BIFURCATION DIAGRAM OF A ROBIN BOUNDARY VALUE PROBLEM ARISING IN MEMS

JONG-SHENQ GUO, NIKOS I. KAVALLARIS, CHI-JEN WANG, AND CHERNG-YIH YU

ABSTRACT. We consider a parabolic problem with Robin boundary condition which arises when the edge of a micro-electro-mechanical-system (MEMS) device is connected with a flexible nonideal support. Then via a rigorous analysis we investigate the structure of the solution set of the corresponding steady-state problem. We show that a critical value (the pull-in voltage) exists so that the system has exactly two stationary solutions when the applied voltage is lower than this critical value, one stationary solution for applying this critical voltage, and no stationary solution above the critical voltage.

1. INTRODUCTION

In this paper, we study the deformation of an elastic membrane inside a micro-electro mechanical system (MEMS). We consider the case when the distance between the plate and the membrane is relative small compared to the length of the device. In the case when we ignore the inertia and the device is embedded in an electric circuit with/without a capacitor, the equation describing the operation of the MEMS is reduced to the following single parabolic equation

$$(1.1) \qquad u_t = \Delta u + \frac{\lambda}{(1-u)^2[1+\alpha \int_\Omega (1-u)^{-1}dx]^2}, \ x \in \Omega, \ t > 0,$$

where λ is a positive constant which proportional to the square of the applied voltage. Besides α is a nonnegative parameter related to the ratio of the applied capacitances; $\alpha = 0$ corresponds to a MEMS device without a capacitor, whilst for $\alpha > 0$ a capacitor is connected in series with the device. Ω is the domain of the plate and $u = u(x, t)$ denotes the displacement of the membrane towards the plate. It is also assumed that the gap between the elastic membrane and the rigid plate is small compared to the dimensions of the elastic membrane.

Due to the support of MEMS device might be nonideal and flexible, we consider the edge of the membrane is connected with a flexible nonideal support so that the following Robin boundary condition is imposed

$$(1.2) \qquad \frac{\partial u}{\partial \nu} + \beta u = 0 \quad \text{on } \partial\Omega,$$

where ν is the unit outer normal of $\partial\Omega$ and β is a given positive constant. For the derivation of this model, we refer the reader to [1] (see also [16, 14, 15, 12]).

We are interested in the structure of stationary solutions of this MEMS problem. We shall only focus on the case when the device is one-dimensional, namely the plate is a rectangle and the deformation u only depends on the horizontal direction. Without loss of generality we may assume that $\beta = 1$ and $\Omega = (-1, 1)$. Therefore, we study the following boundary value problem for $U(x)$:

$$(1.3) \qquad -U''(x) = \frac{\lambda}{[1 - U(x)]^2\{1 + \alpha \int_{-1}^{1}[1 - U(y)]^{-1}dy\}^2}, \quad x \in (-1, 1),$$

$$(1.4) \qquad \pm U'(\pm 1) + U(\pm 1) = 0.$$

Our main theorem on the bifurcation diagram of problem (1.3)-(1.4) in this paper reads

Theorem 1.1. *For each $\alpha \geq 0$, there exists a finite positive constant $\lambda^* = \lambda^*(\alpha)$ such that problem (1.3)-(1.4) has exactly two solutions when $\lambda \in (0, \lambda^*)$, a unique solution when $\lambda = \lambda^*$, and no solution when $\lambda > \lambda^*$.*

Theorem 1.1 includes both the local ($\alpha = 0$) and nonlocal ($\alpha > 0$) cases. It provides the existence of the supremum λ^* of the spectrum of problem (1.3)-(1.4). The existence of a finite λ^* (*pull-in voltage* in MEMS terminology) is vital, since the solution of problem (1.1)-(1.2) supplemented with an initial condition for any nonnegative initial profile quenches for $\lambda > \lambda^*$ (cf. [1]). The case when the boundary of the membrane is clamped so that (1.2) is replaced by the zero Dirichlet boundary condition was studied extensively. We refer the reader to, e.g., [13, 4, 2, 6, 7, 10, 12]. See also [8, 3, 9]. Little work was done for the case with Robin boundary conditions (see, e.g., [5, 1]). In particular, in [1], the bifurcation diagram of problem (1.3)-(1.4) was given, but only numerically. The main purpose of this work is to derive rigorously the bifurcation diagram of problem (1.3)-(1.4).

