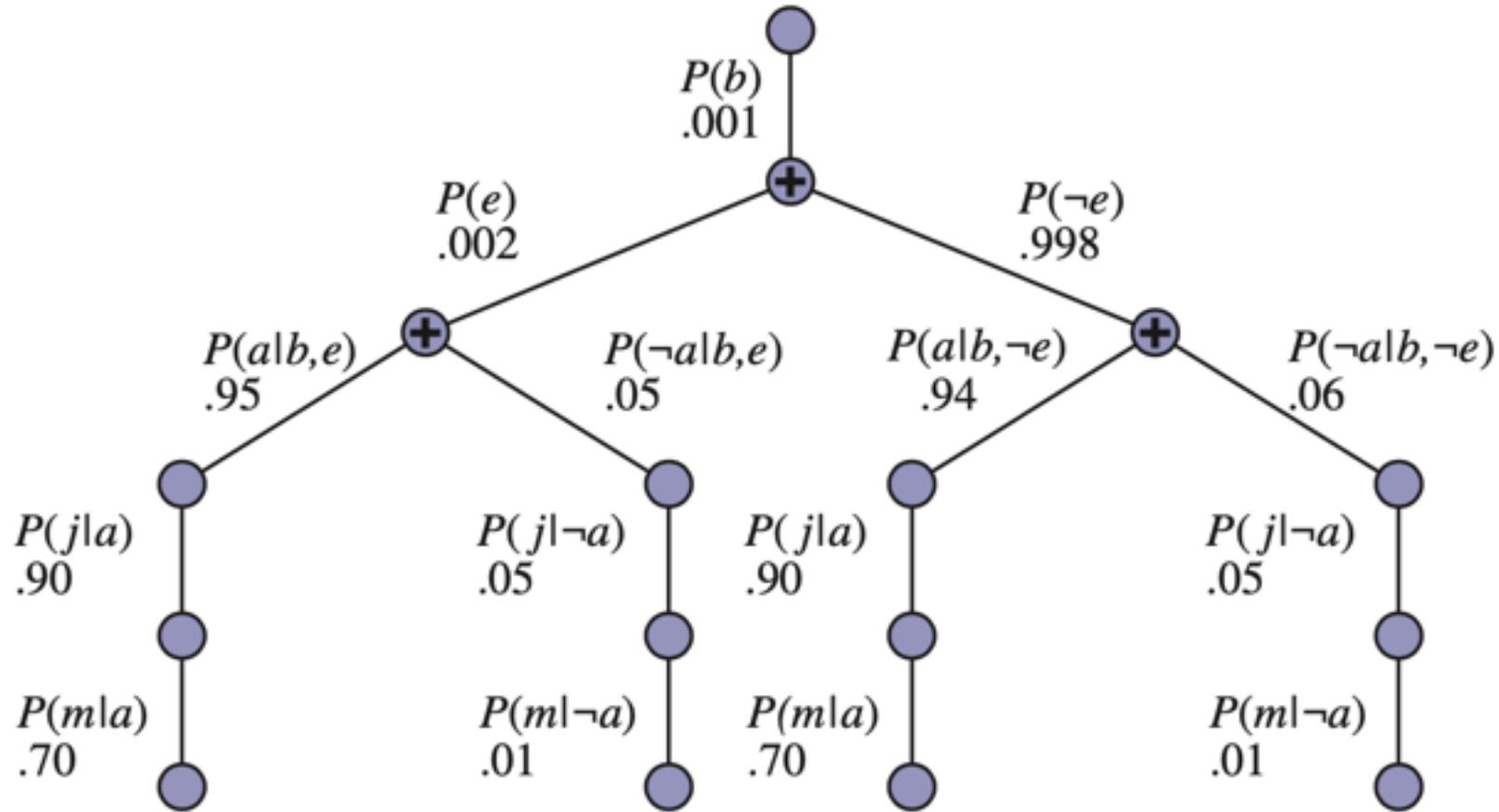
Expit and Probit models for the  
probability of buys given cost

# A Bayesian Network

for evaluating car insurance applications



# The structure of the expression



# The Enumeration Algorithm for Exact Inference in Bayes Nets

**function** ENUMERATION-ASK( $X, \mathbf{e}, bn$ ) **returns** a distribution over  $X$

**inputs:**  $X$ , the query variable

$\mathbf{e}$ , observed values for variables  $\mathbf{E}$

$bn$ , a Bayes net with variables  $vars$

$\mathbf{Q}(X) \leftarrow$  a distribution over  $X$ , initially empty

**for each** value  $x_i$  of  $X$  **do**

$\mathbf{Q}(x_i) \leftarrow$  ENUMERATE-ALL( $vars, \mathbf{e}_{x_i}$ )

where  $\mathbf{e}_{x_i}$  is  $\mathbf{e}$  extended with  $X = x_i$

**return** NORMALIZE( $\mathbf{Q}(X)$ )

**function** ENUMERATE-ALL( $vars, \mathbf{e}$ ) **returns** a real number

**if** EMPTY?( $vars$ ) **then return** 1.0

$V \leftarrow$  FIRST( $vars$ )

**if**  $V$  is an evidence variable with value  $v$  in  $\mathbf{e}$

**then return**  $P(v \mid parents(V)) \times$  ENUMERATE-ALL(REST( $vars$ ),  $\mathbf{e}$ )

**else return**  $\sum_v P(v \mid parents(V)) \times$  ENUMERATE-ALL(REST( $vars$ ),  $\mathbf{e}_v$ )

where  $\mathbf{e}_v$  is  $\mathbf{e}$  extended with  $V = v$

# Pointwise Multiplication

$$f(X, Y) \times g(Y, Z) = h(X, Y, Z)$$

$X$	$Y$	$f(X, Y)$	$Y$	$Z$	$g(Y, Z)$	$X$	$Y$	$Z$	$h(X, Y, Z)$
$t$	$t$	.3	$t$	$t$	.2	$t$	$t$	$t$	$.3 \times .2 = .06$
$t$	$f$	.7	$t$	$f$	.8	$t$	$t$	$f$	$.3 \times .8 = .24$
$f$	$t$	.9	$f$	$t$	.6	$t$	$f$	$t$	$.7 \times .6 = .42$
$f$	$f$	.1	$f$	$f$	.4	$t$	$f$	$f$	$.7 \times .4 = .28$
						$f$	$t$	$t$	$.9 \times .2 = .18$
						$f$	$t$	$f$	$.9 \times .8 = .72$
						$f$	$f$	$t$	$.1 \times .6 = .06$
						$f$	$f$	$f$	$.1 \times .4 = .04$

# The Variable Elimination Algorithm for Exact Inference in Bayes Nets

**function** ELIMINATION-ASK( $X, \mathbf{e}, bn$ ) **returns** a distribution over  $X$

**inputs:**  $X$ , the query variable

$\mathbf{e}$ , observed values for variables  $\mathbf{E}$

$bn$ , a Bayesian network with variables  $vars$

$factors \leftarrow []$

**for each**  $V$  **in** ORDER( $vars$ ) **do**

$factors \leftarrow [\text{MAKE-FACTOR}(V, \mathbf{e})] + factors$

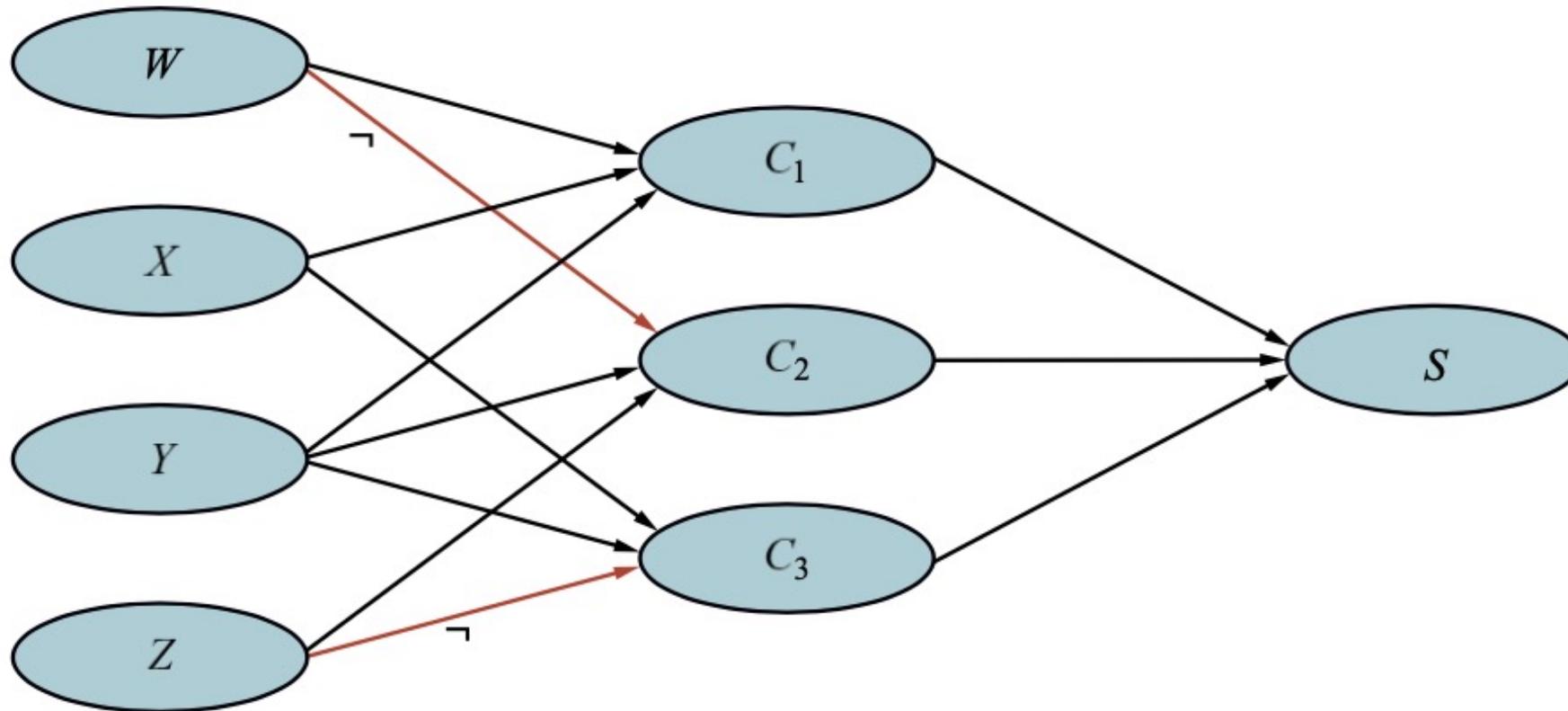
**if**  $V$  is a hidden variable **then**  $factors \leftarrow \text{SUM-OUT}(V, factors)$

**return** NORMALIZE(PPOINTWISE-PRODUCT( $factors$ ))

# Bayes Net Encoding

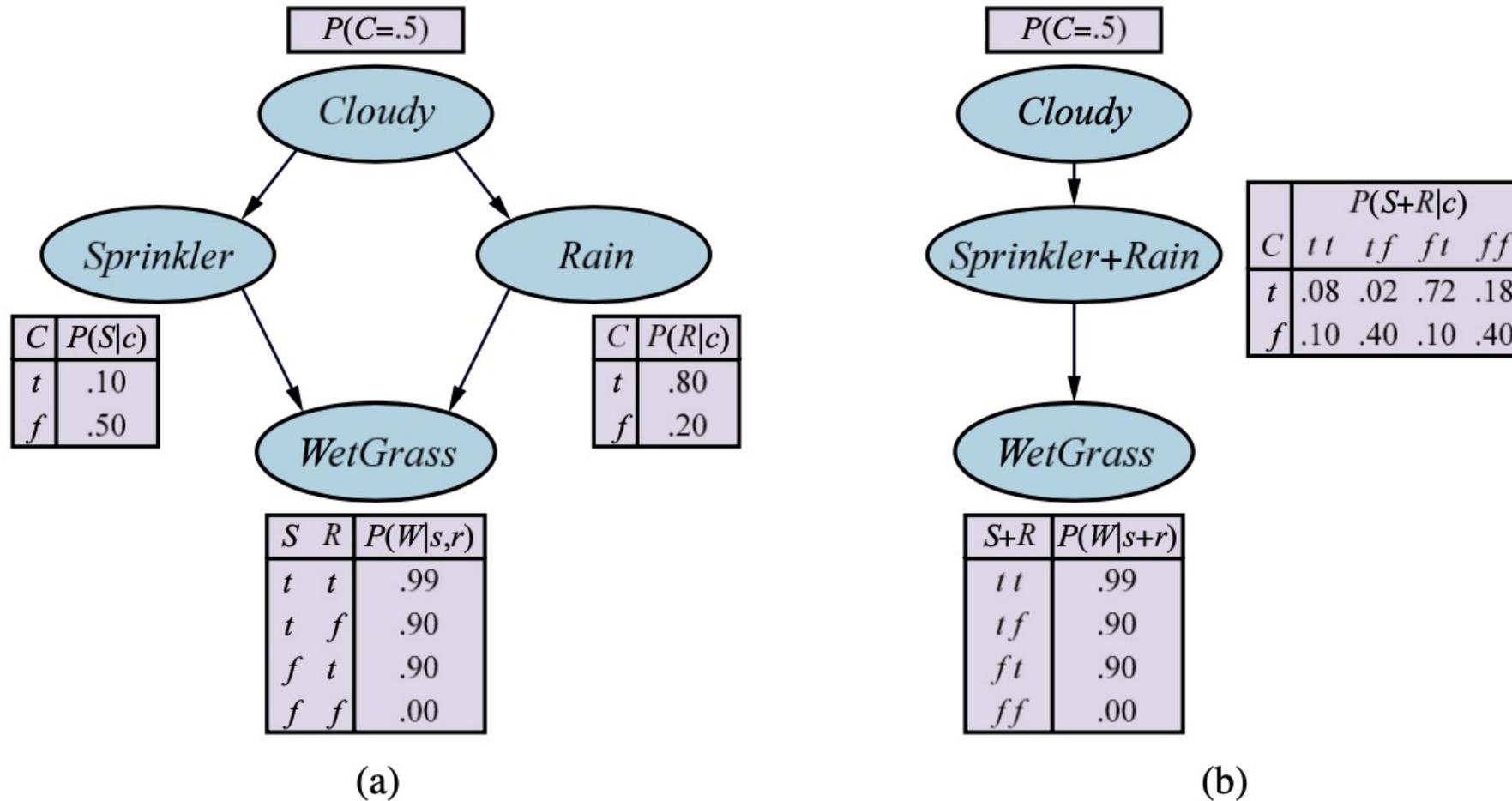
of the 3-CNF (Conjunctive Normal Form) Sentence

$$(W \vee X \vee Y) \wedge (\neg W \vee Y \vee Z) \wedge (X \vee Y \vee \neg Z)$$



# Multiply Connected Network

(b) A clustered equivalent



# A Sampling Algorithm

that generates events from a Bayesian network

**function** PRIOR-SAMPLE( $bn$ ) **returns** an event sampled from the prior specified by  $bn$

**inputs:**  $bn$ , a Bayesian network specifying joint distribution  $\mathbf{P}(X_1, \dots, X_n)$

$\mathbf{x} \leftarrow$  an event with  $n$  elements

**for each** variable  $X_i$  **in**  $X_1, \dots, X_n$  **do**

$\mathbf{x}[i] \leftarrow$  a random sample from  $\mathbf{P}(X_i \mid \text{parents}(X_i))$

**return**  $\mathbf{x}$

# The Rejection-Sampling Algorithm

for answering queries given evidence in a Bayesian network

**function** REJECTION-SAMPLING( $X, \mathbf{e}, bn, N$ ) **returns** an estimate of  $\mathbf{P}(X | \mathbf{e})$

**inputs:**  $X$ , the query variable

$\mathbf{e}$ , observed values for variables  $\mathbf{E}$

$bn$ , a Bayesian network

$N$ , the total number of samples to be generated

**local variables:**  $\mathbf{C}$ , a vector of counts for each value of  $X$ , initially zero

**for**  $j = 1$  **to**  $N$  **do**

$\mathbf{x} \leftarrow$  PRIOR-SAMPLE( $bn$ )

