 Grading Key 分數 **12**

1.(4) $\lim\limits_{(x,y)\to(0,0)} \frac{x^4-y^2}{x^4+y^2}$

若 $y = kx^2$ (+2), 則 原式 $= \lim\limits_{x\to 0} \frac{x^4-k^2x^4}{x^4+k^2x^4} = \frac{1-k^2}{1+k^2}$ (+1) ,
竟與 k 選取有關, 即: 該極限不存在。(+1)

2.(4) $d\big(\arctan(\frac{y}{x})\big)$

$$\begin{aligned}
\text{原式} &= \frac{\partial}{\partial x}\arctan(\tfrac{y}{x})\,dx + \frac{\partial}{\partial y}\arctan(\tfrac{y}{x})\,dy \quad (+1)\\
&= \frac{1}{1+(\frac{y}{x})^2}\frac{\partial}{\partial x}(\tfrac{y}{x})\,dx + \frac{1}{1+(\frac{y}{x})^2}\frac{\partial}{\partial y}(\tfrac{y}{x})\,dy \quad (+1)\\
&= \frac{1}{1+(\frac{y}{x})^2}(\tfrac{-y}{x^2})\,dx + \frac{1}{1+(\frac{y}{x})^2}(\tfrac{1}{x})\,dy \quad (+1)\\
&= \frac{-y}{x^2+y^2}\,dx + \frac{x}{x^2+y^2}\,dy \quad (+1)
\end{aligned}$$

3.(4) $z^3 - xy + yz + y^3 - 2 = 0$. Find $\frac{\partial x}{\partial y}$ and $\frac{\partial y}{\partial z}$ by differentials.

$$\begin{aligned}
\text{原式} &\stackrel{d}{\Longrightarrow} 3z^2\cdot dz - x\cdot dy - y\cdot dx + y\cdot dz + z\cdot dy + 3y^2\cdot dy = 0 \quad (+1)\\
&\Longleftrightarrow (-y)dx + (-x+3y^2+z)dy + (y+3z^2)dz = 0 \quad (+1)
\end{aligned}$$

$\therefore \quad \frac{\partial x}{\partial y} = \frac{-x+3y^2+z}{y}$ (+1), $\frac{\partial y}{\partial z} = \frac{y+3z^2}{x-3y^2-z}$ (+1).

(Tidy up your work above. Anything written below does not count.)