In the sequel, we let $U = U(x; \lambda)$ be a (classical) solution of (1.3)-(1.4) for a given $\lambda > 0$. It is more convenient to use the new dependent variable $w(x) := 1 - U(x)$. Then w satisfies

$$(1.5) \qquad w''(x) = \frac{\lambda}{w^2(x)\left[1 + \alpha \int_{-1}^{1} w^{-1}(y)dy\right]^2}, \quad -1 < x < 1,$$

$$(1.6) \qquad \pm w'(\pm 1) = 1 - w(\pm 1).$$

Since u is the deformation of the membrane and U is classical, we have $0 < U < 1$ in $[-1, 1]$ and U is strictly concave due to the right hand of (1.3) is positive. It also follows that w is strictly convex and $0 < w < 1$ in $[-1, 1]$. In fact, both U and w are symmetric with respect to $x = 0$ (see Lemma 2.1 in §2).

Set $a := w(0)$ and $b := w(1)$ for a solution w of (1.5)-(1.6). To study the bifurcation diagram, the first task is to derive some relations between a, b and λ. Actually, this was done in [1] (see, in particular, (2.8)-(2.10) in [1]). However, these only give some implicit relations between those three parameters. It is not clear, in particular, whether λ is a function of another single parameter
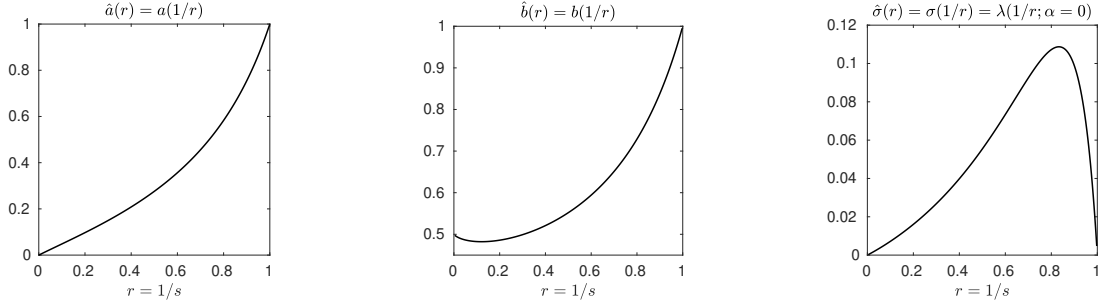
FIGURE 1. For the graph visualization, we set $r = 1/s \in (0,1)$ as the horizontal axis, and plot the curves of $\hat{a}(r) = a(1/r), \hat{b}(r) = b(1/r), \hat{\sigma}(r) = \sigma(1/r) = \lambda(1/r; \alpha = 0)$ derived from (2.12)-(2.15).

(either a or b). Actually, this is the major difficulty of dealing this problem. Luckily, we are able to find a nice transformation to overcome this difficulty. By introducing a new parameter, namely $s := b/a$, we found that all parameters a, b and λ are functions of this single variable s. It turns out that λ is a function of a (but not b), since a is (and b is not) a strictly monotone function of s (see Figure 1 and §2 below). With this piece of information in hand, we then derive the one-to-one correspondence between $\{(a, \lambda(a)) \mid a \in (0,1)\}$ and stationary solutions of (1.5)-(1.6) in §2. Then our main result, Theorem 1.1, is proved in §3, by some delicate analysis of the derivative $\lambda'(s)$. In addition, by the proof of Theorem 1.1, the value of λ^* is directly obtained. In particular, we derive that $\lambda^* \approx 0.10871$ when $\alpha = 0$ and $\lambda^* \approx 2.38709$ when $\alpha = 1$.

2. PRELIMINARIES

In the sequel, we let w be a solution of (1.5)-(1.6) and set

$$(2.1) \qquad \sigma := \lambda \left[1 + \alpha \int_{-1}^{1} \frac{1}{w(y)} dy \right]^{-2}.$$

We first prove the following lemma.

Lemma 2.1. *Any solution w of (1.5)-(1.6) is symmetric with respect to $x = 0$.*

Proof. Since $0 < w < 1$, we have

$$w'(-1) = w(-1) - 1 < 0 < 1 - w(1) = w'(1).$$

Hence there exists $\xi \in (-1, 1)$ such that $w'(\xi) = 0$.

Set $a := w(\xi) \in (0,1)$. We first show that w is symmetric with respect to ξ.