**if**  $\mathbf{x}$  is consistent with  $\mathbf{e}$  **then**

$\mathbf{C}[j] \leftarrow \mathbf{C}[j]+1$  where  $x_j$  is the value of  $X$  in  $\mathbf{x}$

**return** NORMALIZE( $\mathbf{C}$ )

# The Likelihood-Weighting Algorithm for inference in Bayesian networks

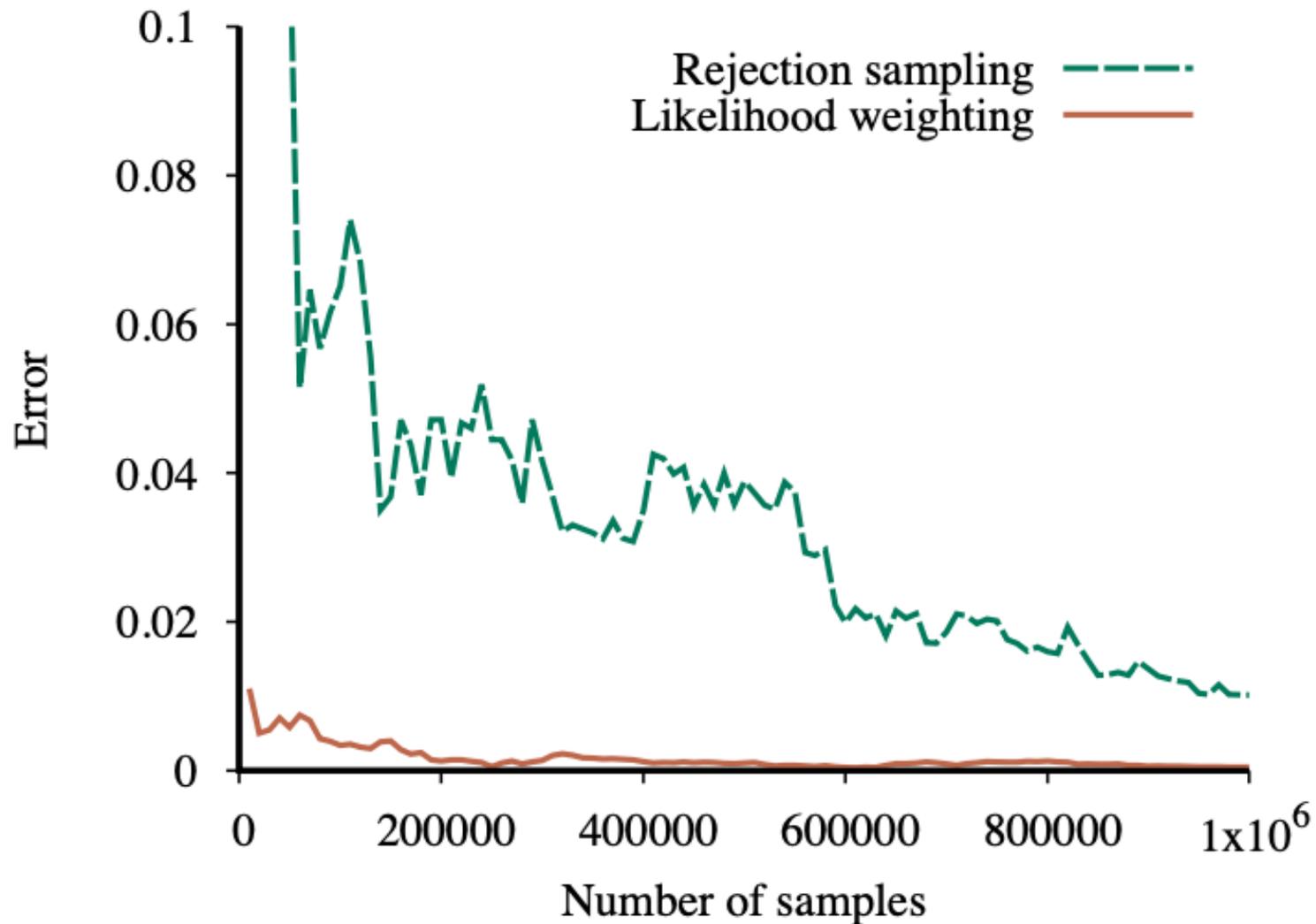
**function** LIKELIHOOD-WEIGHTING( $X, \mathbf{e}, bn, N$ ) **returns** an estimate of  $\mathbf{P}(X | \mathbf{e})$   
**inputs:**  $X$ , the query variable  
           $\mathbf{e}$ , observed values for variables  $\mathbf{E}$   
           $bn$ , a Bayesian network specifying joint distribution  $\mathbf{P}(X_1, \dots, X_n)$   
           $N$ , the total number of samples to be generated  
**local variables:**  $\mathbf{W}$ , a vector of weighted counts for each value of  $X$ , initially zero

**for**  $j = 1$  **to**  $N$  **do**  
     $\mathbf{x}, w \leftarrow$  WEIGHTED-SAMPLE( $bn, \mathbf{e}$ )  
     $\mathbf{W}[j] \leftarrow \mathbf{W}[j] + w$  where  $x_j$  is the value of  $X$  in  $\mathbf{x}$   
**return** NORMALIZE( $\mathbf{W}$ )

**function** WEIGHTED-SAMPLE( $bn, \mathbf{e}$ ) **returns** an event and a weight

$w \leftarrow 1$ ;  $\mathbf{x} \leftarrow$  an event with  $n$  elements, with values fixed from  $\mathbf{e}$   
**for**  $i = 1$  **to**  $n$  **do**  
    **if**  $X_i$  is an evidence variable with value  $x_{ij}$  in  $\mathbf{e}$   
        **then**  $w \leftarrow w \times P(X_i = x_{ij} | \text{parents}(X_i))$   
        **else**  $\mathbf{x}[i] \leftarrow$  a random sample from  $\mathbf{P}(X_i | \text{parents}(X_i))$   
**return**  $\mathbf{x}, w$

# Performance of rejection sampling and likelihood weighting on the insurance network



# The Gibbs Sampling Algorithm

## for approximate inference in Bayes nets

**function** GIBBS-ASK( $X, \mathbf{e}, bn, N$ ) **returns** an estimate of  $\mathbf{P}(X | \mathbf{e})$

**local variables:**  $\mathbf{C}$ , a vector of counts for each value of  $X$ , initially zero  
 $\mathbf{Z}$ , the nonevidence variables in  $bn$   
 $\mathbf{x}$ , the current state of the network, initialized from  $\mathbf{e}$

initialize  $\mathbf{x}$  with random values for the variables in  $\mathbf{Z}$

**for**  $k = 1$  **to**  $N$  **do**

**choose** any variable  $Z_i$  from  $\mathbf{Z}$  according to any distribution  $\rho(i)$

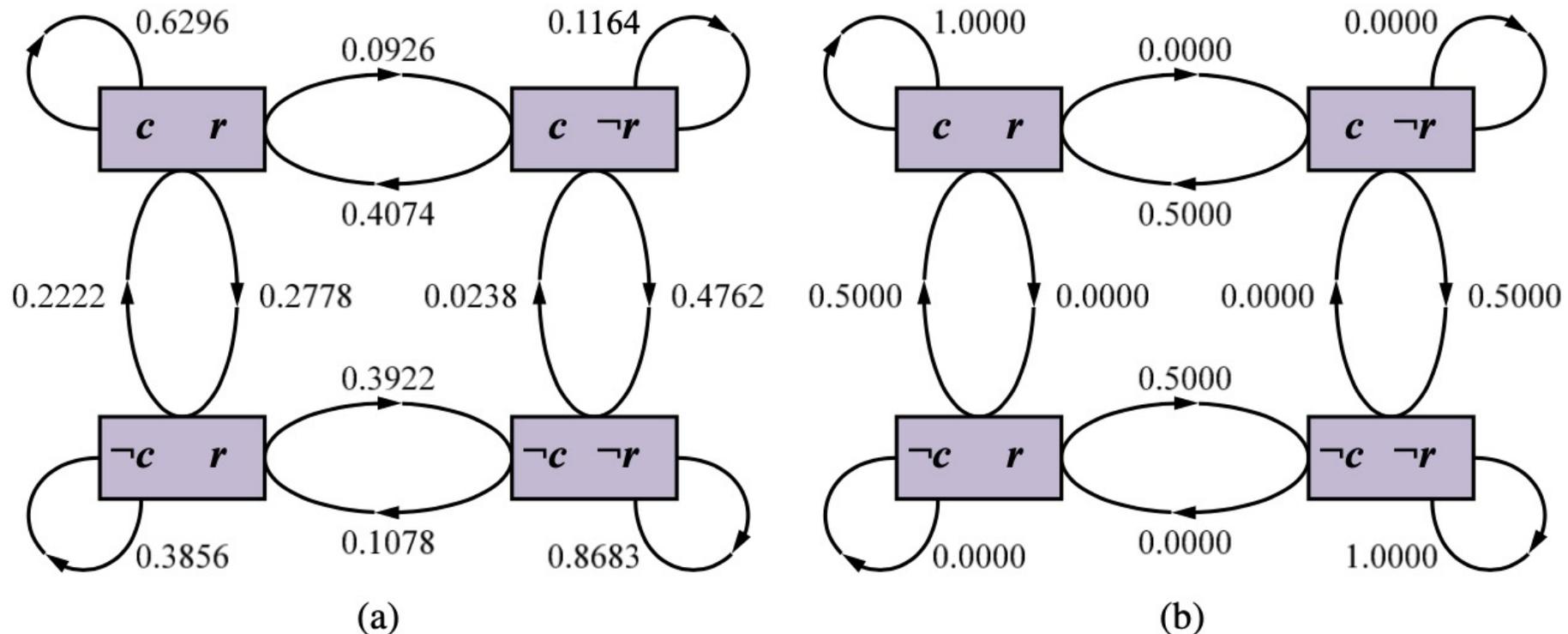
    set the value of  $Z_i$  in  $\mathbf{x}$  by sampling from  $\mathbf{P}(Z_i | mb(Z_i))$

$\mathbf{C}[j] \leftarrow \mathbf{C}[j] + 1$  where  $x_j$  is the value of  $X$  in  $\mathbf{x}$

**return** NORMALIZE( $\mathbf{C}$ )

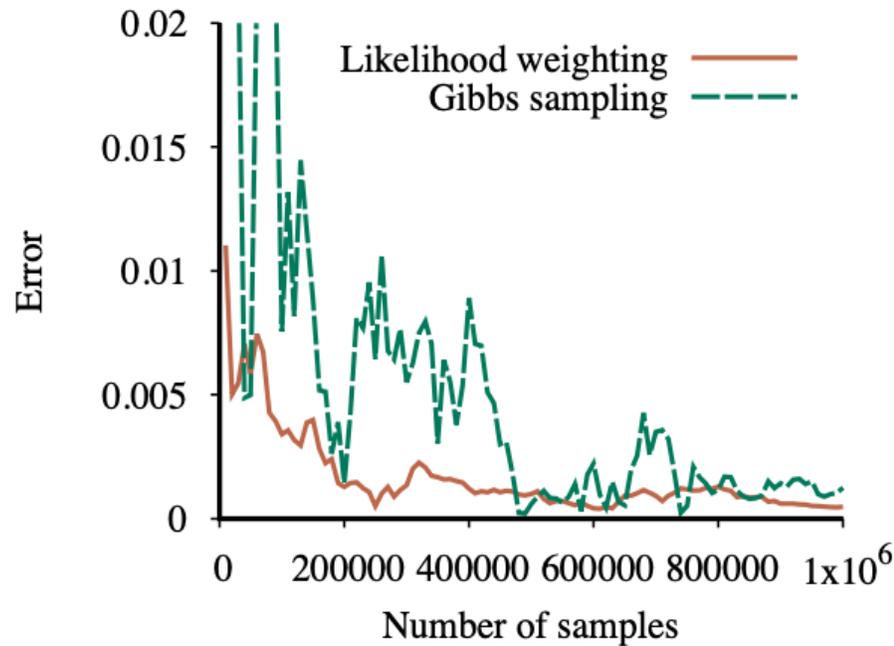
# The States and Transition Probabilities of the Markov Chain

for the query  $\mathbf{P}(\text{Rain} \mid \text{Sprinkler} = \text{true}, \text{WetGrass} = \text{true})$



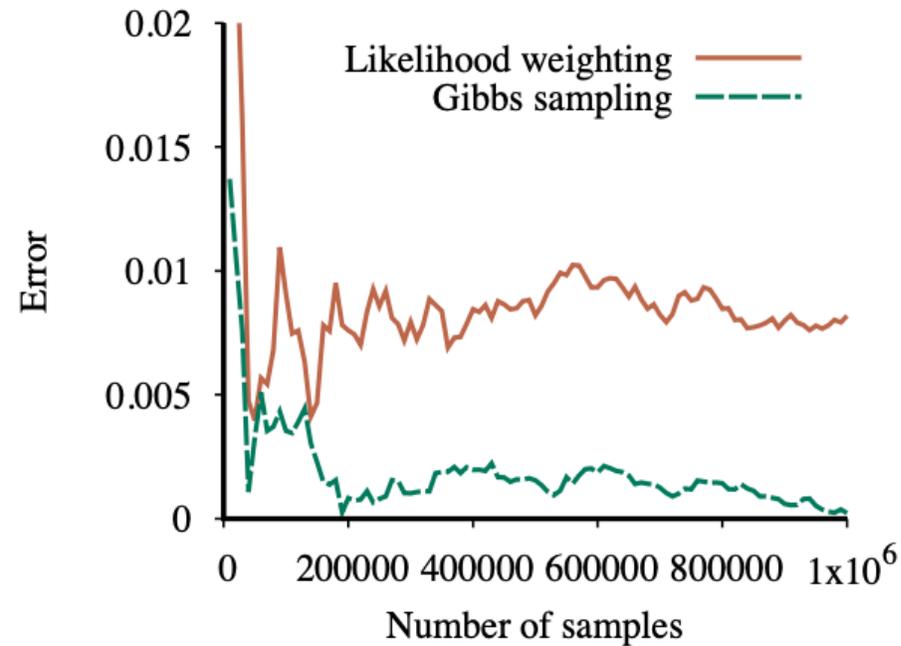
**Transition Probabilities  
when the CPT for Rain constrains it  
to have the same value as Cloudy**

# Performance of Gibbs sampling compared to likelihood weighting on the car insurance network



(a)

for the standard query on PropertyCost

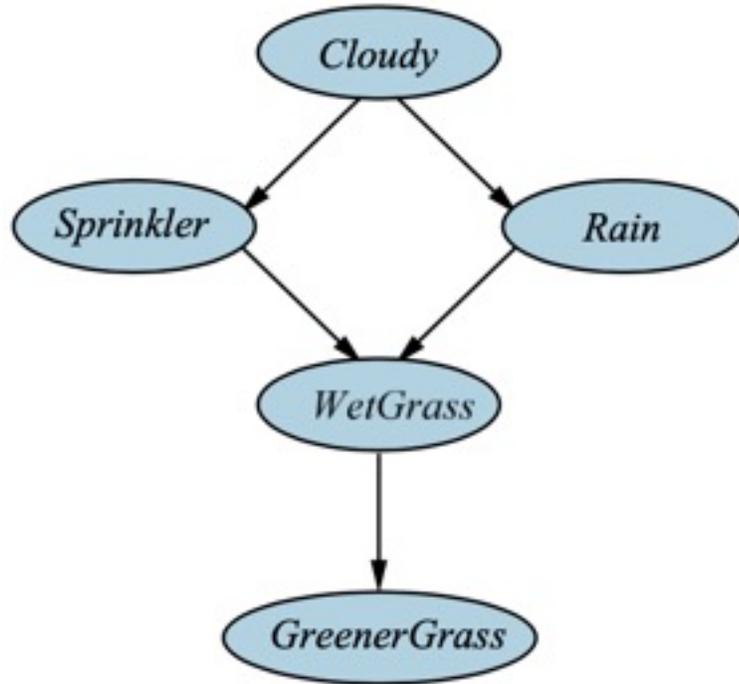


(b)

