Definition K: closed & convex. $N(x) \overset{\text{def}}{=} P_K^{-1}(x) - x$ is called the *normal cone of K at x*. Let σ be a cone with apex 0. $\check{\sigma} \overset{\text{def}}{=} \{x : \langle q, x\rangle \geq 0 \; \forall\, q \in \sigma\}$ is called the *dual cone* of σ.

Lemma 4.10 Let σ be a cone with apex 0, then the normal cone of σ at 0: $N(0) = -\check{\sigma}$ ($\check{\sigma}$ reflected in 0).

(*pf.*) $\boldsymbol{N(0) \subset -\check{\sigma}}$ **:** $u \in N(0)$, $\iff$ $u \in P_\sigma^{-1}(0) - 0$, $\iff$ $P_\sigma(u) = 0$, $\Rightarrow H := \{x : \langle x, u\rangle = 0\}$ is a supporting hyperplane of σ at 0 with $\sigma \in H^-$ by *Lemma 3.5*, i.e. $\langle q, u\rangle \leq 0 \; \forall\, q \in \sigma$, $\iff$ $\langle q, -u\rangle \geq 0 \; \forall\, q \in \sigma$, $\iff$ $-u \in \check{\sigma}$, $\iff$ $u \in -\check{\sigma}$. Hence, $N(0) \subset -\check{\sigma}$.

$\boldsymbol{-\check{\sigma} \subset N(0)}$ **:** $u \in -\check{\sigma}$ $\iff$ $\langle q, u\rangle \leq 0 \; \forall\, q \in \sigma$. Then $H := \{x : \langle x, u\rangle = 0\}$ is a supporting hyperplane with $\sigma \in H^-$ and $\sigma \cap H = \{0\}$ since 0 is the apex of σ. Then, u is an outer normal of σ and $P_\sigma(u) = 0$, $\iff$ $u \in P_\sigma^{-1}(0) - 0 = N(0)$. Hence, $-\check{\sigma} \subset N(0)$. □

The definition of apex is important on the second part.