Multiplying (1.5) by w' and integrating from ξ to $x \in (\xi, 1)$, we obtain

$$(2.2) \qquad \frac{1}{2}(w')^2(x) = \sigma \left[\frac{1}{a} - \frac{1}{w(x)} \right], \quad x \in (\xi, 1].$$

Using $w'(x) > 0$ for $x \in (\xi, 1]$, it follows from (2.2) that

$$(2.3) \qquad \frac{w'(x)}{\sqrt{1/a - 1/w(x)}} = \sqrt{2\sigma}, \ x \in (\xi, 1].$$

Integrating (2.3) from ξ to $x \in (\xi, 1]$ gives

$$(2.4) \qquad \sqrt{a} \left\{ \sqrt{w(x)[w(x) - a]} + a \ln \left(\frac{\sqrt{w(x)} + \sqrt{w(x) - a}}{\sqrt{a}} \right) \right\} = \sqrt{2\sigma}(x - \xi).$$

Similarly, for $x \in [-1, \xi)$, we have

$$(2.5) \qquad \sqrt{a} \left\{ \sqrt{w(x)[w(x) - a]} + a \ln \left(\frac{\sqrt{w(x)} + \sqrt{w(x) - a}}{\sqrt{a}} \right) \right\} = \sqrt{2\sigma}(\xi - x).$$

Since the function $\sqrt{a}\{\sqrt{y(y - a)} + a \ln(\frac{\sqrt{y} + \sqrt{y - a}}{\sqrt{a}})\}$ is strictly increasing for $y \in (a, 1)$, we deduce from (2.4) and (2.5) that $w(\xi + c) = w(\xi - c)$, and consequently $w'(\xi + c) = -w'(\xi - c)$ for all $c \in (0, \min\{1 - \xi, 1 + \xi\}]$.

Next, we show that $\xi = 0$ by a contradiction argument. Without loss of generality we may assume that $\xi \in (-1, 0)$. Then the reflection point of -1 is $d = 2\xi + 1 \in (\xi, 1)$ and we have $w'(d) = -w'(-1) = 1 - w(-1) = 1 - w(d) > 0$, due to $w \in (0, 1)$. Combining with the strict convexity of w, we get that $w'(1) > w'(d)$ and $w(1) > w(d)$. This leads to a contradiction with $w'(1) = 1 - w(1)$. Therefore, $\xi = 0$ and hence any solution w of (1.5)-(1.6) is symmetric with respect to $x = 0$. The lemma is proved. $\qquad \square$

Recall $a = w(0)$ and $b = w(1)$. We now derive some relations between a, b and λ. Note that (2.2), (2.3) and (2.4) hold with $\xi = 0$, by Lemma 2.1.

First, using (1.6), we obtain from (2.2) that

$$(2.6) \qquad (1 - b)^2 = 2\sigma \left(\frac{1}{a} - \frac{1}{b} \right).$$

Moreover, for $x = 1$ (and $\xi = 0$), we obtain from (2.4) that

$$(2.7) \qquad \sqrt{2\sigma} = \sqrt{a} \left[\sqrt{b(b - a)} + a \ln \left(\frac{\sqrt{b} + \sqrt{b - a}}{\sqrt{a}} \right) \right].$$

Note that $0 < a < b < 1$. From (2.6) and (2.7) we deduce that

$$(2.8) \qquad \sqrt{b}(1 - b) = \sqrt{b(b - a)} + a\sqrt{b - a} \ln \left(\frac{\sqrt{b} + \sqrt{b - a}}{\sqrt{a}} \right).$$

This gives the relation between a and b. Note that $\lambda = \sigma$ when $\alpha = 0$.

When $\alpha > 0$, we need to find λ in terms of σ, a and b. Following [1], we first rewrite (2.3) as

$$(2.9) \qquad \sqrt{2\sigma} = \frac{1}{\sqrt{1/a - 1/w(x)}} \frac{dw(x)}{dx}, \ x \in (0, 1].$$

Using (2.9) and (2.7), we compute

$$(2.10) \quad \int_{-1}^{1} \frac{1}{w(y)} dy = \frac{2}{\sqrt{2\sigma}} \int_{a}^{b} \frac{1}{w\sqrt{1/a - 1/w}} dw = \frac{2\sqrt{a}}{\sqrt{2\sigma}} \int_{a}^{b} \frac{1}{\sqrt{w(w-a)}} dw$$

$$= \frac{2}{\sqrt{b(b-a)} + a\ln(\frac{\sqrt{b}+\sqrt{b-a}}{\sqrt{a}})} \left\{ 2\ln\left(\frac{\sqrt{b}+\sqrt{b-a}}{\sqrt{a}}\right) \right\} := I(a,b).$$