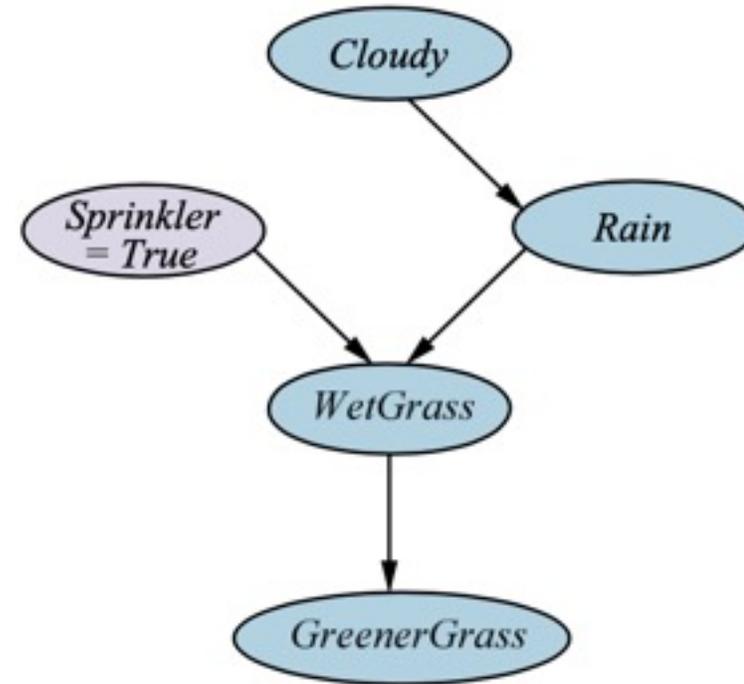
for the case where the output variables are observed and Age is the query variable

# A Causal Bayesian Network

representing cause-effect relations among five variables



(a)



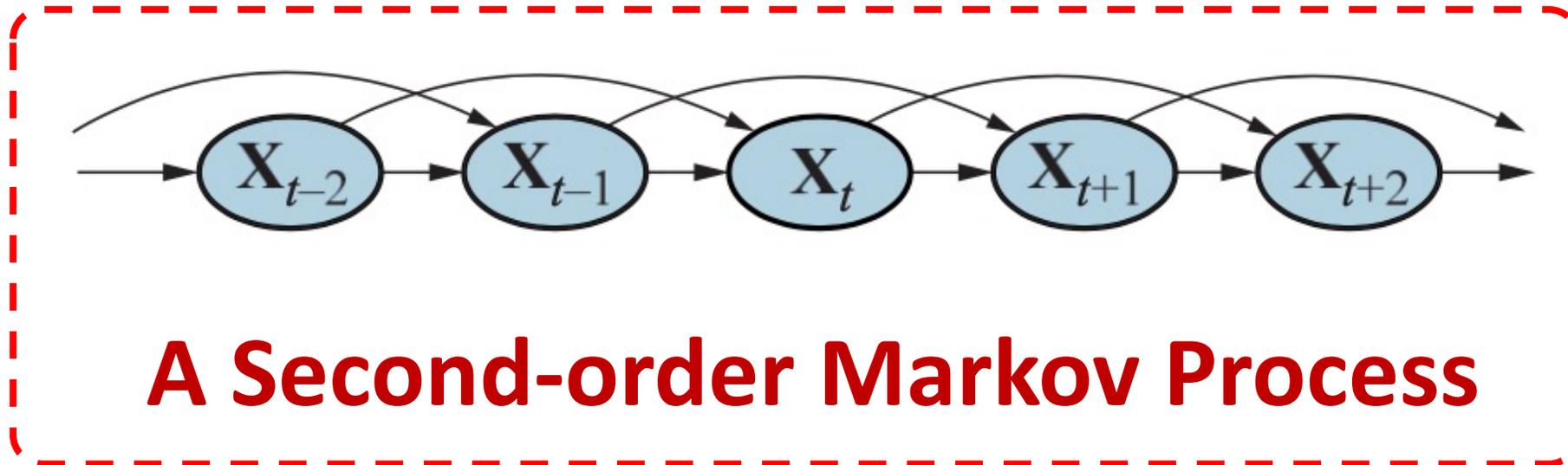
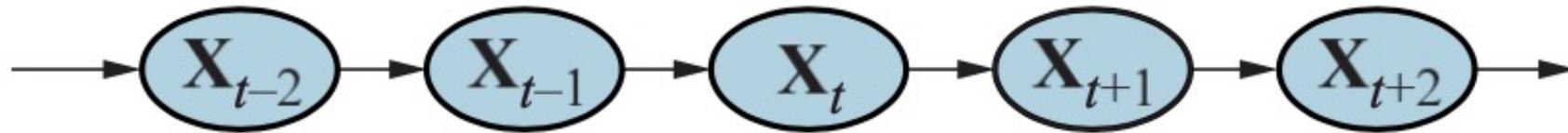
(b)

The network after performing the action  
"turn Sprinkler on."

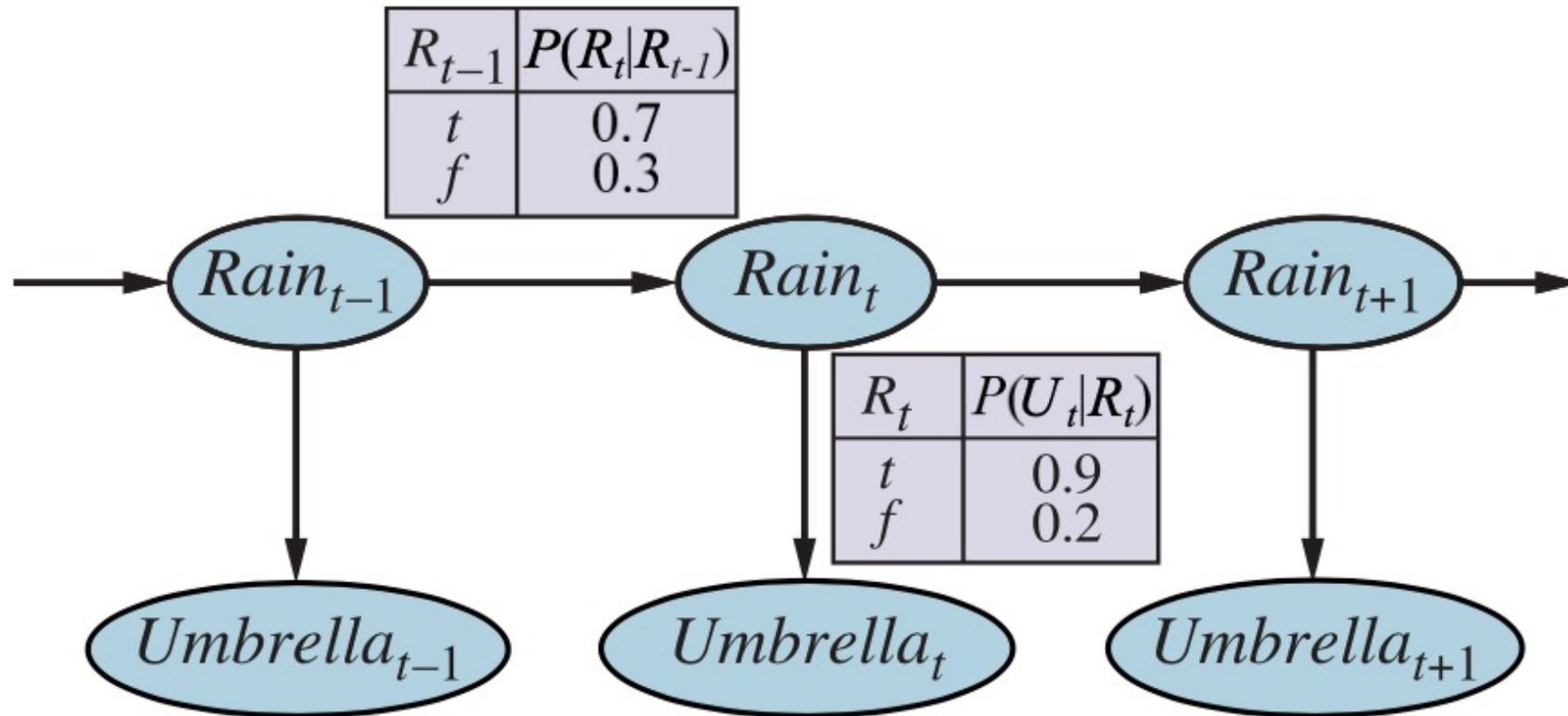
# Probabilistic Reasoning over Time

# Bayesian network structure

corresponding to a First-order Markov Process  
with state defined by the variables  $X_t$ .

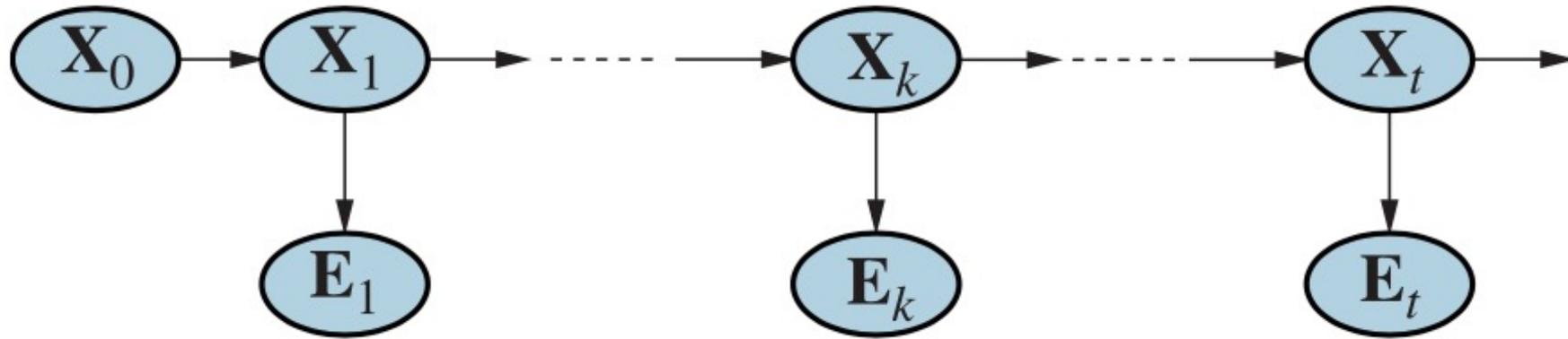


# Bayesian Network Structure and Conditional Distributions describing the umbrella world



# Smoothing computes $P(X_k | e_{1:t})$

the posterior distribution of the state at some past time  $k$  given a complete sequence of observations from 1 to  $t$ .



# The Forward–Backward Algorithm for Smoothing

**function** FORWARD-BACKWARD( $\mathbf{ev}$ ,  $prior$ ) **returns** a vector of probability distributions

**inputs:**  $\mathbf{ev}$ , a vector of evidence values for steps  $1, \dots, t$

$prior$ , the prior distribution on the initial state,  $\mathbf{P}(\mathbf{X}_0)$

**local variables:**  $\mathbf{fv}$ , a vector of forward messages for steps  $0, \dots, t$

$\mathbf{b}$ , a representation of the backward message, initially all 1s

$\mathbf{sv}$ , a vector of smoothed estimates for steps  $1, \dots, t$

$\mathbf{fv}[0] \leftarrow prior$

**for**  $i = 1$  **to**  $t$  **do**

$\mathbf{fv}[i] \leftarrow \text{FORWARD}(\mathbf{fv}[i - 1], \mathbf{ev}[i])$

**for**  $i = t$  **down to**  $1$  **do**

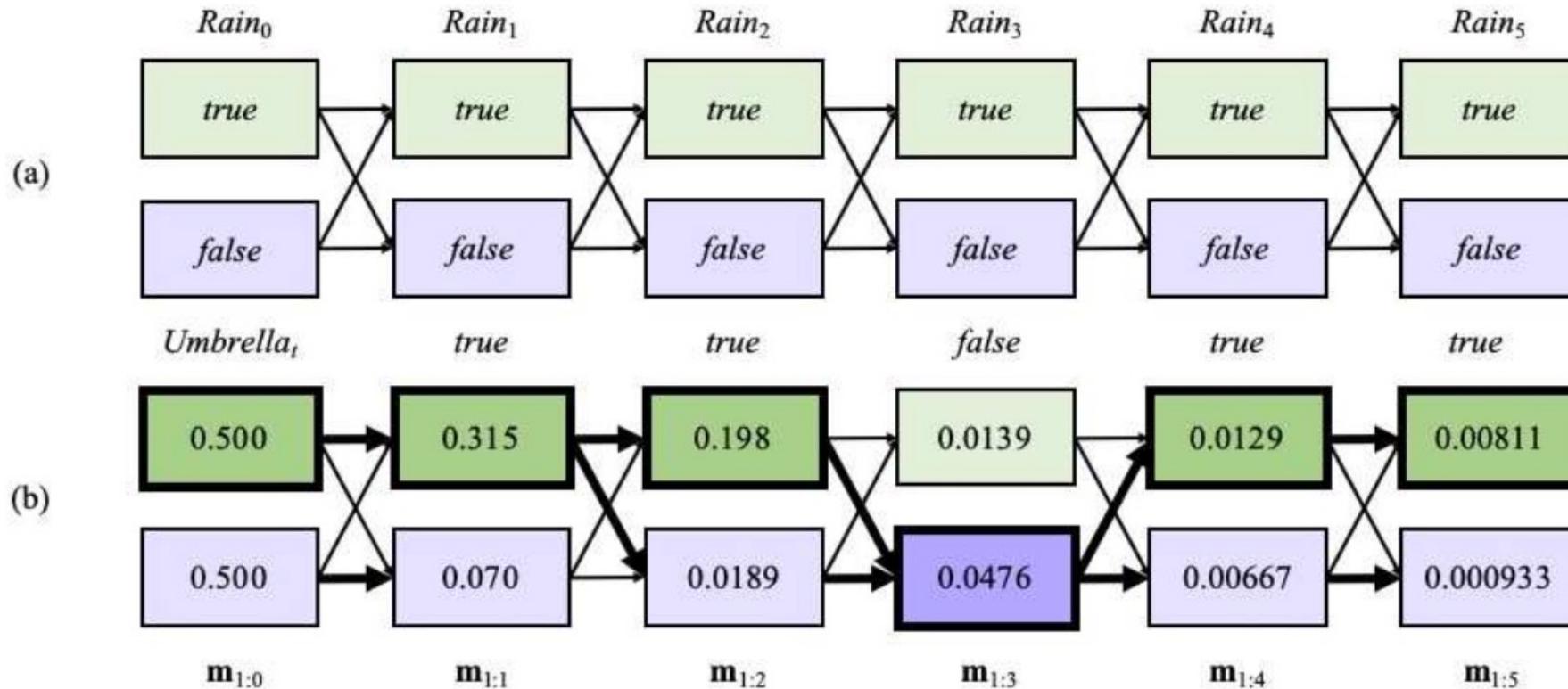
$\mathbf{sv}[i] \leftarrow \text{NORMALIZE}(\mathbf{fv}[i] \times \mathbf{b})$

