1.(6) Find the derivative of $f(x,y,z) = 3e^x\cos(yz)$ at $(0,0,0)$ in the direction of $\vec{u} = (2,1,-2)$.

(sol.) $D_{\vec{u}}f(0,0,0) = \underbrace{3e^x(\cos(yz), -\sin(yz)z, -\sin(yz)y)}_{(1+1+1)}\Big|_{(0,0,0)} \bullet (2,1,-2)/3$ (+1)

$= (1,0,0)$ (+1) $\bullet (2,1,-2) = 2$ (+1)

2.(12) Find all local extrema value(s) of $f(x,y) = 4xy - x^4 - y^4$.

(sol.) $f_x = 4y - 4x^3$ (+1), 令爲 0 : $y = x^3$ $\Rightarrow$ $0 = x^9 - x = x(x-1)(x+1)(x^2+1)(x^4+1) = 0$ (+1)

$f_y = 4x - 4y^3$ (+1), 令爲 0 : $x = y^3$ $\Rightarrow x = 0, \pm 1 \therefore (x,y) = (0,0), (-1,-1), (1,1)$ (+3)

$f_{xx} = -12x^2, f_{xy} = f_{yx} = 4, f_{yy} = -12y^2$ (+1), 令 $D(x,y) := f_{xx}f_{yy} - f_{xy}^2 = 144x^2y^2 - 16$ (+1) ,

$D(0,0) < 0, \therefore (0,0,0)$ 是 saddle pt. (+1); $D(-1,-1) = D(1,1) > 0$ (+1), 且 $f_{xx} < 0$ at $(-1,-1)$, $(1,1)$ (+1),

$\therefore f(-1,-1) = f(1,1) = 2$ local maxima. (+1)

3.(7) Find all extrema value(s) of $f(x,y,z) = x^2 + y^2 + z^2$, where (x,y,z) is on $\underbrace{x^2 - z^2 - 1}_{g(x,y,z)} = 0$。

(sol.) 極值發生處必然 $\nabla f, \nabla g$ 線性相關, 即 $2(x,y,z)$ (+1) $\parallel 2(x,0,-z)$ (+1) , $\Rightarrow y = 0$ (+1) 。
若 $x = 0$ 則 $z^2 + 1 = 0$ 無解 (+1); 若 $z = 0$ 則 $x = \pm 1$ (+1); $f(\pm 1, 0, 0) = 1$ 是 local minimum (+1) 因爲 $f \geq 0$ 沒限制條件時就有下界沒上界了 (+1)。

4.(12) Find all extrema value(s) of $f(x,y,z) = x^2 + 2y - z^2$, where (x,y,z) is on the intersection of $\underbrace{2x - y}_{g(x,y,z)} = 0$ and $\underbrace{y + z}_{h(x,y,z)} = 0$.

(sol.) 極值發生處必然 $\nabla f, \nabla g, \nabla h$ 線性相關, 即 $\det(\nabla f, \nabla g, \nabla h) = 0$, 即 $\begin{vmatrix} 2x & 2 & -2z \\ 2 & -1 & 0 \\ 0 & 1 & 1 \end{vmatrix} = 0$ (+2 +1 +1) $\Leftrightarrow$
$-x - 2 - 2z = 0$ (+2).

將 $y = 2x$ 代入 $y + z = 0$: $\begin{matrix} 2x + z = 0 \\ x + 2z = -2 \end{matrix}$, $x = 2/3$ (+1), $z = -4/3$ (+1), $y = 2x = 4/3$ (+1),

$f(2/3, 4/3, -4/3) = 4/9 + 8/3 - 4/9 = 8/3$ (+1) 是 local maximum 因爲 $f = -3/4y^2 + 2y$ (+2).

(Tidy up your work above. Anything written below does not count.)