Then we deduce from (2.1) that

$$(2.11) \qquad\qquad\qquad \lambda = \sigma[1 + \alpha I(a,b)]^2.$$

Now, we introduce the variable $s := b/a \in (1, \infty)$. Then it follows from (2.8) that

$$(2.12) \qquad\qquad a = a(s) = \left(2s - 1 + \sqrt{\frac{s-1}{s}} A(s)\right)^{-1}.$$

Hereafter $A(s) := \ln(\sqrt{s} + \sqrt{s-1})$. It is clear that $a(s)$ is strictly decreasing in s such that $a(1^+) = 1$ and $a(+\infty) = 0$. This also implies that s is a function of a. Using $b = sa$, we have

$$(2.13) \qquad\qquad b = b(s) = s\left(2s - 1 + \sqrt{\frac{s-1}{s}} A(s)\right)^{-1}.$$

With s being the independent variable, we deduce from (2.7) and (2.12) that

$$(2.14) \qquad \sigma = \sigma(s) = \frac{1}{2}\left[\sqrt{s}\sqrt{s-1} + A(s)\right]^2 \left(2s - 1 + \sqrt{\frac{s-1}{s}} A(s)\right)^{-3}.$$

Moreover, using (2.10), (2.11) and (2.14), we obtain that

$$(2.15) \qquad \lambda = \lambda(s) = \frac{\left[\sqrt{s(s-1)} + A(s) + 4\alpha\left(2s - 1 + \sqrt{\frac{s-1}{s}} A(s)\right) A(s)\right]^2}{2\left[2s - 1 + \sqrt{\frac{s-1}{s}} A(s)\right]^3}.$$

Conversely, given an $s \in (1, \infty)$. Let $a = a(s)$, $\sigma = \sigma(s)$ and $\lambda = \lambda(s)$ be defined by (2.12), (2.14) and (2.15), respectively. Then, with these a, σ and λ, the function $w(x)$ determined uniquely by (2.4) (with $\xi = 0$) for each $x \in (0, 1]$ is the solution of (1.5) with initial condition $w(0) = a$ and $w'(0) = 0$ such that $w(1) = b = 1 - w'(1)$, where $b = b(s)$ is defined by (2.13). By a reflection with respect to $x = 0$, we obtain a solution w of (1.5)-(1.6). We conclude that there is a one-to-one correspondence between $\{(s, \lambda(s)) \mid s \in (1, \infty)\}$ and solutions of (1.5)-(1.6).

Remark 2.2. It is easy to check from (2.13) that the function $b(s)$ is not monotone in s. See also Figure 1.

3. Proof of Theorem 1.1

This section is devoted to the proof of our main theorem, Theorem 1.1. Note that (2.15) is valid for all $\alpha \geq 0$. Note that $\lambda(s) > 0$ for all $s \in (1, \infty)$. Also, it is easy to check from (2.15) that $\lambda(1^+) = \lambda(+\infty) = 0$.

Given a fixed $\alpha \geq 0$. Writing (2.15) as $\lambda = \lambda(s) = [C(s)]^2/\{2[D(s)]^3\}$, where

$$C(s) := \sqrt{s(s-1)} + A(s) + 4\alpha \left(2s - 1 + \sqrt{\frac{s-1}{s}} A(s) \right) A(s),$$

$$D(s) := 2s - 1 + \sqrt{\frac{s-1}{s}} A(s).$$

Then $\lambda'(s) = 0$ if and only if $2C'(s)D(s) = 3C(s)D'(s)$. To proceed further, we first compute

$$A'(s) = \frac{1}{2\sqrt{s(s-1)}}, \quad D'(s) = 2 + \frac{1}{2s} + \frac{A(s)}{2s\sqrt{s(s-1)}},$$

$$C'(s) = \frac{s}{\sqrt{s(s-1)}} + 4\alpha \left[2 + \frac{1}{2s} + \frac{A(s)}{2s\sqrt{s(s-1)}} \right] A(s)$$

$$+ 2\alpha \left[2s - 1 + \sqrt{\frac{s-1}{s}} A(s) \right] \frac{1}{\sqrt{s(s-1)}}.$$

