 Grading Key 分數 **30**

1.(6) Evaluate $\int_0^1 x\ln x\,dx$.

$(sol.)$ 原式是瑕積分 $\lim\limits_{a\to0^+}\int_a^1 x\ln x\,dx$

$$\begin{aligned}\int x\ln x\,dx &= \tfrac{1}{2}\int \ln x\,d(x^2) \quad (+1)\\ &= \tfrac{1}{2}(\ln x\cdot x^2-\int x\,dx) \quad (+1)\\ &= \tfrac{1}{2}(\ln x\cdot x^2-\tfrac{1}{2}x^2) \quad (+1)\end{aligned}$$

$$\lim_{a\to0^+}\ln a\cdot a^2=\lim_{x\to\infty}\frac{-\ln x}{x^2}=0 \quad (+2)$$

$$\therefore \lim_{a\to0^+}\int_a^1 x\ln x\,dx=\frac{-1}{4} \quad (+1)$$

2.(5) Evaluate $\int_0^{\ln 2} x^{-2}e^{-1/x}\,dx$

$(sol.)$ 原式是瑕積分 $\lim\limits_{a\to0^+}\int_a^{\ln 2} x^{-2}e^{\frac{-1}{x}}\,dx$

$$\begin{aligned}\int x^{-2}e^{\frac{-1}{x}}\,dx &= \int e^{\frac{-1}{x}}\,d(\tfrac{-1}{x}) \quad (+1)\\ &= e^{\frac{-1}{x}} \quad (+1)\end{aligned}$$

$$\therefore \lim_{a\to0^+}\int_a^{\ln 2}x^{-2}e^{\frac{-1}{x}}\,dx=\lim_{a\to0^+}[e^{\frac{-1}{\ln 2}}-e^{\frac{-1}{a}}] \quad (+1)$$

$$=e^{\frac{-1}{\ln 2}}-e^{\frac{-1}{0^+}}=e^{\frac{-1}{\ln 2}} \quad (+2)$$

3.(6) $a_n=\left(\frac{3n+1}{3n-1}\right)^n$. Find $\lim\limits_{n\to\infty}a_n$ if it converges.

$(sol.)$ 原式 $=\left(1+\frac{2}{3n-1}\right)^n$ (+2)

$$=\left[\left(1+\frac{2}{3n-1}\right)^{\frac{3n-1}{2}}\right]^{\frac{2n}{3n-1}} \quad (+2)$$

$$\xrightarrow{n\to\infty} e^{2/3} \quad (+2)$$

4.(7) Determine the convergence: $\sum\limits_{n=1}^{\infty}(1-\frac{1}{n})^n$

$(sol.)$ $\lim\limits_{n\to\infty}(1-\frac{1}{n})^n=\lim\limits_{n\to\infty}\frac{1}{(1+\frac{-1}{n})^{(-n)}}$ (+2)

$$=\lim_{x\to0}\frac{1}{(1+x)^{\frac{1}{x}}} \ (+2) \ =\frac{1}{e} \ (+2),\neq 0$$

The ∞-sum diverges by n^{th} term test. (+1)

5.(6) [C]onverges or [D]iverges:

$\sum\frac{1}{n(1+\ln^2 n)}$ [C] $\sum\frac{8\arctan n}{1+n^2}$ [C] $\sum\frac{\ln n}{\sqrt{n}}$ [D] $\sum\frac{1}{\sqrt{n}(\sqrt{n}+1)}$ [D] $\sum\frac{1/n}{\ln n\sqrt{\ln^2 n-1}}$ [C] $\sum\frac{5^n}{4^n+3}$ [D]

(Tidy up your work above. Anything written below does not count.)

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