CHAP 10 Inventory Model

10.1 Introduction

A. Inventory: any idle goods that are mainting to be used

— cycle stock: inventory in curved by ordering process

— safety stock: inventory used to protected stockout

B. Inventory carrying cost

— cost associated with maintaining a given level of inventories:

(1) cost of finance

· interest - if it is borrowed

· opportunity cost -if it is owned

(2) other cost

· insurance

· taxes

· breakage

C. Reasons

— periodic order

(1) cycle stock inventory

(2) advantage

· time utility

· economic of scale

· quantity discount

— fluctuation in demand and delivery

(1) protect stockout

(2) maintain a level of customer service

D. Objective

— minimum total inventory cost to maintain sufficient inventories.

E. Cost components

— inventory carrying cost (holding cost)

— order cost

(1) order -to other

(2) setup own

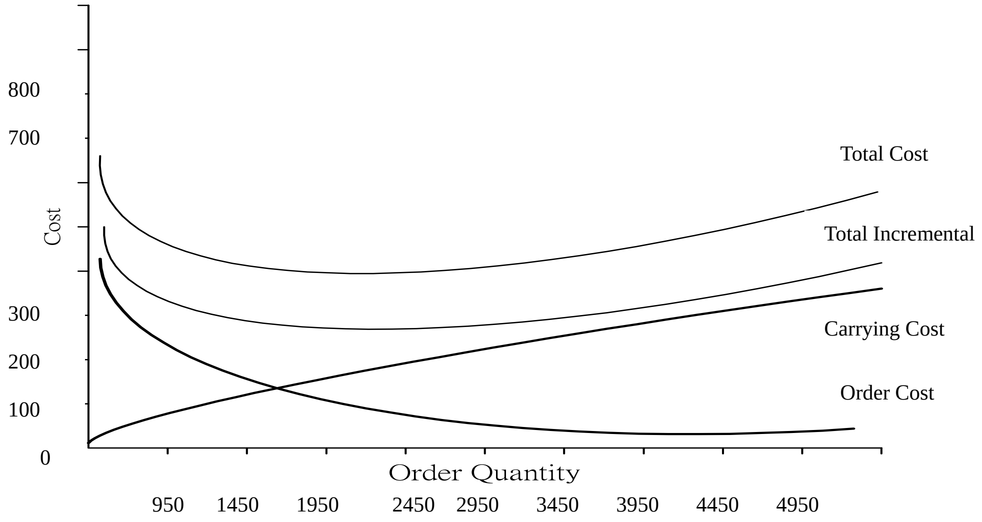
— stockout cost

(1) loss of scale

(2) loss of goodwill

(3) backorder

— purchase cost



F. Inventory model

— deterministic

(1) static

(2) dynamic

— probabilitic

(1) stationary

(2) unstationary

10.2 Economic Order Quantity(EOQ) Model

A. Basic assumptions

— constant demand rate

— uniform purchase cost

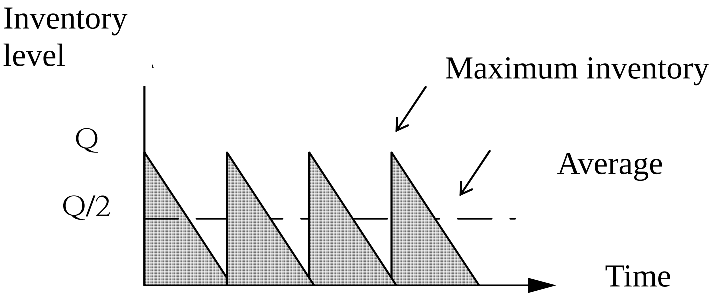
— no stockout

B. Basic question

— how much to order

— when to order - reorder point

C. Inventory pattern



D. Total cost method

T.C.=purchase cost + inventory carrying cost + order cost

$$= C \bullet D + C_h \bullet \frac{Q}{2} + C_0 \bullet \frac{D}{Q}$$

$$\frac{dTC(Q)}{d(Q)} = \frac{C_h}{2} - \frac{C_0 D}{Q^2} = 0$$

$$\frac{d^2 T(Q)}{dQ} = \frac{2C_0 D}{Q^3} > 0$$

$$Q^* = \sqrt{\frac{2C_0 D}{C_h}}$$

E. Reorder point

— r = d ・ m

r = reorder time

d = daily demand

m = lead time = the time between place an order and receive the order

F. Cycle time

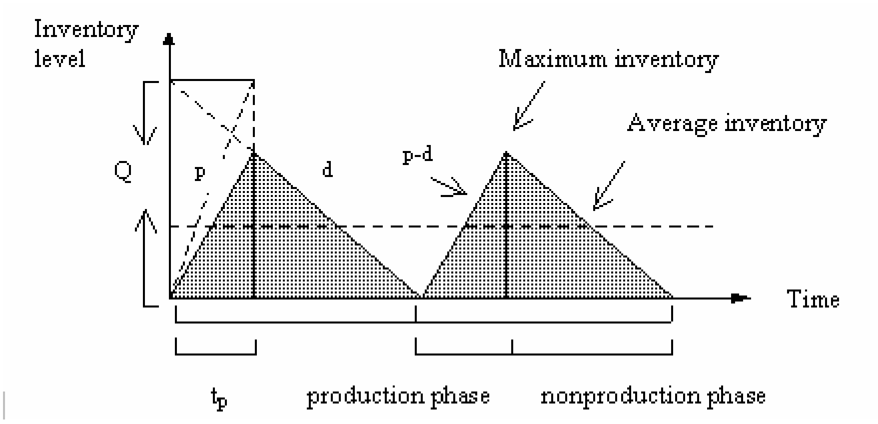
— the time between two replenishments

$$T = \frac{250Q^*}{D}$$