Hence $\lambda'(s) = 0$ if and only if $P_\alpha(s) := E(s) + \alpha F(s) = 0$, where

$$E(s) := \frac{3}{2s\sqrt{s(s-1)}} A^2(s) + \left(4 + \frac{3}{s} \right) A(s) + \frac{4s^2 - 5s - 3}{2\sqrt{s(s-1)}},$$

$$F(s) := \frac{2}{s^2} A^3(s) + \frac{2(4s-3)}{\sqrt{s(s-1)}} A^2(s) + \frac{2(2s-1)(4s-3)}{s} A(s) - \frac{4(2s-1)^2}{\sqrt{s(s-1)}}.$$

Lemma 3.1. *Both $E(s)$ and $F(s)$ are strictly increasing for $s > 1$.*

Proof. First, we compute

$$E'(s) = \frac{8s^3 - 4s^2 + 9s - 9}{4s(s-1)\sqrt{s(s-1)}} - \frac{6s-9}{2s^2(s-1)} A - \frac{3(4s-3)}{4s^2(s-1)\sqrt{s(s-1)}} A^2$$

$$:= \frac{1}{4s^2(s-1)\sqrt{s(s-1)}} G(s), \quad \text{where}$$

$$G(s) := (8s^4 - 4s^3 + 9s^2 - 9s) - 2(6s-9)\sqrt{s(s-1)} A - 3(4s-3)A^2.$$

Using the property that $A(s) < \sqrt{s-1} < \sqrt{s(s-1)}$ for $s > 1$, we have

$$G(s) > (8s^4 - 4s^3 + 9s^2 - 9s) + 6\sqrt{s(s-1)}A - 2(6s-6)[s(s-1)] - 3(4s-3)(s-1)$$

$$> 8s^4 - 16s^3 + 21s^2 - 9 = 8s^2(s-1)^2 + (13s^2 - 9) > 0, \ \forall s > 1.$$

Hence $E'(s) > 0$ for $s > 1$.

Next, we consider the function $F(s)$. As before, we first compute

$$
\begin{aligned}
F'(s) \;=\; & -\frac{4}{s^3}A^3 + \frac{2s^2 - 3}{s^2(s-1)\sqrt{s(s-1)}}A^2 \\
& + \frac{2(2s-1)(4s^2-3)}{s^2(s-1)}A - \frac{(4s-5)(2s-1)(s+1)}{s(s-1)\sqrt{s(s-1)}}.
\end{aligned}
$$

We rewrite $F'(s) = G_1(s) + G_2(s)$, where

$$
G_1(s) := -\frac{4}{s^3}A^3 - \frac{1}{s^2(s-1)\sqrt{s(s-1)}}A^2 + \frac{2(2s-1)}{s^2(s-1)}A,
$$

$$
G_2(s) := \frac{2s^2-2}{s^2(s-1)\sqrt{s(s-1)}}A^2 + \frac{2(2s-1)(4s^2-4)}{s^2(s-1)}A - \frac{(4s-5)(2s-1)(s+1)}{s(s-1)\sqrt{s(s-1)}}.
$$

We claim that both $G_1(s)$ and $G_2(s)$ are positive for $s > 1$.

For G_1, we use again $A(s) < \sqrt{s-1} < \sqrt{s(s-1)}$ and obtain

$$
G_1(s) > \frac{A}{s^3(s-1)}\{2s(2s-1) - 4(s-1)^2 - s\} = \frac{A}{s^3(s-1)}(5s-4) > 0
$$

for $s > 1$. For G_2, we rewrite it as

$$
G_2(s) = \frac{2s^2-2}{s^2(s-1)\sqrt{s(s-1)}}A^2 + \frac{8(2s-1)(s+1)}{s^2}g(s), \; g(s) := A - \frac{s(4s-5)}{8(s-1)\sqrt{s(s-1)}}.
$$

We compute

$$
g'(s) = \frac{8s^2 - 14s + 3}{16(s-1)^2\sqrt{s(s-1)}}.
$$

It is easy to see that $g(s)$ has a global minimum at $s = 3/2$ for $s \in (1, \infty)$. Since $g(3/2) > 0$, so $g(s) > 0$ for $s > 1$. Hence $G_2(s) > 0$ for $s > 1$. We conclude that $F'(s) > 0$ for $s > 1$ and the lemma is proved. $\qquad\square$

Now, we are ready to prove Theorem 1.1 as follows.

