1.(6) Evaluate $\int_0^8 \int_{\sqrt[3]{x}}^2 \frac{dy}{y^4+1}\, dx$.

($sol.$) 原式 $= \int_0^2 \int_0^{y^3} \frac{dx}{y^4+1}\, dy$ $(1+\frac{1}{1})$
$= \int_0^2 \frac{y^3}{y^4+1}\, dy$ (+1) $= \frac{1}{4}\int_0^2 \frac{d(y^4+1)}{y^4+1}$ (+1)
$= \frac{1}{4}\ln(y^4+1)|_0^2 = \frac{1}{4}\ln 17$ (+1)

2.(6) R is the upper semi unit disc centered at the origin, $f(x,y) := e^{x^2+y^2}$. Evaluate $\int_R f\, dA$.

($sol.$) 原式 $= \int_0^1 \underbrace{\pi r}_{\text{長度 }(+1)} \underbrace{e^{r^2}}_{\text{密度 }(+1)} \underbrace{dr}_{\text{寬度 }(+1)}$
$= \frac{\pi}{2}\int_0^1 e^{r^2}\, d(r^2)$ (+1)
$= \frac{\pi}{2} e^{r^2}|_0^1$ (+1) $= \frac{\pi}{2}(e-1)$ (+1)

3.(11) Compute the volume of the region in the first octant bounded by $x+y=4$ and $y^2+4z^2=16$.

($sol.$) Vol $= \int_0^4 \underbrace{(4-y)\frac{1}{2}\sqrt{16-y^2}}_{\text{面積 }(+2)} \underbrace{dy}_{\text{厚度 }(+1)}$
$= \int_0^4 2\underbrace{\sqrt{16-y^2}}_{\text{令 } y=4\sin t}\, dy + 4\int_0^4 (-2y)\sqrt{16-y^2}\, dy$ (+1)
$= 32\int_0^{\frac{\pi}{2}} \frac{1+\cos 2t}{2}\, dt$ (+2) $+ 4\int_0^4 \sqrt{16-y^2}\, d(16-y^2)$ (+1)
$= 32(\frac{t}{2}+\frac{\sin 2t}{4})|_0^{\frac{\pi}{2}}$ (+2) $+ 4\cdot\frac{2}{3}(16-y^2)^{\frac{3}{2}}|_0^4$ (+1) $= 8\pi - \frac{32}{3}$ (+1)

4.(7) Compute the volume of the region between $r=\cos\theta$ and $r=2\cos\theta$, and between $z=0$ and $z=3-y$.

($sol.$) Vol $= \int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \int_{\cos\theta}^{2\cos\theta} (3-r\sin\theta)\, r\, dr\, d\theta$ (1+1+1)
$= \frac{9}{2}\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \cos^2\theta\, d\theta$ (+1) $- \frac{7}{3}\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \underbrace{\cos^3\theta\sin\theta}_{\text{odd}}\, d\theta$ (+1)
$= \frac{9}{2}\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \frac{1+\cos 2\theta}{2}\, d\theta$ (+1) $= \frac{9}{2}\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \frac{1}{2}\, d\theta = \frac{9}{4}\pi$ (+1)

5.(7) Compute the volume of the region above xy-plane, below $\phi = \frac{\pi}{3}$ and inside $\rho = 2$.

($sol.$) Vol $= \int_0^{2\pi} \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \int_0^2 \rho^2 \sin\phi\, d\rho\, d\phi\, d\theta$ (1+1+1+1)
$= 2\pi\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{8}{3}\sin\phi\, d\phi$ (+1) $= \frac{16}{3}\pi(\cos\frac{\pi}{3} - \cos\frac{\pi}{2})$ (+1) $= \frac{8}{3}\pi$ (+1)

(Tidy up your work above. Anything written below does not count.)