G. Sensitivity of EOQ

— the Q^* is insensitive to small variation or errors in the cost estimated

H. Manager's adjustment



— to adjust to move practical order quantity

— to adjust the reorder point to protect stockout by adding safe stock

Example：某工廠爲了滿足生產的需要，定期向外單位訂購一種零件。這種零件平均每日需求量爲100個，每個零件一天的儲存費爲0.02元，訂購一次的費用爲100元。假設不允許缺貨，求最佳訂購量Q^*、訂購間隔期t^*和單位時間總費用TC^*。

Sol：

$$Q^* = \sqrt{\frac{2C_0D}{C_h}} = \sqrt{\tfrac{2\times100\times100}{0.02}} = 1000\text{個}$$

$$t^* = \frac{Q^*}{D} = \tfrac{1000}{100} = 10\text{天}$$

$$TIC^* = \sqrt{2DC_oC_h} = \sqrt{2\times100\times100\times0.02} = 20\text{元}/\text{天}$$

Example：某公司有擴充業務的計劃，每年需要招聘和培訓新的工作人員60名，開班培訓一次需費用1000元，每位應聘的員工年薪爲540元，所以公司不願意在不需要時招聘訓練這些人員。另一方面，在需要時卻又不能延誤，所以人員需事先完成訓練。在訓練期間，雖未正式錄用仍需支付全薪。問每次需訓練幾名人員才經濟？隔多久辦一期訓練班？全年的總費用爲多少？

Sol：

$$Q^* = \sqrt{\frac{2C_0D}{C_h}} = \sqrt{\tfrac{2\times60\times1000}{540}} = 15\text{人}$$

$$t^* = \frac{Q^*}{D} = \tfrac{15}{60} = 0.25\text{年} = 3\text{月}$$

$$TC^* = \sqrt{2DC_oC_h} = \sqrt{2\times1000\times60\times540} = 8050\text{元}/\text{年}$$

10.3 Economic Production Lot Size Model

A. Basic assumption

— constant supply rate

B. Production lot size

— the optimal production quantity per production

C. Cost components

— C_h=inventory carrying cost

— C_o=production set up cost - laber, material, lost production

D. Maximum inventory

$$(p-d)\bullet t=(p-d)\bullet \tfrac{Q}{p}$$
$$=\left(1-\tfrac{d}{p}\right)\bullet Q$$
$$=\left(1-\tfrac{D}{P}\right)\bullet Q$$

where p:daily production rate

d:daily demend

P:annual production

D:annual demand

E. Total cost model

$$T.C.=\frac{1}{2}\left(1-\frac{D}{p}\right)Q\bullet C_h+\frac{D}{Q}\bullet C_0$$

$$\frac{dTC}{dQ}=\frac{1}{2}\left(1-\frac{D}{p}\right)C_h-\frac{D}{Q^2}\bullet C_0=0$$

$$\Rightarrow Q^*=\sqrt{\frac{2DC_0}{\left(1-\frac{D}{p}\right)C_h}}$$

Example ：美容相皂一條生產線的年產能爲60000箱，年需求估計26000箱，需求率固定、清潔、準備及設立等費用每次135元。製造成本每箱4.5元，年持有成本爲24%，$C_h=IC=0.24\times 4.5=1.08$元 。建議生產量爲何?

