

Linearity of the derivative/differential operators is trivial. Besides that, you must know how to derive the derivatives of basic functions such as (already shown in class)

$$x^r, \ln x, e^x, \sin x, \cos x, \text{ (then } \tan x, \sec x, \cot x, \csc x \text{)}.$$

Then simply apply *product rule* (*quotient rule*) or *chain rule* to obtain the derivatives of varied types like

$$[f(x)]^r, \log_b f(x), a^{f(x)}, \sin\{f(x)\}, \cos\{f(x)\}, \text{ (then } \tan\{f(x)\}, \sec\{f(x)\}, \dots \text{)}.$$

For instance,

$$\begin{aligned} & \frac{d}{dx} [\log_b f(x)] \\ &= \frac{d}{dx} \left[\frac{\ln f(x)}{\ln b} \right] && \text{by "change of base"} \\ &= \frac{1}{\ln b} \frac{d}{dx} [\ln \overbrace{f(x)}^u] && \text{by linearity} \\ &= \frac{1}{\ln b} \frac{d(\ln u)}{dx} \frac{du}{dx} = \frac{1}{\ln b} \frac{1}{f(x)} f'(x) && \text{by chain rule} \\ &= \frac{1}{\ln b} \frac{f'(x)}{f(x)} \end{aligned}$$

To deal with a seemed complicated one, simply apply one of those rules repeatedly, for example,

$$\begin{aligned} & \frac{d}{dx} \left[f(x)^{g(x)} \right] \\ &= \frac{d}{dx} \left[\overbrace{e^{g(x) \ln f(x)}}^u \right] && \text{"exponentiate"} \\ &= \frac{d}{du} [e^u] \frac{d}{dx} [u] && \text{by chain rule} \\ &= e^u \frac{d}{dx} [g(x) \ln f(x)] \\ &= f(x)^{g(x)} \left\{ g'(x) \ln f(x) + g(x) \frac{d}{dx} [\ln f(x)] \right\} && \text{by product rule} \\ &= f(x)^{g(x)} \left\{ g'(x) \ln f(x) + g(x) \frac{f'(x)}{f(x)} \right\} && \text{by chain rule} \end{aligned}$$

$$\begin{aligned} & \frac{d}{dx} [(\sin x)^{\cos x}] \\ &= \frac{d}{dx} [e^{\cos x \ln(\sin x)}] \\ &= e^{\cos x \ln(\sin x)} \frac{d}{dx} [\cos x \ln(\sin x)] \\ &= (\sin x)^{\cos x} \left[-\sin x \ln(\sin x) + \cos x \frac{\cos x}{\sin x} \right] \end{aligned}$$

Generalize product rule: *find the derivative of* $(f_1(x) \cdots f_n(x))$ *with respect to* x .

You don't need to do innumerable exercises, as long as you know every in and out of limit and those derivative rules ... Learn things with full understanding and nothing needs to be crammed into brains!