$\mathbf{b} \leftarrow \text{BACKWARD}(\mathbf{b}, \mathbf{ev}[i])$

**return**  $\mathbf{sv}$

# Possible state sequences for $Rain_t$ can

be viewed as paths through a graph of the possible states at each time step



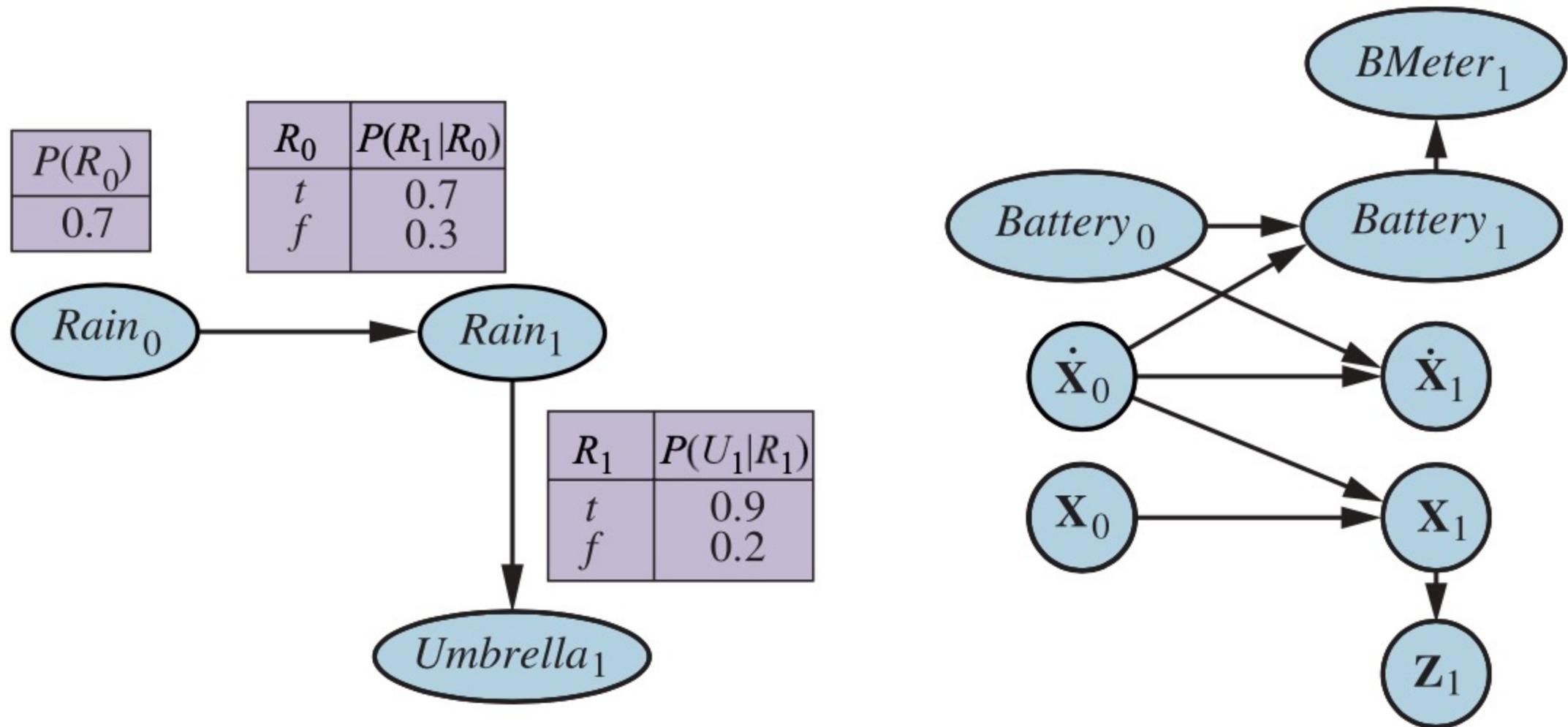
Operation of the Viterbi algorithm  
for the umbrella observation sequence  $[true, true, false, true, true]$

# Algorithm for Smoothing with a Fixed Time Lag of $d$ Step

**function** FIXED-LAG-SMOOTHING( $e_t, hmm, d$ ) **returns** a distribution over  $\mathbf{X}_{t-d}$   
**inputs:**  $e_t$ , the current evidence for time step  $t$   
 $hmm$ , a hidden Markov model with  $S \times S$  transition matrix  $\mathbf{T}$   
 $d$ , the length of the lag for smoothing  
**persistent:**  $t$ , the current time, initially 1  
 $\mathbf{f}$ , the forward message  $\mathbf{P}(X_t | e_{1:t})$ , initially  $hmm.PRIOR$   
 $\mathbf{B}$ , the  $d$ -step backward transformation matrix, initially the identity matrix  
 $e_{t-d:t}$ , double-ended list of evidence from  $t - d$  to  $t$ , initially empty  
**local variables:**  $\mathbf{O}_{t-d}, \mathbf{O}_t$ , diagonal matrices containing the sensor model information

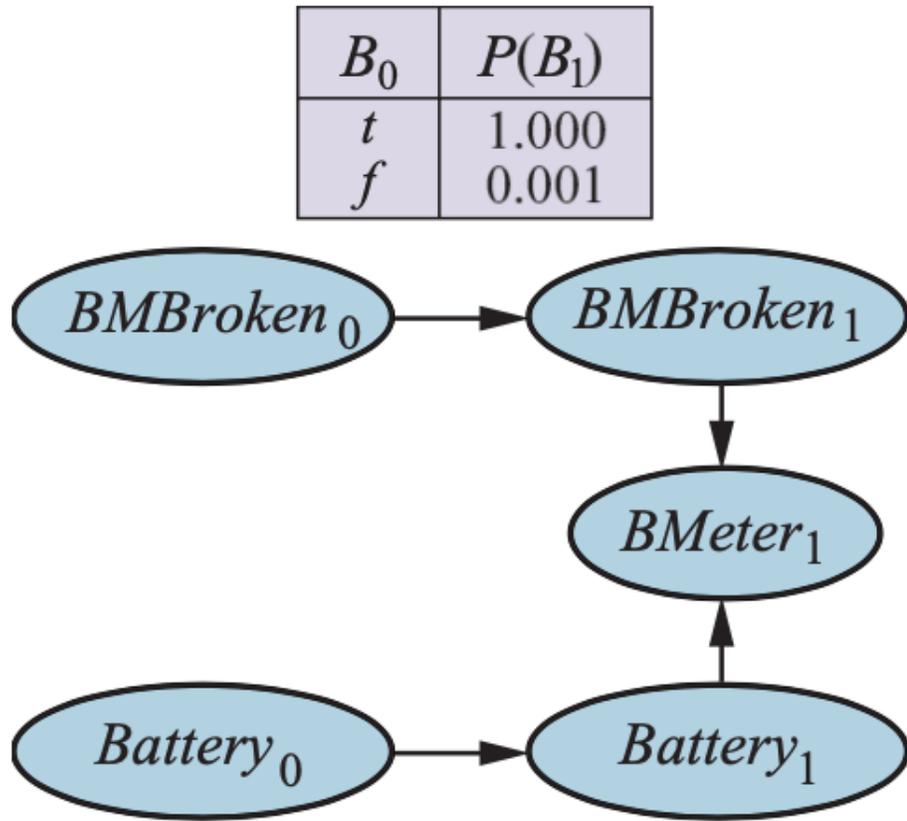
add  $e_t$  to the end of  $e_{t-d:t}$   
 $\mathbf{O}_t \leftarrow$  diagonal matrix containing  $\mathbf{P}(e_t | X_t)$   
**if**  $t > d$  **then**  
     $\mathbf{f} \leftarrow$  FORWARD( $\mathbf{f}, e_{t-d}$ )  
    remove  $e_{t-d-1}$  from the beginning of  $e_{t-d:t}$   
     $\mathbf{O}_{t-d} \leftarrow$  diagonal matrix containing  $\mathbf{P}(e_{t-d} | X_{t-d})$   
     $\mathbf{B} \leftarrow \mathbf{O}_{t-d}^{-1} \mathbf{T}^{-1} \mathbf{B} \mathbf{T} \mathbf{O}_t$   
**else**  $\mathbf{B} \leftarrow \mathbf{B} \mathbf{T} \mathbf{O}_t$   
 $t \leftarrow t + 1$   
**if**  $t > d + 1$  **then return** NORMALIZE( $\mathbf{f} \times \mathbf{B} \mathbf{1}$ ) **else return** null

# Specification of the prior, transition model, and sensor model for the umbrella DBN

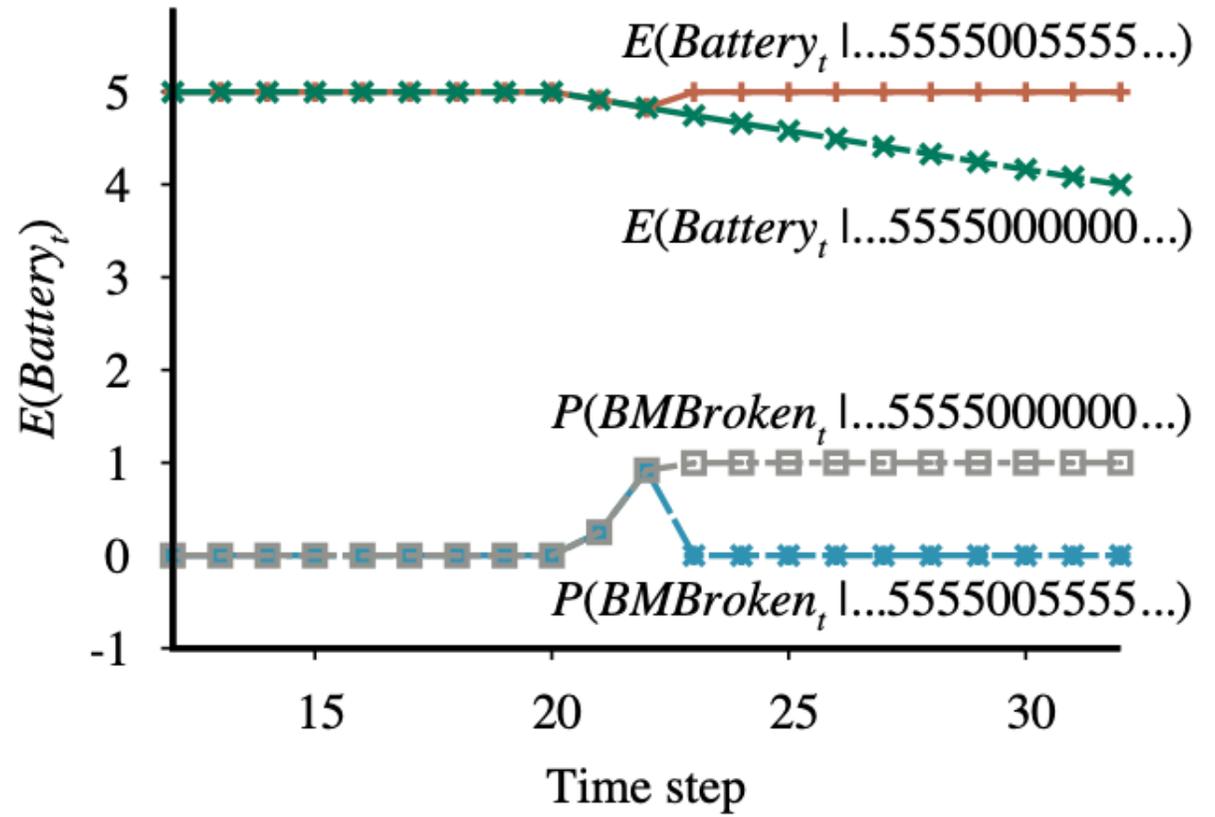


# A DBN fragment

the sensor status variable required for modeling persistent failure of the battery sensor

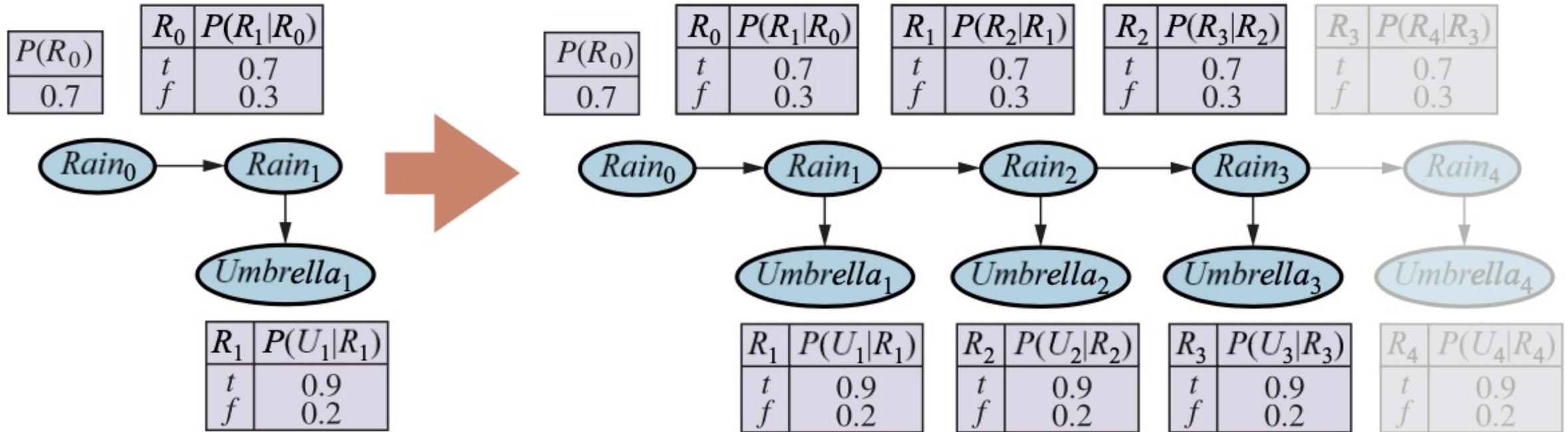


(a)



(b)

# Unrolling a Dynamic Bayesian Network



# The Particle Filtering Algorithm

**function** PARTICLE-FILTERING( $\mathbf{e}$ ,  $N$ ,  $dbn$ ) **returns** a set of samples for the next time step

**inputs:**  $\mathbf{e}$ , the new incoming evidence

$N$ , the number of samples to be maintained

$dbn$ , a DBN defined by  $\mathbf{P}(\mathbf{X}_0)$ ,  $\mathbf{P}(\mathbf{X}_1 | \mathbf{X}_0)$ , and  $\mathbf{P}(\mathbf{E}_1 | \mathbf{X}_1)$

**persistent:**  $S$ , a vector of samples of size  $N$ , initially generated from  $\mathbf{P}(\mathbf{X}_0)$

**local variables:**  $W$ , a vector of weights of size  $N$

**for**  $i = 1$  to  $N$  **do**

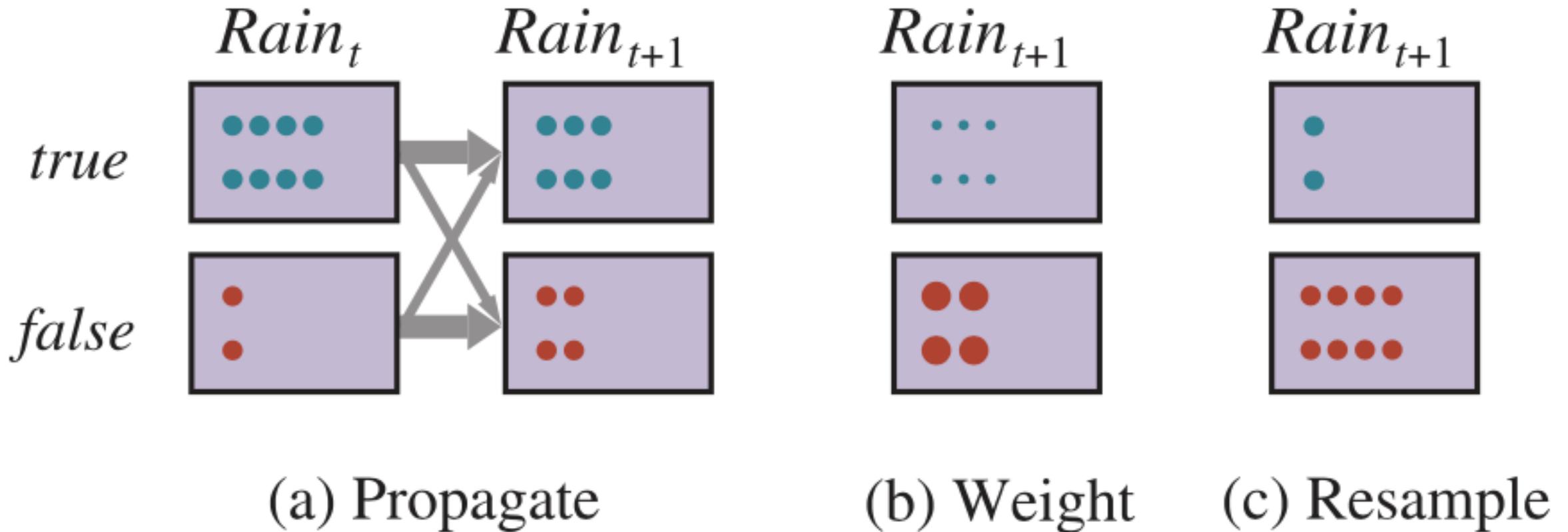
$S[i] \leftarrow$  sample from  $\mathbf{P}(\mathbf{X}_1 | \mathbf{X}_0 = S[i])$  // step 1