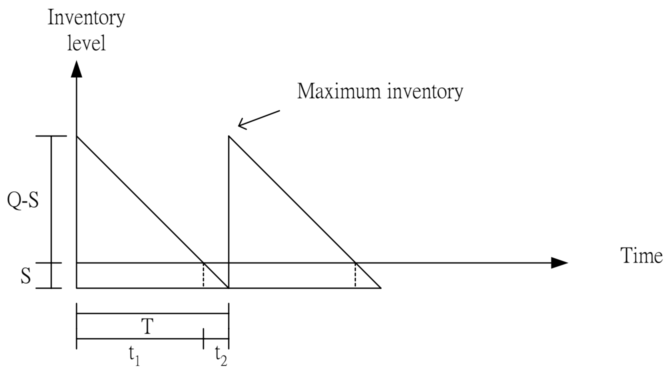
Sol:

$$Q^*=\sqrt{\frac{2DC_0}{\left(1-\frac{D}{p}\right)C_h}}=\sqrt{\frac{2\times 26000\times 135}{\left(1-\frac{26000}{60000}\right)\times 1.08}}=3387$$

年總成本$T.C.=\frac{1}{2}\left(1-\frac{D}{p}\right)Q\bullet C_h+\frac{D}{Q}\bullet C_0$=2073元

生產週期間隔$T=\frac{250Q^*}{D}=\frac{250\times 3387}{26000}\cong 33$個工作天 ，這表示每隔33個工作天要規劃生產3387箱。

10.4 Inventory Model with Planned Stockout



A. Desirable shortout

— high inventory carrying cost

B. Backorder

— if out-of-stock, the consumer doesn't withdraw the order

— short backorder period

— immediately delivery

— average inventory $= \dfrac{\frac{1}{2}(Q-S)t_1 + 0t_2}{t_1+t_2}$

$$= \frac{\frac{1}{2}(Q-S)t_1}{T}$$

$$= \frac{(Q-S)^2}{2Q}$$

$$t_1 = \frac{Q-S}{d}, t_2 = \frac{S}{d}, T = \frac{Q}{d}$$

— average backorder level $= \dfrac{\left(\frac{S}{2}\right)\left(\frac{S}{d}\right)}{\frac{Q}{d}}$

$$= \frac{S^2}{2Q}$$

C. Total Cost

— $T.I.C. = \frac{(Q-S)^2}{2Q}C_h + \frac{D}{Q}C_0 + \frac{S^2}{2Q}C_b$

$$= \frac{D}{Q}\left(\frac{C_h}{2} \times \frac{(Q-S)^2}{D} + C_0 + \frac{C_0}{2} \times \frac{S^2}{D}\right)$$

$$= \frac{Q^2 - 2QS + S^2}{2Q} \times C_h + \frac{D}{Q}C_0 + \frac{S^2}{2Q}C_b$$

$$= \frac{C_h + C_b}{2Q} \times S^2 - S \times C_h + \frac{QC_h}{2} + \frac{DC_0}{Q}$$

$$\frac{\partial TC(Q,S)}{\partial S} = 0 \Rightarrow \frac{C_h + C_b}{Q} \times S - C_h = 0$$

$$S^* = Q \times \frac{C_h}{C_h + C_b}$$

$$\frac{\partial TC(Q,S)}{\partial Q} = 0 \Rightarrow \frac{-(C_h + C_b)S^2}{2Q^2} + \frac{C_h}{2} - \frac{DC_0}{Q^2} = 0$$

$$\Rightarrow \frac{-(C_h + C_b)Q^2 C_h^2}{2Q^2 (C_h + C_b)^2} - \frac{C_h}{2} - \frac{DC_0}{Q^2} = 0$$

$$\Rightarrow \frac{-C_h^2}{2(C_h + C_b)} + \frac{C_h}{2} - \frac{DC_0}{Q^2} = 0$$

$$\Rightarrow \frac{-C_h^2 + C_h (C_h + C_b)}{2(C_h + C_b)} - \frac{DC_0}{Q^2} = 0$$

$$\Rightarrow \frac{C_h C_b}{2(C_h + C_b)} - \frac{DC_0}{Q^2} = 0$$

$$\Rightarrow Q^2 = \frac{2DC_0 (C_h + C_b)}{C_h C_b} = \left(\frac{2DC_0}{C_h}\right)\left(\frac{C_h + C_b}{C_b}\right)$$

$$Q^* = \sqrt{\left(\frac{2DC_0}{C_h}\right)\left(\frac{C_h + C_b}{C_b}\right)}$$

$$T.I.C.(Q,S) = \sqrt{2DC_0 C_h \left(\frac{C_b}{C_b + C_h}\right)}$$

$$T.I.C.(Q) = \sqrt{2DC_0 C_h}$$

D. Observations

— $TC(Q,S) \le TC(Q)$

— $Q^* \ge Q^*$ *for* EOQ

— if C_b become larger relative to $C_h \Rightarrow S^* = 0$

if C_h become larger relative to $C_b \Rightarrow S^*$ *increase*

Example:

$D = 2000,\ C_h = 10,\ C_o = 25,\ C_b = 30$

$$Q^* = \sqrt{\frac{2(2000)25}{10} \times \left(\frac{10+30}{30}\right)} = 115$$

$$S^* = 115 \times \frac{10}{10+30} = 29$$

$$T = \frac{115}{2000}(250) = 14.4$$

$$T.I.C = \sqrt{2(2000)(25)(10)\left(\frac{30}{10+30}\right)} = 866$$

Example:某工廠爲了滿足生產的需要，定期向外單位訂購一種零件。這種零件平均每日需求量爲100個，每個零件一天的儲存費爲0.02元，訂購一次的費用爲100元。假設允許缺貨，每個零件缺貨一天的損失費爲0.08元，求最佳訂購量Q^*、求最佳缺貨量S^*、訂購間隔期t^*和單位時間總費用TC^*。

Sol:

$$Q^* = \sqrt{\left(\frac{2DC_0}{C_h}\right)\left(\frac{C_h+C_b}{C_b}\right)} = 1000 \times \sqrt{\frac{0.08+0.02}{0.08}} = 1118\text{個}$$

$$S^* = \sqrt{\left(\frac{2DC_0}{C_b}\right)\left(\frac{C_h}{C_h+C_b}\right)} = 500 \times \sqrt{0.2} = 224\text{個}$$

$$T.C.(Q,S) = \sqrt{2DC_0C_h\left(\frac{C_b}{C_b+C_h}\right)} = 20\sqrt{0.8} \approx 18\text{元}/\text{天}$$

$$t^* = \frac{Q^*}{D} = \frac{1118}{100} = 11.2\text{天}$$

10.5 Quantity Discounts for EOQ Model

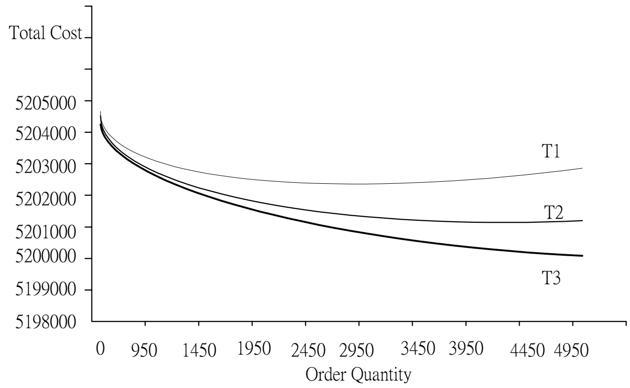
A. Supplier provide incentives for large purchase quantity by offering discount

B. Procedures

— step 1 : Based on discount prices, compute EOQ(Q^*) for each category i

— step 2 : If $Q_i^* \le \min Q_i \Rightarrow Let \quad Q_i^* = \min Q_i$

— step 3 : identify Q_i^* with smallest total cost



Example :

$$Q_1^* = \sqrt{\frac{2(5000)(48)}{(0.2)(5.00)}} = 700$$

$$Q_2^* = \sqrt{\frac{2(5000)(48)}{(0.2)(4.85)}} = 711$$

$$Q_3^* = \sqrt{\frac{2(5000)(48)}{(0.2)(4.75)}} = 718$$

$$T.C. = \sqrt{2DC_0C_h}$$

$$Q_1^* = 700 \Rightarrow TC = 25700$$

$$Q_2^* = 1000 \Rightarrow TC = 24980$$

$$Q_3^* = 2500 \Rightarrow TC = 25036$$

Example: 體育用品店某運動鞋年需求量10000雙，訂購成本每次90元，每雙鞋購進價格200元，存貨成本每年每雙約爲購進價格之10%。今有如下所列之數量折扣，求最佳訂購量？

數量	折扣	價格	存貨成本
0~249	0%	200	20
250~999	10%	180	18
1000以上	20%	160	16

Sol:

訂購範圍1000以上：

$$Q_3^* = \sqrt{\tfrac{2\times10000\times90}{16}} = 335 < 1000$$

$$TC_3 = 90\times\tfrac{10000}{1000} + 16\times\tfrac{1000}{2} + 10000\times160 = 1608900$$

訂購範圍250~999：

$$Q_2^* = \sqrt{\tfrac{2\times10000\times90}{18}} = 316$$

$$250 < Q_2^* < 999$$

$$TC_2 = 90\times\tfrac{10000}{316} + 18\times\tfrac{316}{2} + 10000\times180 = 1805692$$

