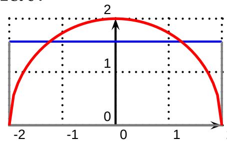
1.(7) Fill in the blanks:

$\int x^r\,dx = \underline{\frac{1}{r+1}x^{r+1}}$, $\int b^x\,dx = \underline{\frac{1}{\ln b}b^x}$, $\int \ln x\,dx = \underline{x\ln x - x}$, $\int \tan x\,dx = \underline{-\ln(\cos x)}$,

$\int \cot\,dx = \underline{\ln(\sin x)}$, $\int \sec x\,dx = \underline{\ln(\sec x + \tan x)}$, $\int \csc x\,dx = \underline{-\ln(\csc x + \cot x)}$

2.(6) Find the average value of $f(x) = \sqrt{4-x^2}$ on $[-2, 2]$. Draw a picture to show that.



≈1.5708 (+2)

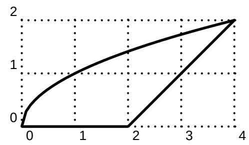
(*sol.*) $\text{avg}(f) = \dfrac{\int_{-2}^{2}\sqrt{4-x^2}\,dx}{4}$ (+2) $= \dfrac{\frac{1}{2}(\pi\cdot 2^2)}{4} = \dfrac{\pi}{2}$ (+2) .

3.(5) $\int x^3\sqrt{x^2+1}\,dx =$

(*sol.*) 原式 $= \int (x^2+1)^{\frac{1}{2}}x^2\cdot x\,dx$ (+1) $= \frac{1}{2}\int (x^2+1)^{\frac{1}{2}}\left[(x^2+1)-1\right]d(x^2+1)$ (+1)

$= \frac{1}{2}\int \left[()^{\frac{3}{2}} - ()^{\frac{1}{2}}\right]d()$ (+1) $= \frac{1}{2}\left[\frac{2}{5}()^{\frac{5}{2}} - \frac{2}{3}()^{\frac{3}{2}}\right]$ (+1) $= \frac{1}{5}(x^2+1)^{\frac{5}{2}} - \frac{1}{3}(x^2+1)^{\frac{3}{2}}$ (+1)

4.(8) Find the area bounded by $y=0$, $y=\sqrt{x}$, and $y=x-2$ in two ways:



(a) integrating with respect to x:

$\int_0^2[\sqrt{x}-0]\,dx$ (+1) $+ \int_2^4[\sqrt{x}-x+2]\,dx$ (+1)

$= \frac{2}{3}x^{\frac{3}{2}}\Big|_0^2 + \frac{2}{3}x^{\frac{3}{2}}\Big|_2^4$ (+1) $+ \left[-x^2/2+2x\right]_2^4$ (+1)

$= 16/3 + (0-2) = 10/3$ (+1)

(b) integrating with respect to y:

$\int_0^2[y+2-y^2]\,dy$ (+1)

$= \left[y^2/2 + 2y - y^3/3\right]_0^2$ (+1)

$= 2 + 4 - 8/3 = 10/3$ (+1)

(Tidy up your work above. Anything written below does not count.)