$W[i] \leftarrow \mathbf{P}(\mathbf{e} | \mathbf{X}_1 = S[i])$  // step 2

$S \leftarrow$  WEIGHTED-SAMPLE-WITH-REPLACEMENT( $N, S, W$ ) // step 3

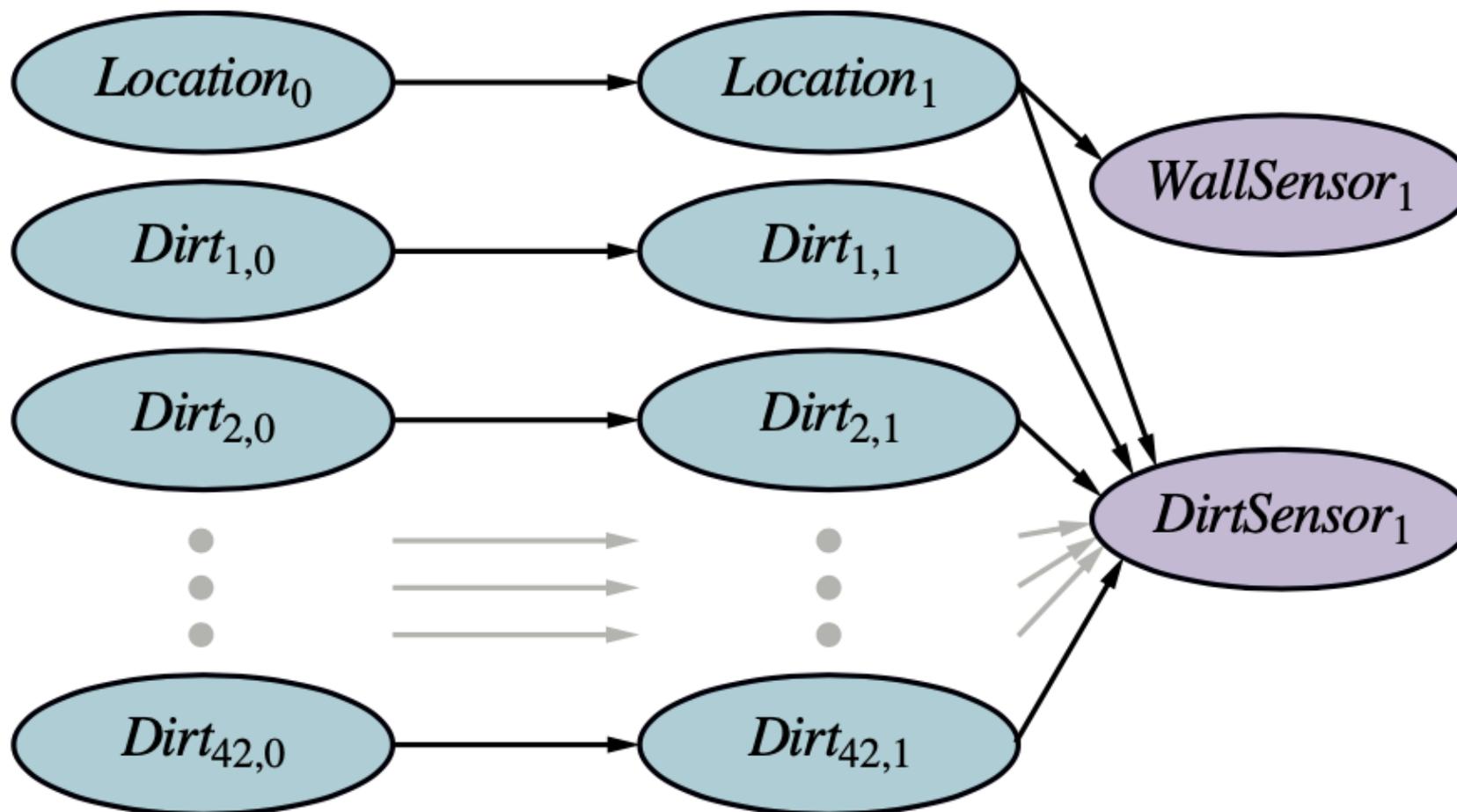
**return**  $S$

# The Particle Filtering Update Cycle for the Umbrella DBN



# A Dynamic Bayes Net

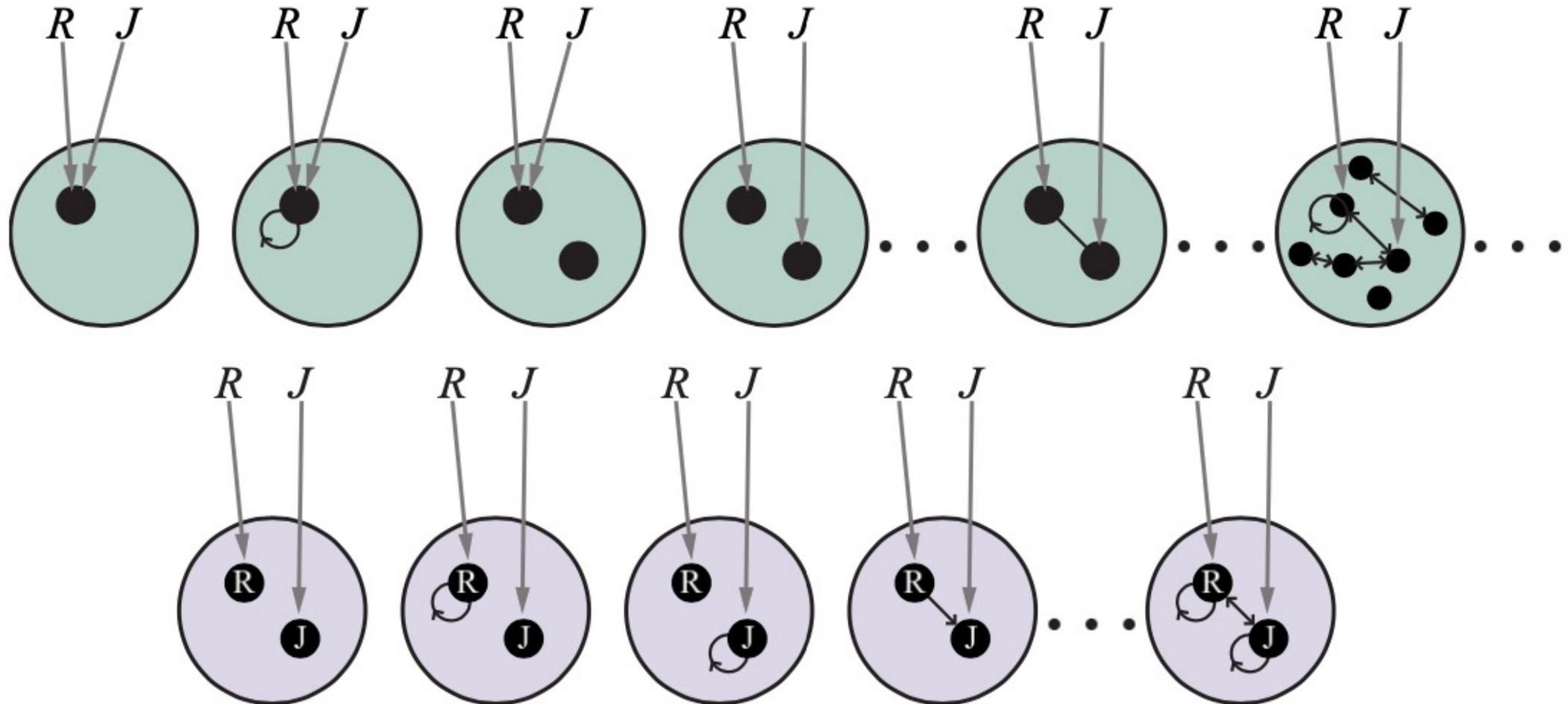
for simultaneous localization and mapping  
in the stochastic-dirt vacuum world



# Probabilistic Programming

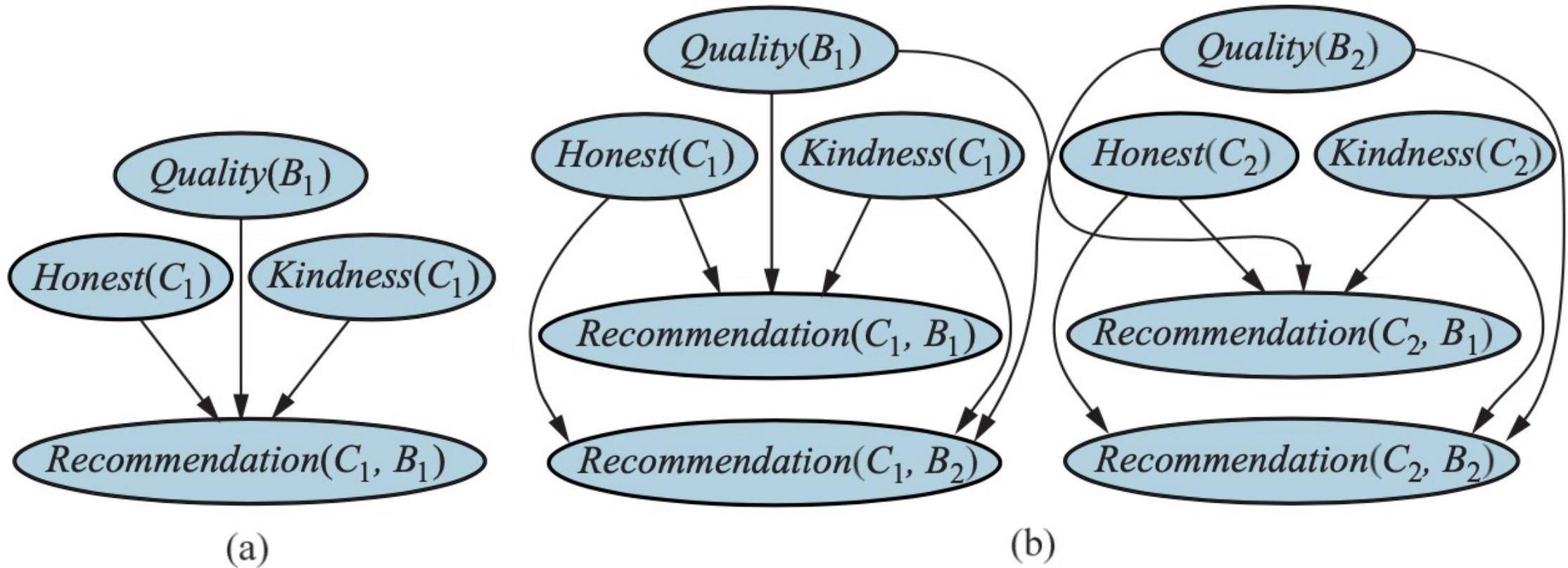
# Possible Worlds

for a language with two constant symbols, R and J



# Bayes Net for a Single customer C1

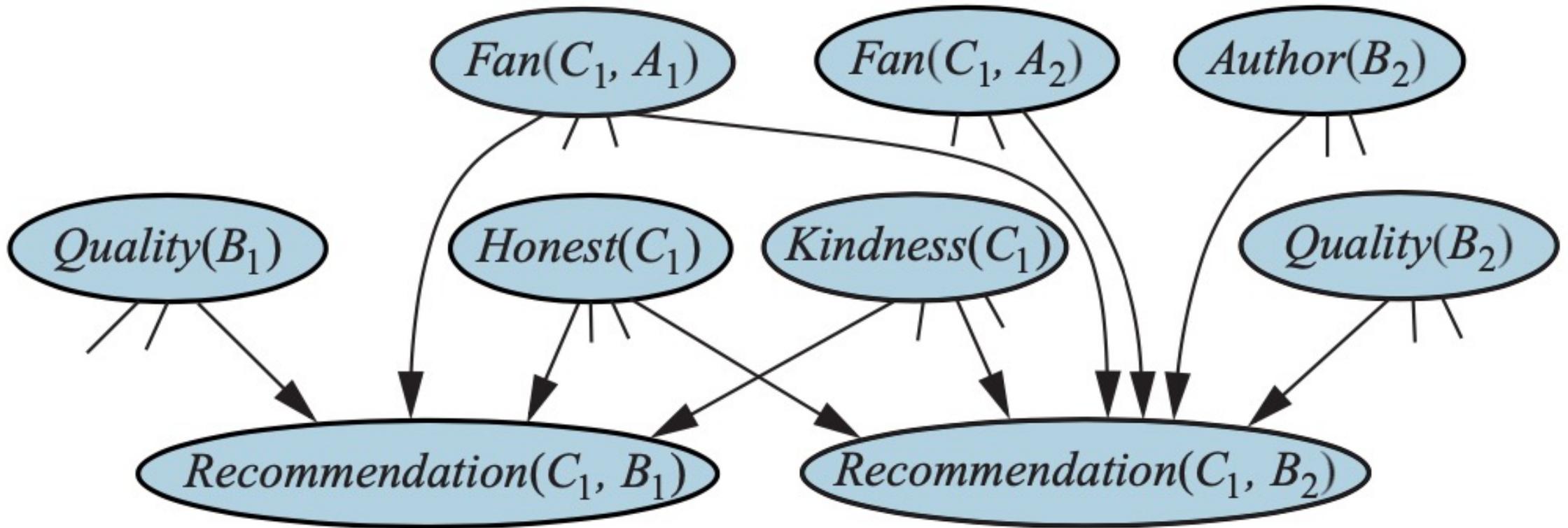
recommending a single book B1. Honest(C1) is Boolean



Bayes net with two customers and two books

# Bayes Net

for the book recommendation when Author(B2) is unknown



# One particular world for the book recommendation OUPM

Variable	Value	Probability
<i>#Customer</i>	2	0.3333
<i>#Book</i>	3	0.3333
<i>Honest</i> <sub>(Customer, 1)</sub>	<i>true</i>	0.99
<i>Honest</i> <sub>(Customer, 2)</sub>	<i>false</i>	0.01
<i>Kindness</i> <sub>(Customer, 1)</sub>	4	0.3
<i>Kindness</i> <sub>(Customer, 2)</sub>	1	0.1
<i>Quality</i> <sub>(Book, 1)</sub>	1	0.05
<i>Quality</i> <sub>(Book, 2)</sub>	3	0.4
<i>Quality</i> <sub>(Book, 3)</sub>	5	0.15
<i>#LoginID</i> <sub>(Owner, (Customer, 1))</sub>	1	1.0
<i>#LoginID</i> <sub>(Owner, (Customer, 2))</sub>	2	0.25
<i>Recommendation</i> <sub>(LoginID, (Owner, (Customer, 1)), 1), (Book, 1)</sub>	2	0.5
<i>Recommendation</i> <sub>(LoginID, (Owner, (Customer, 1)), 1), (Book, 2)</sub>	4	0.5
<i>Recommendation</i> <sub>(LoginID, (Owner, (Customer, 1)), 1), (Book, 3)</sub>	5	0.5
<i>Recommendation</i> <sub>(LoginID, (Owner, (Customer, 2)), 1), (Book, 1)</sub>	5	0.4
<i>Recommendation</i> <sub>(LoginID, (Owner, (Customer, 2)), 1), (Book, 2)</sub>	5	0.4
<i>Recommendation</i> <sub>(LoginID, (Owner, (Customer, 2)), 1), (Book, 3)</sub>	1	0.4
<i>Recommendation</i> <sub>(LoginID, (Owner, (Customer, 2)), 2), (Book, 1)</sub>	5	0.4
<i>Recommendation</i> <sub>(LoginID, (Owner, (Customer, 2)), 2), (Book, 2)</sub>	5	0.4
<i>Recommendation</i> <sub>(LoginID, (Owner, (Customer, 2)), 2), (Book, 3)</sub>	1	0.4