Proof of Theorem 1.1. Given a fixed $\alpha \geq 0$. First, we note that $P_\alpha(1^+) = -\infty$ and $P_\alpha(\infty) = \infty$. Since $P_\alpha(s)$ is strictly increasing in s, by Lemma 3.1, there is a unique $s^* \in (1, \infty)$ (depending on α) such that $P_\alpha(s^*) = 0$. Hence $\lambda'(s) = 0$ if and only if $s = s^*$. Therefore, Theorem 1.1 follows immediately with $\lambda^* := \lambda(s^*)$, since $\lambda(s) > 0$ for $s > 1$ and $\lambda(1^+) = \lambda(+\infty) = 0$. $\qquad\square$

REFERENCES

[1] O. Drosinou, N.I. Kavallaris, C.V. Nikolopoulos, *A study of a nonlocal problem with Robin boundary conditions arising from MEMS technology*, preprint (arXiv:1906.12093).

[2] P. Esposito and N. Ghoussoub, *Uniqueness of solutions for an elliptic equation modeling MEMS*, Methods Appl. Anal., 15 (2008), 341–354.

[3] G. Flores, G. Mercado, J.A. Pelesko and N. Smyth, *Analysis of the dynamics and touchdown in a model of electrostatic MEMS*, SIAM J. Appl. Math., 67 (2007), 434–446.

[4] N. Ghoussoub and Y. Guo, *On the partial differential equations of electrostatic MEMS devices: stationary case*, SIAM J. Math. Anal., 38 (2007), 1423–1449.

[5] J.-S. Guo, *On a quenching problem with the Robin boundary condition*, Nonlinear Anal., 179 (1991), 803-809.

[6] J.-S. Guo, B. Hu and C.-J. Wang, *A nonlocal quenching problem arising in micro-electro mechanical systems*, Quart. Appl. Math., 67 (2009), 725-734.

[7] J.-S. Guo and N.I. Kavallaris, *On a nonlocal parabolic problem arising in electrostatic MEMS control*, Discrete Contin. Dyn. Syst., 32 (2012), 1723-1746.

[8] Y. Guo, Z. Pan and M.J. Ward, *Touchdown and pull-in voltage behavior of a MEMS device with varying dielectric properties*, SIAM J. Appl. Math., 166 (2006), 309–338.

[9] Z. Guo and J. Wei, *Asymptotic Behavior of touch-down solutions and global bifurcations for an elliptic problem with a singular nonlinearity*, Commun. Pure Appl. Anal., 7 (2008), 765–786.

[10] N.I. Kavallaris, A.A. Lacey and C.V. Nikolopoulos, *On the quenching of a nonlocal parabolic problem arising in electrostatic MEMS control*, Nonlinear Anal., 138 (2016), 189–206.

[11] N.I. Kavallaris, T. Miyasita and T. Suzuki, *Touchdown and related problems in electrostatic MEMS device equation*, NoDEA Nonlinear Differential Equations Appl., 15 (2008), 363–385.

[12] N.I. Kavallaris and T. Suzuki, *"Non-Local Partial Differential Equations for Engineering and Biology: Mathematical Modeling and Analysis"*, Math. Ind. (Tokyo), Vol. 31, Springer Nature, 2018.

[13] H.A. Levine, *Quenching, nonquenching, and beyond quenching for solutions of some parabolic equations*, Ann. Mat. Pura Appl., 155 (1989), 243-260.

[14] J.A. Pelesko and D.H. Bernstein, *"Modeling MEMS and NEMS"*, Chapman Hall and CRC Press, 2002.

[15] J.A. Pelesko and T.A. Driscoll, *The effect of the small-aspect-ratio approximation on canonical electrostatic MEMS models*, J. Engrg. Math, 53 (2005), 239-252.

[16] J.A. Pelesko and A.A. Triolo, *Nonlocal problems in MEMS device control*, J. Engrg. Math., 41 (2001), 345–366.

DEPARTMENT OF MATHEMATICS, TAMKANG UNIVERSITY, TAMSUI, NEW TAIPEI CITY 251301, TAIWAN
Email address: jsguo@mail.tku.edu.tw

DEPARTMENT OF MATHEMATICAL AND PHYSICAL SCIENCES, UNIVERSITY OF CHESTER, THORNTON SCIENCE PARK, POOL LANE, INCE, CHESTER CH2 4NU, UK
Email address: n.kavallaris@chester.ac.uk

DEPARTMENT OF MATHEMATICS, NATIONAL CHUNG CHENG UNIVERSITY, MINHSIUNG, CHIAYI 62102, TAIWAN
Email address: cjwang@ccu.edu.tw

DEPARTMENT OF MATHEMATICS, TAMKANG UNIVERSITY, TAMSUI, NEW TAIPEI CITY 251301, TAIWAN
Email address: cherngyi@mail.tku.edu.tw