微積分 I 之『凹凸性 & 極值』 ***Grading Key*** 分數 **30**

$f(x) = -2\cos x - \cos^2 x$. Only consider $x \in [-\pi, \pi]$:

$f'(x) = 2\sin x - 2(\cos x)(-\sin x)$ (+2)
$= 2\sin x(1+\cos x)$ (+1)
$= 2\sin x + \sin 2x$

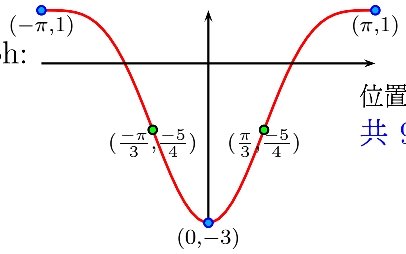
crit.pts: let $f(x) = 0 \Leftrightarrow \sin x(1+\cos x) = 0$
$\sin x = 0 \Rightarrow x = 0, \pm\pi$ (+1)
$\cos x = -1 \Rightarrow x = \pm\pi$ (+1)
, ∴ 三個奇點 $-\pi, 0, \pi$ (+1)

$f''(x) = 2\cos x + \cos 2x \cdot 2$ (+1)
$= 2(\cos x + 2\cos^2 x - 1)$ (+1) ..... even
$= 2(2\cos x - 1)(\cos x + 1)$ (+2)

infl.pts: let $(2\cos x - 1)(\cos x + 1) = 0 \Rightarrow x = \pm\frac{\pi}{3}, \pm\pi$ (+2)
$f''(\frac{\pi}{3}^+) = 2\cdot 0^-(\frac{1}{2}^- + 1) < 0$ (+1)
$f''(\frac{\pi}{3}^-) = 2\cdot 0^+(\frac{1}{2}^+ + 1) > 0$ (+1)
,
$f''(\pi^+) = 2(2\cdot -1^+ + 1)0^+ < 0$ (+1)
$f''(\pi^-) = 2(2\cdot -1^+ + 1)0^+ < 0$ (+1)
∴ 兩個反曲點 $-\frac{\pi}{3}, \frac{\pi}{3}$ (+1)

local extrema:
$f''(0^-) > 0$
$f''(0^+) > 0$ (+1) , $f(0) = -3$ (+1) local min
$f''(\pi^-) < 0$
$f''(\pi^+) < 0$ (+1) , $f(\pm\pi) = 1$ (+1) local max

Graph:



位置一點一分, 凹凸性一段一分
共 9 分

(Tidy up your work above. Anything written below does not count.)

禁止交談、傳遞物品、掀示考卷、四處張望 .... 做弊者、疑似做弊 警告後再犯者, 以零分計算。