

Results for unbounded star-graphs



Theorem 3

Assume $m > n$. Then the followings hold:

(1) If $0 < a < \frac{n}{m+n}$ or $\frac{1}{2} \leq a < 1$, (1) does not possess an equilibrium solution.

(2) If $a = \frac{n}{m+n}$, (1) possesses a unique equilibrium solution u of the form
$$u(x) = \frac{n}{m+n} e^x \quad (x \leq 0), \quad 1 - \frac{m}{m+n} e^{-x} \quad (x > 0).$$

(3) If $\frac{n}{m+n} < a < \frac{1}{2}$, (1) possesses exactly two equilibrium solutions $u_- < u_+$ of the form

$$u_-(x) = \begin{cases} \frac{2a-1}{2} e^{x+x_0} + \frac{1}{2} e^{x-x_0} & (x \leq 0) \\ \frac{2a-1}{2} e^{-x+x_0} + \frac{1}{2} e^{x-x_0} & (0 < x \leq x_0) \\ 1 - (1-a) e^{-x+x_0} & (x > x_0) \end{cases}$$
$$u_+(x) = \begin{cases} a e^{x+x_0} & (x \leq -x_0) \\ 1 + \frac{2a-1}{2} e^{x+x_0} - \frac{1}{2} e^{-x-x_0} & (-x_0 < x \leq 0) \\ 1 + \frac{2a-1}{2} e^{-x+x_0} - \frac{1}{2} e^{x-x_0} & (x > 0) \end{cases}$$

$$\text{with } x_0 = \frac{1}{2} \log \frac{n-m}{(2a-1)(m+n)}.$$