# An OUPM for Citation Information Extraction

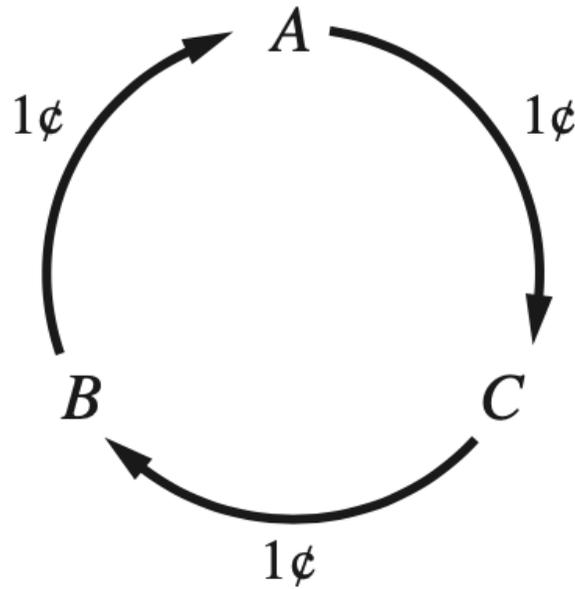
```
type Researcher, Paper, Citation
random String Name(Researcher)
random String Title(Paper)
random Paper PubCited(Citation)
random String Text(Citation)
random Boolean Professor(Researcher)
origin Researcher Author(Paper)
```

```
#Researcher ~ OM(3, 1)
Name(r) ~ NamePrior()
Professor(r) ~ Boolean(0.2)
#Paper(Author = r) ~ if Professor(r) then OM(1.5, 0.5) else OM(1, 0.5)
Title(p) ~ PaperTitlePrior()
CitedPaper(c) ~ UniformChoice({Paper p})
Text(c) ~ HMMGrammar(Name(Author(CitedPaper(c))), Title(CitedPaper(c)))
```

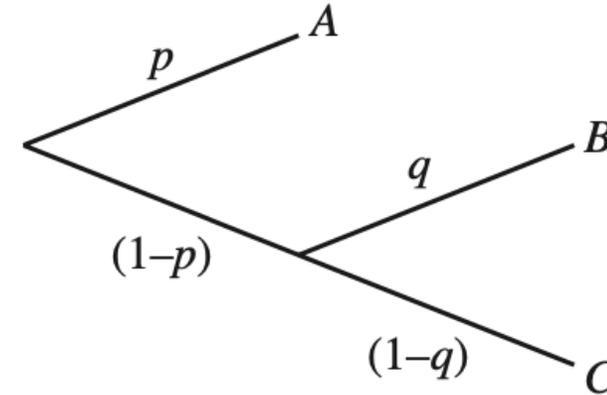
# Making Simple Decisions

# Nontransitive preferences $A \succ B \succ C \succ A$

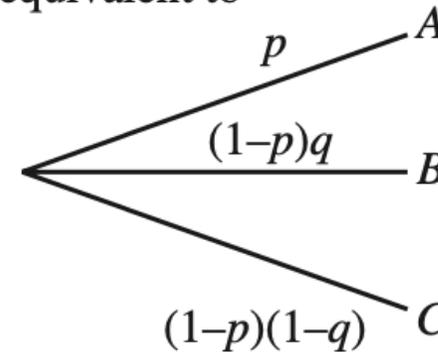
can result in irrational behavior:  
a cycle of exchanges each costing one cent



(a)



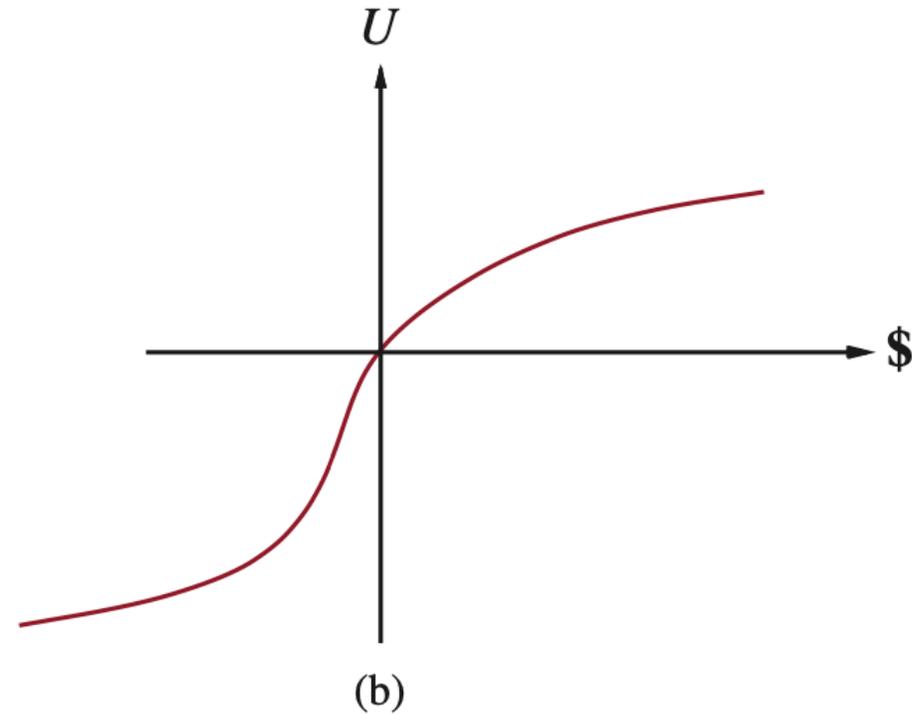
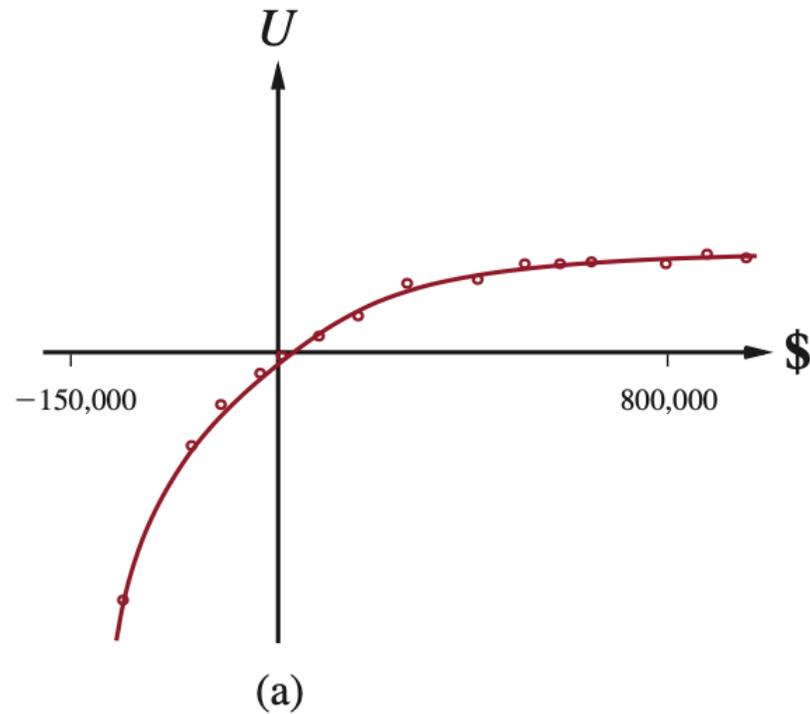
is equivalent to



(b)

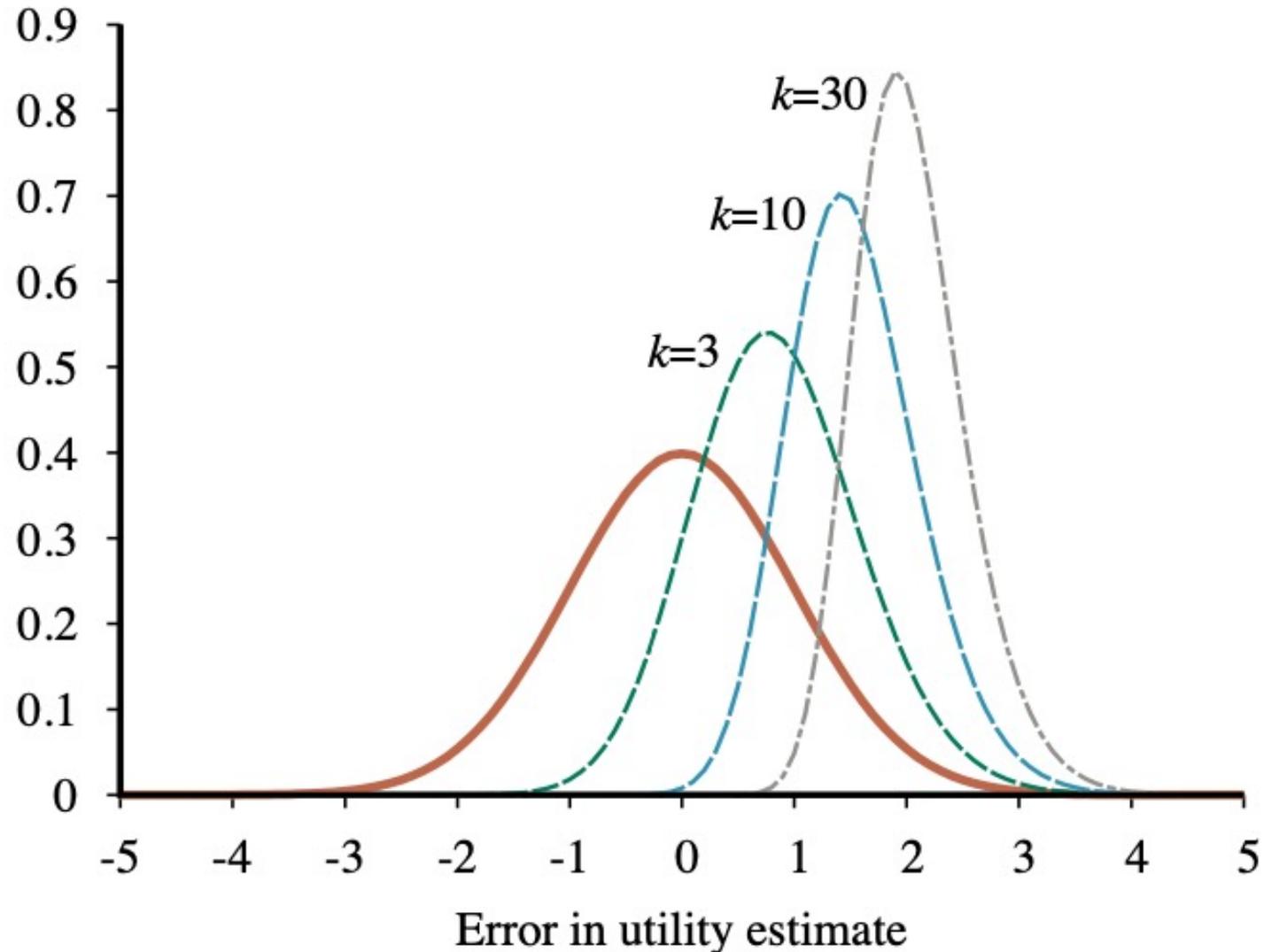
**The decomposability axiom**

# The Utility of Money



# Unjustified optimism

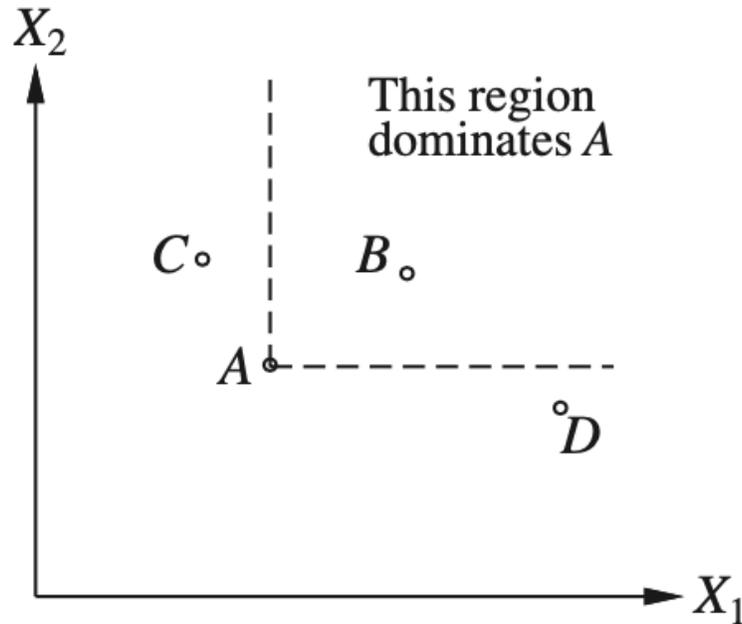
caused by choosing the best of  $k$  options



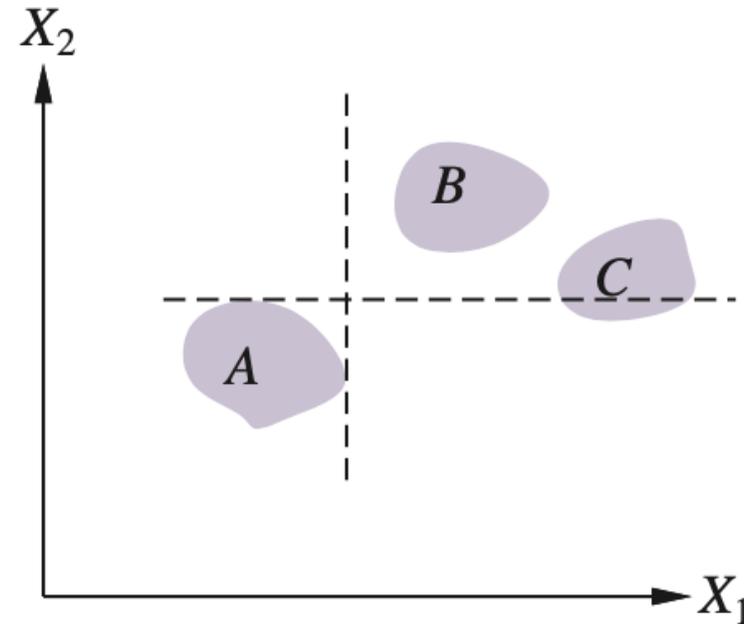
# Strict dominance

(a) Deterministic

(b) Uncertain

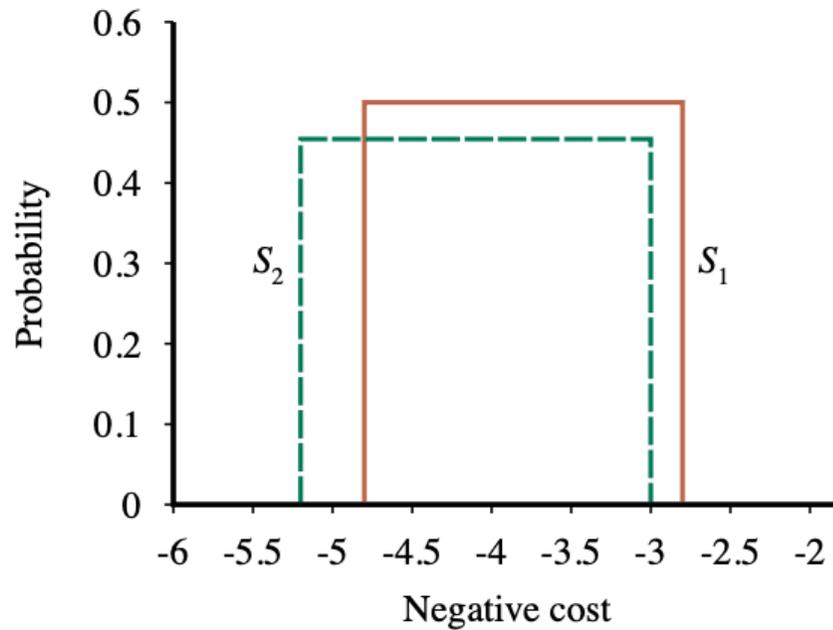


(a)

