1. Let $f(x) = x^2/2 + 1/2$. Find the volume of " $\substack{f(x)\le y\le 1\\ x\in[0,1]}$ revolving about $y$-axis "

by disc/washer method

(*sol.*) "$\int \pi x^2\, dy$" $= \int_0^1 \pi x^2 \cdot x\, dx = \frac{\pi}{4}$ (+1) (+1) (+1)

by shell method

(*sol.*) "$\int 2\pi x(1-y)\, dx$" $= \int_0^1 \pi x(1-x^2)\, dx = \pi(\frac{1}{2}-\frac{1}{4}) = \frac{\pi}{4}$ (+1) (+1) (+1)

Find the arc length of $\substack{y=f(x)\\ x\in[0,1]}$

(*sol.*) $\int \sqrt{1+x^2}\, dx$ , let $x = \tan t$

$= \int \sec^3 t\, dt = \int \sec t\, d(\tan t)$ (+1)

$= \sec t \tan t - \int \tan^2 t \sec t\, dt$ (+1)

$= \sec t \tan t + \int \sec t\, dt - \int \sec^3 t\, dt$ (+1) , $\therefore \int \sec^3 t\, dt = \frac{1}{2}[\sec t \tan t + \ln|\sec t + \tan t|]$ (+2) .

$\therefore$ 弧長 $= \int_0^1 \sqrt{1+x^2}\, dx = \frac{1}{2}\left[x\sqrt{1+x^2} + \ln(x+\sqrt{1+x^2})\right]_0^1 = \frac{1}{2}[\sqrt{2} + \ln(1+\sqrt{2})]$ (+1) (+2) (+1)

Find the area of " $\substack{y=f(x)\\ x\in[0,1]}$ revolving about $y$-axis ".

(*sol.*) $\int_0^1 2\pi x\sqrt{1+x^2}\, dx = \pi\frac{2}{3}(1+x^2)^{\frac{3}{2}}\Big|_0^1 = \frac{4}{3}\pi\sqrt{2} - \frac{2}{3}\pi$ (+1) (+2) (+1)

2. $\int \frac{dx}{2\sqrt{x}+2x}$

(*sol.*) 原式 $= \int \frac{dx}{\sqrt{x}(2+2\sqrt{x})}$ (+1)

$= \int \frac{d(2+2\sqrt{x})}{2+2\sqrt{x}}$ (+1)

$= \ln(2+2\sqrt{x})$ (+1)

3. $\int \frac{dx}{1+e^x}$

(*sol.*) 原式 $= \int \frac{1+e^x-e^x}{1+e^x}\, dx$ (+1)

$= x - \int \frac{d(1+e^x)}{1+e^x}$ (+1)

$= x - \ln(1+e^x)$ (+1)

(Tidy up your work above. Anything written below does not count.)