訂購範圍0~249：

$$Q_1^* = \sqrt{\tfrac{2\times10000\times90}{20}} = 300$$

$$250 < Q_1^* < 999$$

$$TC_1 = 90\times\tfrac{10000}{249} + 20\times\tfrac{249}{2} + 10000\times200 = 2006000$$

故選擇一次訂購1000雙。

10.6 EOQ Under Constraint in storage

A. Limited on storage space

— One type of commodity

(1) use EOQ model to find Q^*

(2) if $Q^* \le K$ (Quantity limit) then use Q^* else let $Q^* = K$

— Multiple commodities

(1) use linear programming to solve the problem

(2) objective function

$$\min TC = \sum\left(C_{oi}\tfrac{D_i}{Q_i} + C_{hi}\tfrac{Q_i}{2} + W_i K_i Q_i\right)$$

$$s.t. \sum_i^n K_i Q_i = K$$

where W_i=warehouse rate for commodity i

K_i =size of commodity per unit

K =total space allowable

(3) solution approach

· step 1 : formulating the model

· step 2 : use unconstrained EOQ model to identify each Q^*

· step 3 : if $\sum K_i Q_i^* \le K$ stop, otherwise go to step 4.

· step 4 : use (Lagrangian function)

$$L(Q,\lambda) = \sum \left(C_{oi} \frac{D_i}{Q_i} + C_{hi} \frac{Q_i}{2} + W_i K_i Q_i \right) + \lambda \left(\sum K_i Q_i - K \right)$$

· step 5 : partial differential of $L(Q,\lambda)$ with respect to Q_i and λ

$$\frac{\partial L(Q_I,\lambda)}{\partial Q_i} = 0 \quad \text{and} \quad \frac{\partial L(Q_I,\lambda)}{\partial \lambda} = 0$$

$$\Rightarrow Q_i = \sqrt{\frac{2D_i C_{oi}}{C_{hi} + 2K_i(\lambda + W_i)}}$$

· step 6 : use trial -and-error identify λ^* such that $\sum K_i \sqrt{\frac{2D_i C_{oi}}{C_{hi} + 2K_i(\lambda^* + W_i)}} - K = 0$

· step 7 : substitute λ^* into Q_i equation in step 5

Example:某工廠有一面積爲25m^{2}的倉庫，它儲存三種物資，每種物資不能缺貨，且能及時得到供貨。假設每種物資每單位佔用的面積均爲1 m^2，第 i 種物資的訂購量C_{oi}，單位時間的需求量D_i，單位物資單位時間儲存費C_{hi}。求每種物資的最佳批量？

物資 i	C_{oi}	D_i	C_{hi}
1	10	2	0.3
2	5	4	0.1
3	15	4	0.2

Sol:

$$\min C(Q_1,Q_2,Q_3)=\sum_{i=1}^{3}\left(C_{oi}\tfrac{D_i}{Q_i}+C_{hi}\tfrac{Q_i}{2}\right)$$

$$s.t.\sum_{i=1}^{3}1\times Q_i\le 25$$

$$Q_1,Q_2,Q_3\ge 0$$

首先不管倉庫面積的約束條件，用EOQ算出各物資的經濟批量：

$Q_1=11.5, Q_2=20.0, Q_3=24.5$　　佔用面積爲11.5+20+24.5=56>25，因此上述約束條件是起作用的。

$$L(\lambda,Q_1,Q_2,Q_3)=\sum_{i=1}^{3}\left(C_{oi}\tfrac{D_i}{Q_i}+C_{hi}\tfrac{Q_i}{2}\right)-\lambda\left(\sum_{i=1}^{3}Q_i-25\right)$$

其中$\lambda<0$

令

$$\frac{\partial L}{\partial Q}=-\frac{R_iD_i}{Q_i^2}+\frac{C_{hi}}{2}-\lambda=0 \qquad \text{i=1,2,3}$$

$$\frac{\partial L}{\partial \lambda}=-\sum_{i=1}^{3}Q_i+25=0$$

$$\Rightarrow Q_i=\sqrt{\tfrac{2C_{oi}D}{C_{hi}-2\lambda}}$$

經過試算如下表：

λ	Q_1	Q_2	Q_3	$-\sum_{i=1}^{3}Q_i+25$
0	11.5	20.0	24.5	+31.0
-0.25	7.1	8.2	11.3	+1.6
-0.30	6.7	7.6	10.6	-0.1