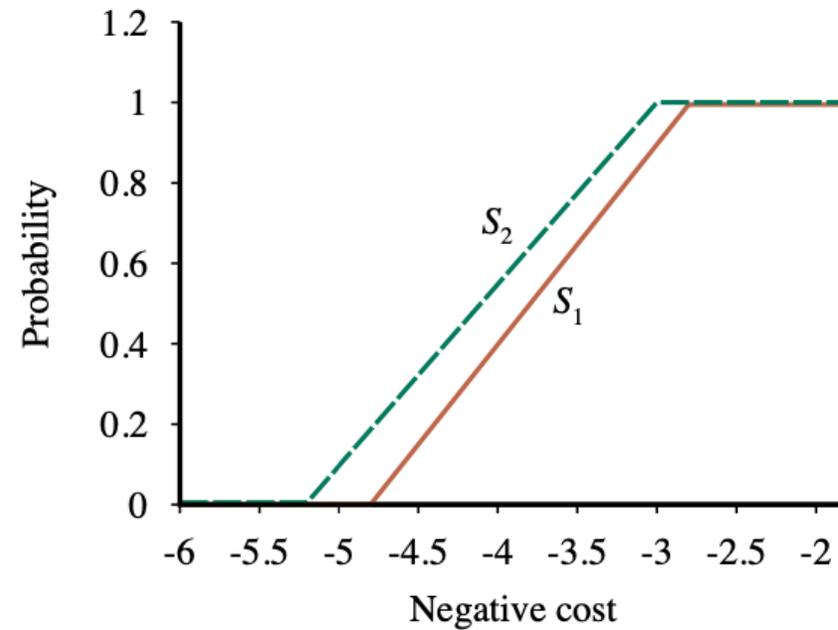


(b)

# Stochastic dominance



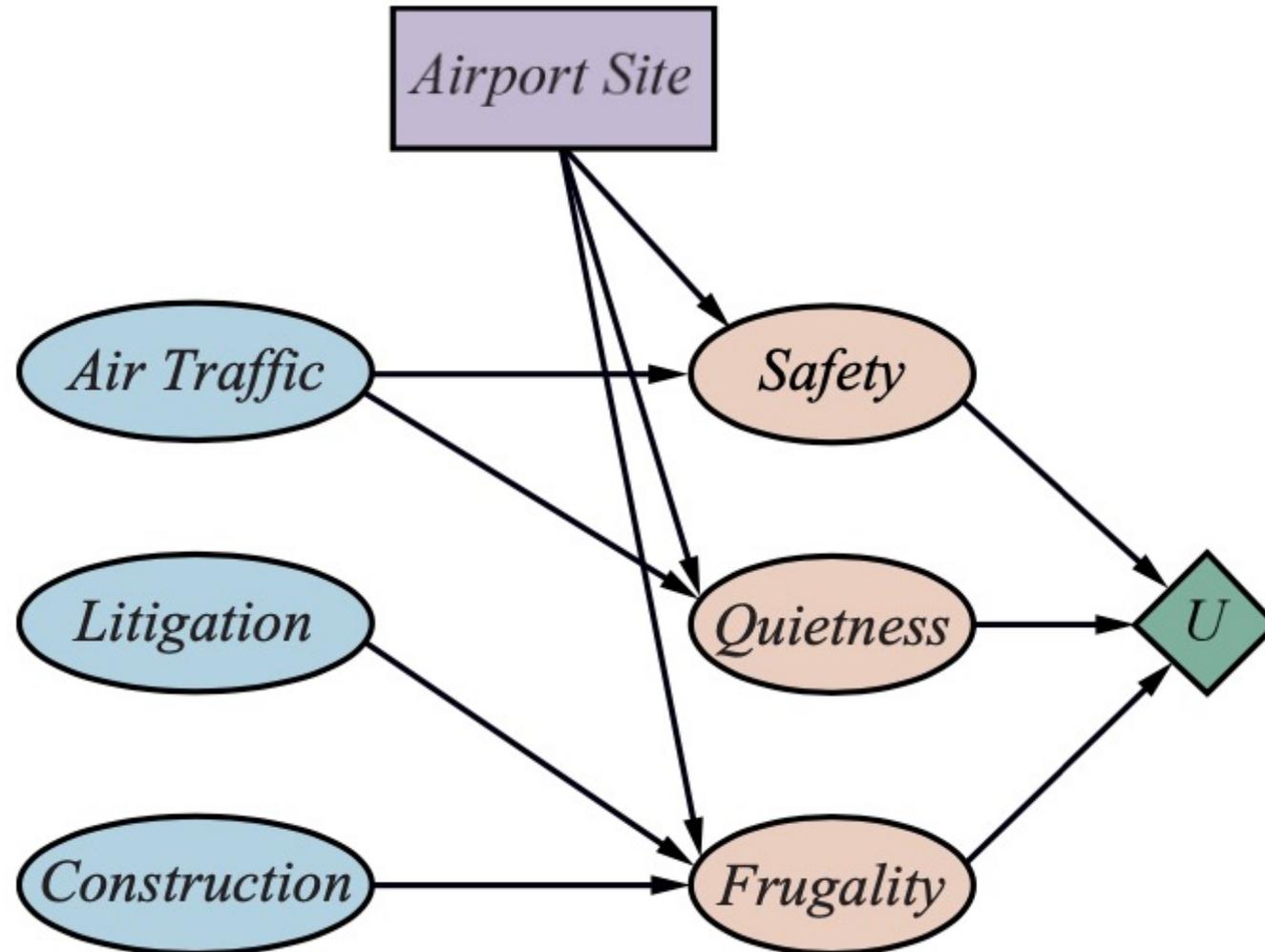
(a)



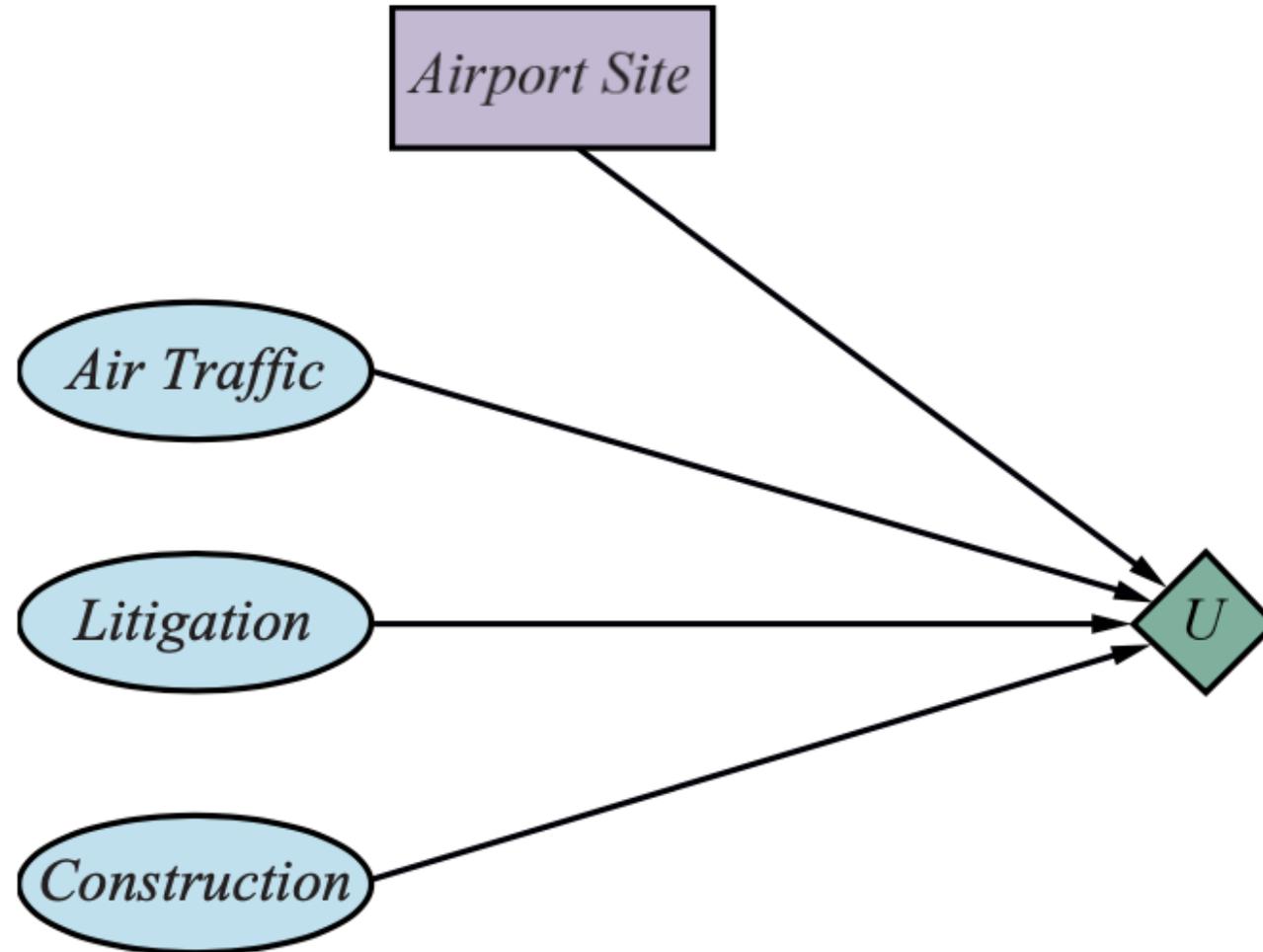
(b)

Cumulative distributions for the frugality of  $S_1$  and  $S_2$ .

# A decision network for the airport-siting problem



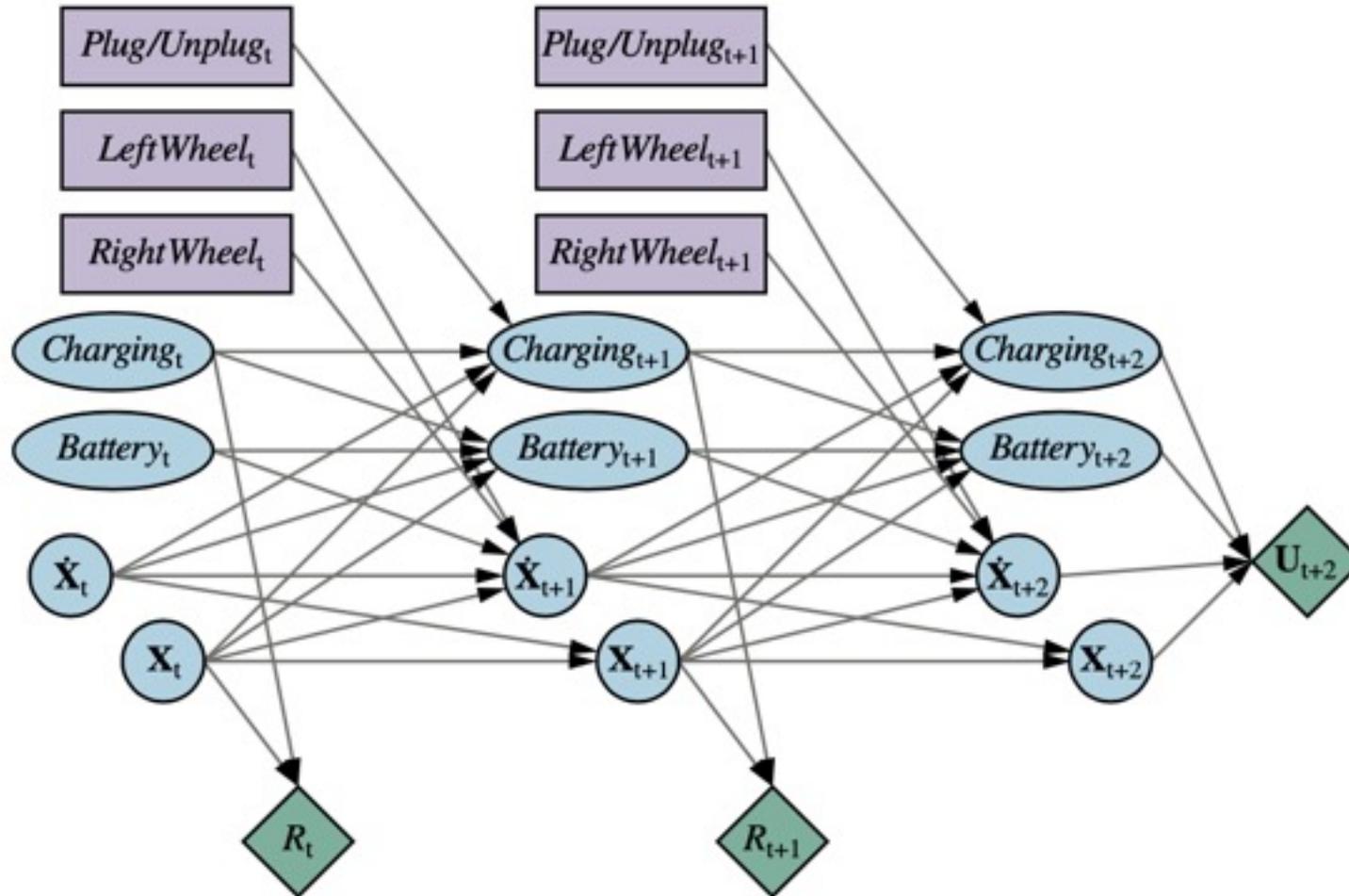
# A simplified representation of the airport-siting problem



# Making Complex Decisions

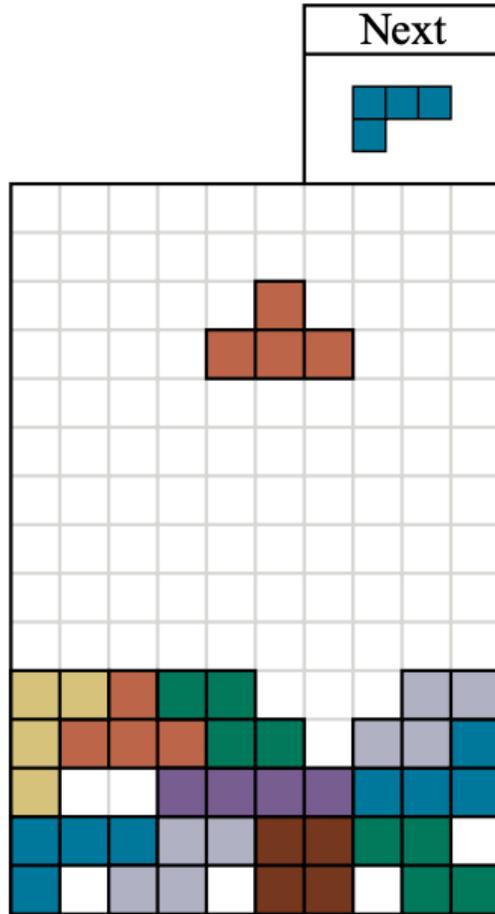
# A dynamic decision network

for a mobile robot with state variables for battery level, charging status, location, and velocity, and action variables for the left and right wheel motors and for charging.

