微積分 I 之『微分』 學號＿＿＿＿＿＿＿ 系級＿＿＿＿ 姓名 ***Grading Key*** 分數 **24**

1.(12) Fill in the blanks:

$\frac{d}{dx}[\sin x] = \cos x$ (+.5) $\frac{d}{dx}[\tan x] = \sec^2 x$ (+1) $\frac{d}{dx}[\sec x] = \tan x \sec x$ (+1) $\frac{d}{dx}\left[\frac{p(x)}{q(x)}\right] = p'(x)q(x) + p(x)q'(x)$ (+1)

$\frac{d}{dx}[\cos x] = -\sin x$ (+.5) $\frac{d}{dx}[\cot x] = -\csc^2 x$ (+1) $\frac{d}{dx}[\csc x] = -\cot x \csc x$ (+1) $\frac{d}{dx}[f(x)\cdot g(x)] = \frac{f'(x)g(x)-f(x)g'(x)}{g^2(x)}$ (+1.5)

$\frac{d}{dx}[x^r] = rx^{r-1}$ (+.5) $\frac{d}{dx}[\ln x] = \frac{1}{x}$ (+1) $\frac{d}{dx}[e^x] = e^x$ (+1) $\frac{d}{dx}\left[u\big(v(w)\big)\right] = \frac{du}{dv}\frac{dv}{dw}\frac{dw}{dx}$ (+2)

2.(3) Find $\frac{d}{dt}[\tan(5-\sin(2t))]$

(*sol.*) 原式 $= \sec^2(5-\sin(2t))$ (+1.5) $\cdot(-\cos(2t))$ (+1) $\cdot 2$ (+.5)

3.(4) $xy = \cot(xy)$. Find $\frac{dy}{dx}$.

(*sol.*) 原式 $\stackrel{d}{\Longrightarrow} (dx\cdot y + x\cdot dy)$ (+1) $= -\csc^2(xy)$ (+1) $(dx\cdot y + x\cdot dy)$ (+1) , 注意括號
$\Longleftrightarrow (dx\cdot y + x\cdot dy)(1+\csc^2 x) = 0$ (+1) , $\Longleftrightarrow (dx\cdot y + x\cdot dy) = 0$ (+1) , $\Rightarrow \frac{dy}{dx} = -\frac{y}{x}$ (+1)。

4.(5) Estimate $\sqrt{24.21} = \sqrt{(3.9)^2+3^2}$ by linearization of $f(x) = \sqrt{x^2+9}$.

(*sol.*) $f'(x) = \frac{1}{2}(x^2+9)(2x) = \frac{x}{\sqrt{x^2+9}}$ (+2) , $f(x) \approx f(4) + f'(4)(x-4)$ (+1) , $f(3.9) \approx f(4) + f'(4)(3.9-4)$ (+1) $= 5 + \frac{4}{5}(3.9-4) = 4.92$ (+1) $(\sqrt{24.21} \approx 4.92036584)$

(Tidy up your work above. Anything written below does not count.)

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