可見，$\lambda^*\approx -0.30, Q_1^*=6.7, Q_2^*=7.6, Q_3^*=10.6$

10.7 Inventory Model with Probability Demand

A.Q-system

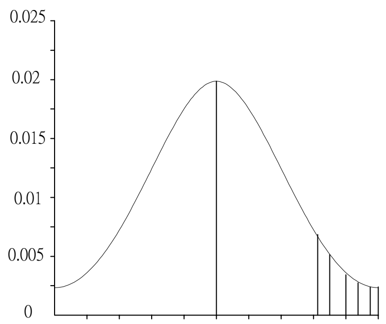
— fixed order quantity(EOQ)

— continuous review (variable order interral)

B.Lead time

— the time between order is placed and the goods is received

C.To specify the allowable probability - reorder point of stockout



D.Stockout cost

— assume a fixed penalty cost for stockout(ρ)

— the expected number of stockout

$$B(r) = \int_0^r 0 f(x)dx + \int_r^\infty (x-r) f(x)dx = \int_r^\infty (x-r) f(x)dx$$

— stockout cost function

$$SC = \rho\left(\tfrac{D}{Q}\right)B(r)$$

— order cost

$$OC = C_o \tfrac{D}{Q}$$

— inventory cost

$$IVC = C_h\left(\tfrac{Q}{2} + r - \bar{x}\right)$$

E.Total cost

$$TC(Q,r) = C_o \tfrac{D}{Q} + \rho\left(\tfrac{D}{Q}\right)B(r) + C_h\left(\tfrac{Q}{2} + r - \bar{x}\right)$$

$$\tfrac{TC}{\partial Q} = -C_o \tfrac{D}{Q^2} - \rho\left(\tfrac{D}{Q^2}\right)B(r) + \tfrac{C_h}{2}$$

F.Solution approach-Henristic

— step1: let $B(r) = 0 \Rightarrow Q_1^* = \sqrt{\tfrac{2DC_o}{C_h}} = EOQ$ (for deterministic)

— step2: use $Q_i^* \Rightarrow r_i^*$

— step3: use $r_i^* \Rightarrow B(r_i)^*$

－step4: use $B(r_i)^* \Rightarrow Q_{i+1}^*$

－step4: set i=i+1 repeat step2, 3, 4 until $Q_{i+1}^* - Q_i^* < \varepsilon \quad and \quad r_{i+1}^* - r_i^* < \varepsilon$, stop

Example:

The lead time is 5.2 weeks(1/10 years) with demand mean(μ) of 1,000 units and

$\sigma = 250, C_0 = 100, C_h = 0.15$ per unit and $\rho = 1.00$ per unit

sol:

$$Q_1 = \sqrt{\frac{2DC_o}{C_h}} = \sqrt{\frac{2(10000)(100)}{0.15}} = 3651.5$$

$$\frac{d(B_1)}{dr} = \int_r^\infty f(x)dx = \frac{Q}{\rho}\frac{C_h}{D} = \frac{3651.5(0.15)}{1(1000)} = 0.05$$

$$\Rightarrow \frac{r_1 - 1000}{250} = 1.60, r_1 = 1400$$

$$B(r_1) = \rho f\left(\frac{r-\mu}{\sigma}\right) + (\mu - r)G\left(\frac{r-\mu}{\sigma}\right)$$

$$= 250f(1.60) + (1000 - 1400)G(1.60)$$

$$= 250(0.111) - 400(0.055)$$

$$= 5.822$$

$$\Rightarrow Q_2^* = \sqrt{\frac{2D(C_o + \rho B(r_1))}{C_h}} = \sqrt{\frac{2(10000)(100+5.822)}{0.15}} = 3756.27$$

$$\int_{r_2}^\infty f(x)dx = \frac{Q_2 C_h}{\rho D} = 0.0563$$

$$\frac{r_2 - 1000}{250} = 1.586, r_2 = 1396.5$$

$$B(r_2) = 250f(1.586) - 396.5G(1.586)$$

$$= 5.99$$

$$\Rightarrow Q_3^* = 3759.86$$

Example:某商品年需求量爲1000個，定購成本每次100元，存貨成本2元/個/年，其需求於前置時間內爲0~100個之連續均等分配，缺貨的懲罰成本每單位10元，求最佳訂購量及訂購點？