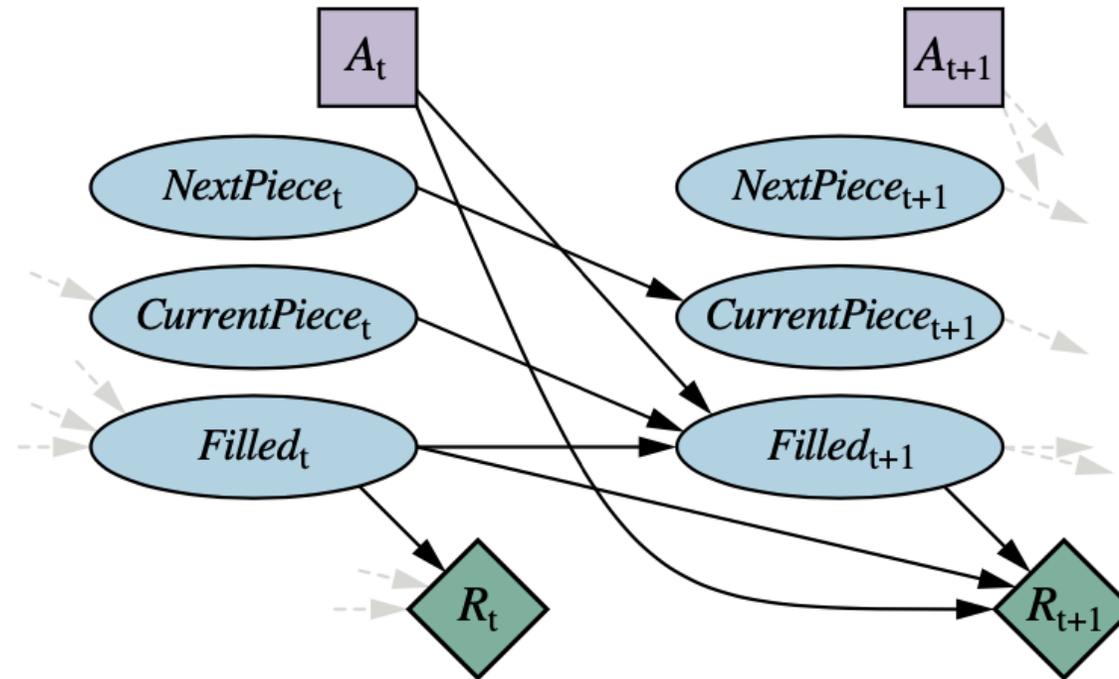


# The game of Tetris

## The DDN for the Tetris MDP



(a)



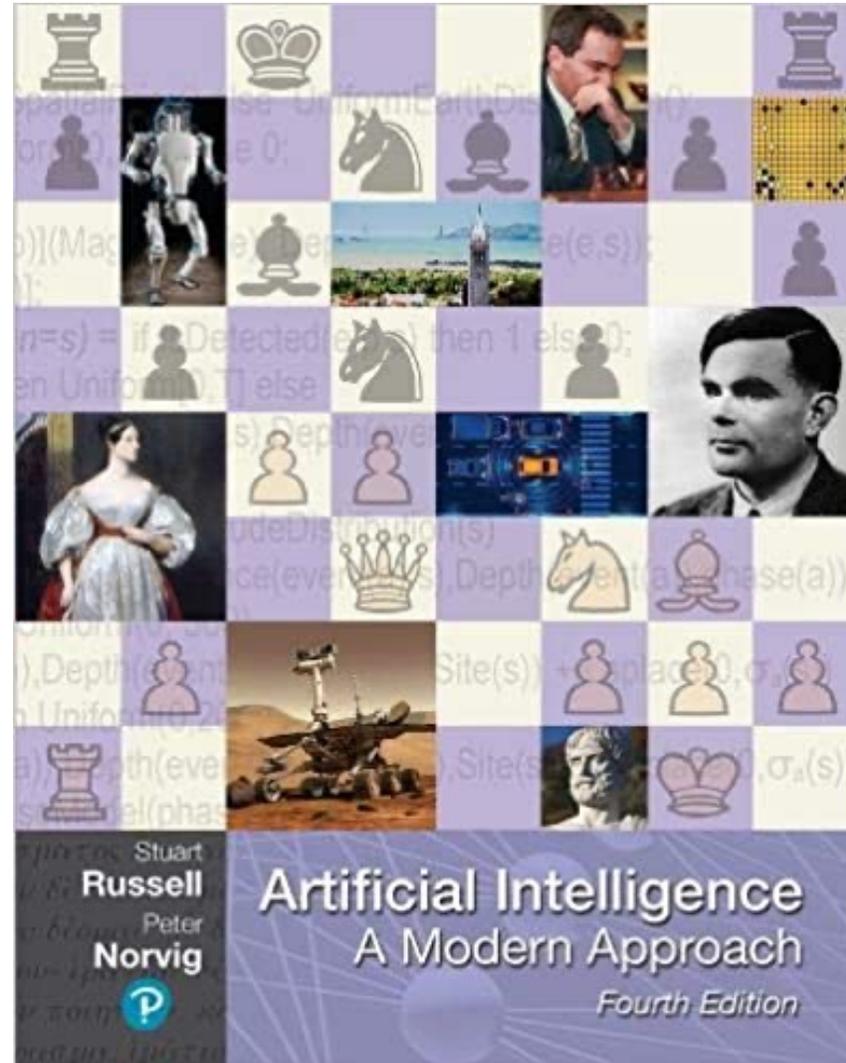
(b)

# The Value Iteration Algorithm for calculating utilities of states

**function** VALUE-ITERATION( $mdp, \epsilon$ ) **returns** a utility function  
**inputs:**  $mdp$ , an MDP with states  $S$ , actions  $A(s)$ , transition model  $P(s' | s, a)$ ,  
rewards  $R(s, a, s')$ , discount  $\gamma$   
 $\epsilon$ , the maximum error allowed in the utility of any state  
**local variables:**  $U, U'$ , vectors of utilities for states in  $S$ , initially zero  
 $\delta$ , the maximum relative change in the utility of any state

**repeat**  
     $U \leftarrow U'; \delta \leftarrow 0$   
    **for each** state  $s$  **in**  $S$  **do**  
         $U'[s] \leftarrow \max_{a \in A(s)} \text{Q-VALUE}(mdp, s, a, U)$   
        **if**  $|U'[s] - U[s]| > \delta$  **then**  $\delta \leftarrow |U'[s] - U[s]|$   
**until**  $\delta \leq \epsilon(1 - \gamma)/\gamma$   
**return**  $U$

Stuart Russell and Peter Norvig (2020),  
**Artificial Intelligence: A Modern Approach,**  
4th Edition, Pearson



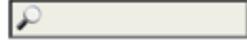
Source: Stuart Russell and Peter Norvig (2020), Artificial Intelligence: A Modern Approach, 4th Edition, Pearson

<https://www.amazon.com/Artificial-Intelligence-A-Modern-Approach/dp/0134610997/>

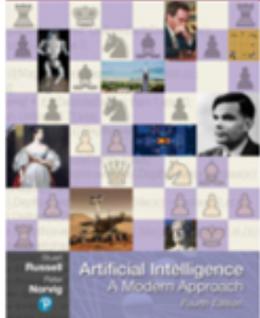
# Artificial Intelligence: A Modern Approach (AIMA)

- **Artificial Intelligence: A Modern Approach (AIMA)**
  - <http://aima.cs.berkeley.edu/>
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  - <http://aima.cs.berkeley.edu/python/readme.html>
  - <https://github.com/aimacode/aima-python>
- **Logic, KB Agent**
  - <http://aima.cs.berkeley.edu/python/logic.html>
- **Probability Models (DTAgent)**
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- **Markov Decision Processes (MDP)**
  - <http://aima.cs.berkeley.edu/python/mdp.html>

# Artificial Intelligence: A Modern Approach (AIMA)



- △ US Edition
- △ Global Edition
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- Courses
- Editions
- Errata
- Exercises
- Figures
- Instructors Page
- Pseudocode
- Reviews



## Artificial Intelligence: A Modern Approach, 4th US ed.

by Stuart Russell and Peter Norvig

The authoritative, most-used AI textbook, adopted by over 1500 schools.

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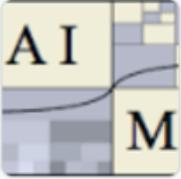
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# AIMA Code



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Code for the book "Artificial Intelligence: A Modern Approach"

358 followers Berkeley, CA <http://aima.cs.berkeley.edu> [peter@norvig.com](mailto:peter@norvig.com)

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mcventur Fixed bug in treatment of repeated nodes in frontier... 61d695b on Dec 5, 2021 1,190 commits

aima-data @ f6cbea6	updating submodule (#994)	4 years ago
gui	fixed tests (#1191)	2 years ago
images	add perception and tests (#1091)	3 years ago
js	Added TicTacToe to notebook (#213)	7 years ago
notebooks	Image Rendering problem resolved (#1178)	3 years ago
tests	fixed tests (#1191)	2 years ago
.coveragerc	Added coverage report generation to Travis (#1058)	3 years ago
.flake8	Fix flake8 warnings (#508)	5 years ago
.gitignore	Reworked PriorityQueue and Added Tests (#1025)	4 years ago
.gitmodules	Updating Submodule (#647)	5 years ago
.travis.yml	fixed svm for not posdef kernel matrix, updated .travis.yml wi...	2 years ago

### About

Python implementation of algorithms from Russell And Norvig's "Artificial Intelligence - A Modern Approach"

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**Machine Translation**



**Language Modelling**



**Question Answering**



**Sentiment Analysis**

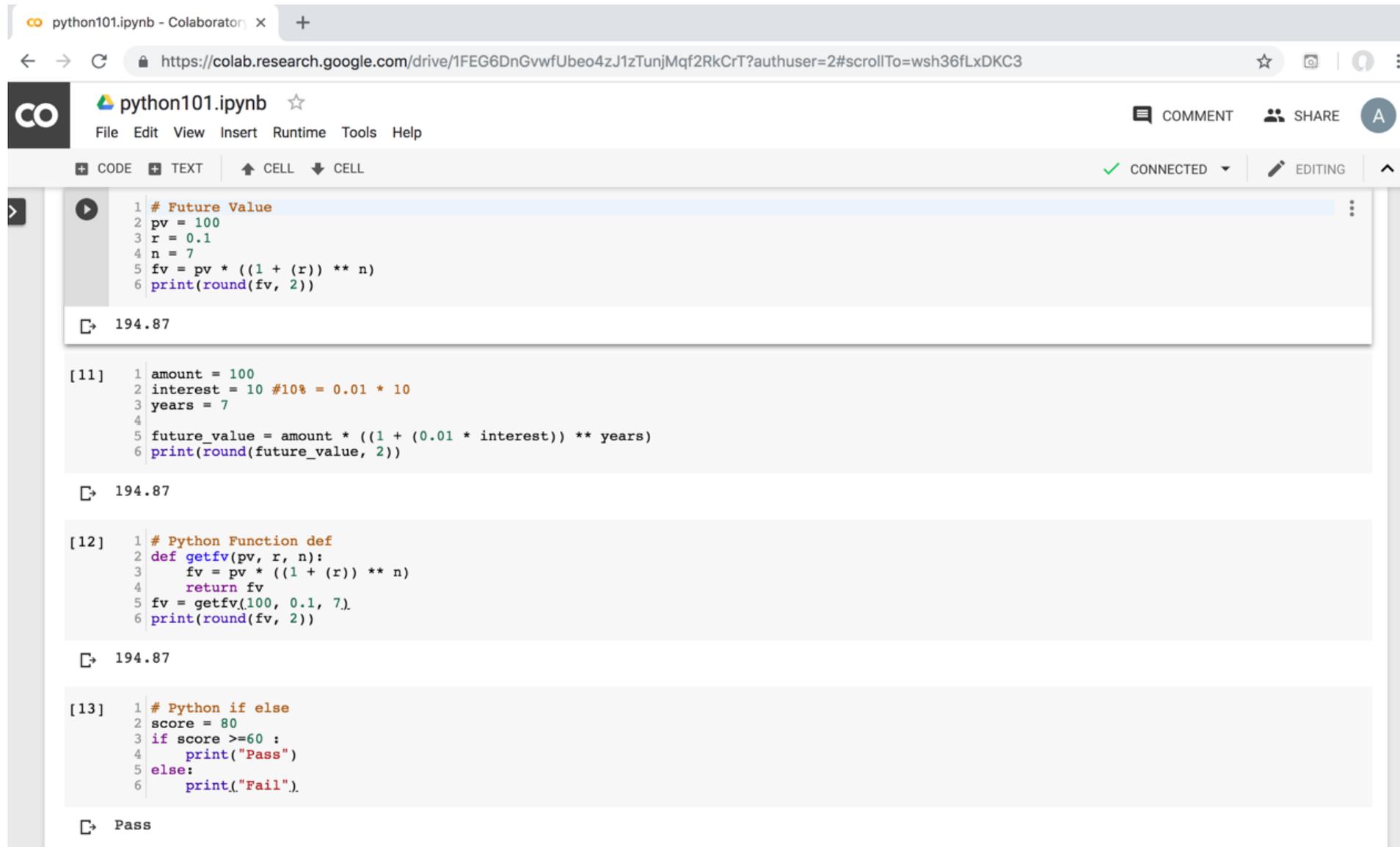


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# Python in Google Colab (Python101)

<https://colab.research.google.com/drive/1FEG6DnGvwfUbeo4zJ1zTunjMqf2RkCrT>



The screenshot shows a Google Colab notebook interface. The browser address bar displays the URL: <https://colab.research.google.com/drive/1FEG6DnGvwfUbeo4zJ1zTunjMqf2RkCrT?authuser=2#scrollTo=wsh36fLxDKC3>. The notebook title is "python101.ipynb". The interface includes a menu bar (File, Edit, View, Insert, Runtime, Tools, Help) and a toolbar with options for CODE, TEXT, CELL, and a status indicator showing "CONNECTED" and "EDITING".

The notebook contains four code cells:

- Cell 1:** A code cell with the following Python code:

```
1 # Future Value
2 pv = 100
3 r = 0.1
4 n = 7
5 fv = pv * ((1 + (r)) ** n)
6 print(round(fv, 2))
```

The output is "194.87".
- Cell [11]:** A code cell with the following Python code:

```
1 amount = 100
2 interest = 10 #10% = 0.01 * 10
3 years = 7
4
5 future_value = amount * ((1 + (0.01 * interest)) ** years)
6 print(round(future_value, 2))
```

The output is "194.87".
- Cell [12]:** A code cell with the following Python code:

```
1 # Python Function def
2 def getfv(pv, r, n):
3     fv = pv * ((1 + (r)) ** n)
4     return fv
5 fv = getfv(100, 0.1, 7)
6 print(round(fv, 2))
```

The output is "194.87".
- Cell [13]:** A code cell with the following Python code:

```
1 # Python if else
2 score = 80
3 if score >=60 :
4     print("Pass")
5 else:
6     print("Fail").
```

The output is "Pass".

<https://tinyurl.com/aintpupython101>

# Summary

- **Knowledge and Reasoning**
  - **Logical Agents**
  - **First-Order Logic**
  - **Inference in First-Order Logic**
  - **Knowledge Representation**
  - **Knowledge Graph (KG)**
- **Uncertain Knowledge and Reasoning**
  - **Quantifying Uncertainty**
  - **Probabilistic Reasoning**
  - **Making Complex Decisions**

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- Stuart Russell and Peter Norvig (2020), Artificial Intelligence: A Modern Approach, 4th Edition, Pearson.
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