Sol: $D = 1000, C_o = 100, C_h = 2, \rho = 10, f(x) = \frac{1}{100}$

$$Q_1 = \sqrt{\frac{2DC_o}{C_h}} = \sqrt{\frac{2\times1000\times100}{2}} = 316.227$$

$$\int_{r_1}^{100} f(x)dx = \frac{C_h Q_1}{\rho D} = \int_{r_1}^{100} \frac{1}{100} = \frac{2\times316}{10\times1000}$$

$$\left.\frac{x}{100}\right|_{r_1}^{100} = 0.0632 \quad \Rightarrow r_1 = 93.68$$

$$B(r_1) = \int_{r_1}^{100} (x - r) f(x)dx = \int_{94}^{100} (x - 94)\times\frac{1}{100}dx = 0.18$$

$$Q_2 = \sqrt{\frac{2D(C_o+\rho B(r))}{C_h}} = \sqrt{\frac{2\times1000\times(100+10\times0.18)}{2}} = 319.06$$

$$\int_{r_2}^{100} f(x)dx = \frac{C_h Q_2}{\rho D} = \int_{r_2}^{100} \frac{1}{100} = \frac{2\times319}{10\times1000}$$

$$\left.\frac{x}{100}\right|_{r_2}^{100} = 0.0638 \quad \Rightarrow r_2 = 93.62$$

$$B(r_2) = \int_{r_2}^{100} (x-r) f(x)dx = \int_{93.6}^{100} (x-93.6)\times\frac{1}{100}\,dx = 0.204$$

$$Q_3 = \sqrt{\frac{2D(C_o+\rho B(r))}{C_h}} = \sqrt{\frac{2\times1000\times(100+10\times0.204)}{2}} = 319.437$$

$$\int_{r_3}^{100} f(x)dx = \frac{C_h Q_3}{\rho D} = \int_{r_3}^{100} \frac{1}{100} = \frac{2\times319.437}{10\times1000}$$

$$\left.\frac{x}{100}\right|_{r_3}^{100} = 0.0639 \quad \Rightarrow r_3 = 93.61$$

$$Q_3 - Q_2 = 0.163$$

$$r_3 - r_2 = 0.010 \qquad \Rightarrow r_3 \approx r_2 \Rightarrow stop$$

當商品庫存量達到約94個時需發出訂購320個的訂單。

10.8 The Newsboy Problem

A.A single period model

B.One order for entire process

C.No items are transfer to next period

D.Unknown demand

E.Trade-off between under too much and too little

F.Incremental approach

— cost comparison between order one additional unit and not ordering one additional unit

— the optimal Q^* is reached

when $EL(Q^*+1) = C_{ov}\times p(x\le Q^*) = C_u\times p(x\ge Q^*) = EL(Q^*)$ since

$$p(x\le Q^*) + p(x\ge Q^*) = 1$$

$$\Rightarrow C_{ov}\times p(x\le Q^*) = C_u\times p(x\ge Q^*) = C_u(1-p(x\le Q^*))$$

$$\Rightarrow p(x\le Q^*) = \frac{C_u}{C_U+C_{OV}}$$

G.Key element

— probability distribution of demand

— over estimation cost C_{ov}

— under estimation cost C_u

Example:

$$\mu = 1000, \sigma = 100, C_{ov} = 10, C_u = 4$$

sol:

$$p\left(z \le Q^*\right) = \tfrac{4}{4+10} = 0.29 \Rightarrow Z = 0.55$$
$$Q^* = \mu - z \times \sigma = 1000 - 0.55(100) = 945$$

Example:報童根據經驗，每天報紙的銷售量是服從 $N(150{,}625)$ 的分配，假設報紙進價每份3元，售價5元，如果賣剩了每份可以1元處理掉；如果缺貨則不會有直接損失。問報童每天應進貨多少份才能獲得的利潤期望值最大？

Sol:

$$P(x \le Q) = \frac{C_u}{C_u + C_{ov}} = \frac{2}{2+2} = 0.5$$
$$P\left(Z \le \tfrac{Q-150}{25}\right) = 0.5$$
$$\Rightarrow Q = 150$$

Example:某商品銷售根據經驗，季節銷售量是服從指數分配 $P(x) = \begin{cases} \frac{1}{10000} e^{-x/10000} & x \ge 0 \\ 0 & o.w. \end{cases}$，假設批發站商品進價每件10元，零售價35元，賣剩商品儲存費每件1元；如果缺貨則只有從市場上以零售價進貨。求批發站最佳進貨量？

Sol:

設c…..進貨價，p….. 單位缺貨損失費，h….. 單位儲存費。根據單時期無訂貨費的儲存模型，最佳訂貨量 Q^* 滿足方程式

$$\int_0^Q p(x)dx = \frac{p-c}{p+h} \qquad (p \ge c)$$
$$q\text{臨界值} = \frac{p-c}{p+h} = \frac{35-10}{35+1} = 0.6944$$
$$\Rightarrow \int_0^Q \frac{1}{10000} e^{-x/10000} dx = 1 - e^{-Q/10000} = 0.6944$$
$$\Rightarrow Q^* = 11856$$

即進貨11856件，比期望需求量 $\lambda = 10000$ 稍大些。

10.9 Variable Lead Time

A.To allow variability of both demand and delivery process

B.The lead-time demand density function(p.d.f)

$$f(x) = \int_0^\infty y(x:t)g(t)dt$$

$y(x:t) =$ conditional p.d.f. for lead time given lead time

$g(t) =$ p.d.f. for lead time t

C.Based on central limitation theorem, we assume f(x) is normal distribution

$$\mu = E(x)E(L)$$

$$\sigma^2 = E(L)\sigma^2(x) + E(x)^2\sigma^2(L)$$

Example:

$$E(x) = 50, \sigma(x) = 5, E(L) = 16, \sigma(L) = 2$$

$$\sigma^2(x, L) = (16)(5)^2 + (50)^2(2)^2 = 10{,}400$$

$\sigma^2(x) = 16(5)^2 = 400$ (lead time is not variable $\Rightarrow \sigma^2(L) = 0$

練習題：

1.某產品平均每日需求量約為 100 件，每個零件一天的儲存費 h=0.02 元，訂購一次的費用 k=100 元。如前置時間為 12 天，求最佳訂購量 Q*、訂購週期 t*、補貨點 r。

Sol:

$$Q^* = \sqrt{\frac{2kd}{h}} = \sqrt{\frac{2\times100\times100}{0.02}} = 1000(\text{件})$$

$$t^* = \frac{Q^*}{d} = \frac{1000}{100} = 10(\text{天})$$

＃因為前置時間為 12 天，而訂購週期為 10 天，因此當存貨尚數 2 天（＝12-10）時，即應訂貨。即當存貨水準為 200 件（100×2）時應訂貨 1000 件。

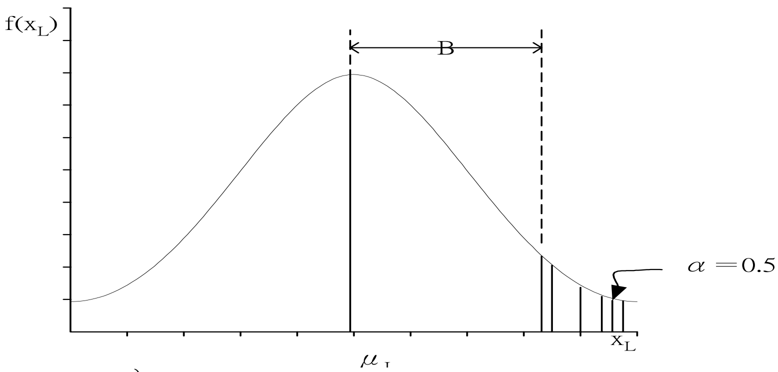
$$r = d \times m = 100 \times 2 = 200(\text{件})$$

2.承上題，當前置時間為 15 天，求補貨點。（Ans:500 件）
當前置時間為 23 天，求補貨點。（Ans:300 件）
當前置時間為 80 天，求補貨點。（Ans:800 件）
當前置時間為 10 天，求補貨點。（Ans:0 件）

3.假定在第 1 題中每天需求量近似於常態分配，其平均數 $\mu = 100$，標準差 $\sigma = 10$。如果希望在前置時間發生缺貨的機率最大為 0.05，求預期缺貨量 B。

Sol:

前置時間＝2 天，平均數 $\mu_L = 2\times100 = 200$ 件，標準差 $\sigma_L = \sqrt{2\times10^2} = 14.14$



$$P\{x_L \ge \mu_L + B\} \le \alpha$$

或

$$P\left\{\frac{x_L - \mu_L}{\sigma_L} \ge \frac{B}{\sigma_L}\right\} \le \alpha$$　　由標準常態查表得 $B/14.14 \ge 1.64$ 或 $B \ge 23.2$

或

$$P\left\{\frac{x_L - \mu_L}{\sigma_L} \ge \frac{B}{14.14}\right\} \le 0.05$$

4.例題 2 中，如每天需求量為常態分配，平均數 100 件，變異數 30。求解下列情況之缺貨量 B。
當前置時間為 15 天。（Ans: $\mu_L = 500, \sigma_L = 12.25, B \ge 20.1$）
當前置時間為 23 天。（Ans: $\mu_L = 300, \sigma_L = 9.49, B \ge 15.6$）
當前置時間為 80 天。（Ans: $\mu_L = 800, \sigma_L = 15.49, B \ge 25.4$）
當前置時間為 10 天。（Ans: $\mu_L = \sigma_L = 0, B = 0$）