

Chapter 9 Project Management CPM and PERT

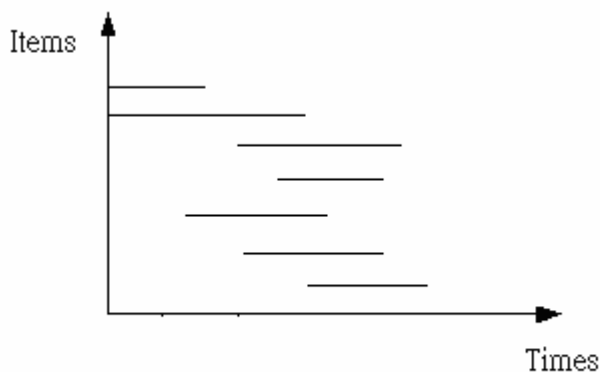
9.1 Introduction

A. Each project is consist of numerous jobs or tasks performed by a variety of departments or individuals

B. Project management

- Identify project goals or objectives
- Work breakdown structure
- Establish activities relationship
- Estimate activities times
- Estimate project completion time
- Post completion evaluation
- Resource allocations

C. Gantt chart



- Disadvantage of Gantt chart
 - Unable to reflect the interrelation among activities
 - No revelation of the delay of activity on other activities

D. Disadvantage of Gantt chart

- a. PERT (Program Evaluation and Review Technique) : Probabilistic model (stochastical model)
- b. CPM (Critical Path Model) : deterministic model

E. Nations in PERT

- a. Activity : effort need resource and time
- b. Event : milestones or checkpoints
- c. Project : a collection of activity
- d. Network : a logical and chronological set of activities and events
- e. Critical activity
- f. Path

g. Critical path

— Advantage of PERT/CPM

- Commitments and communications
- Detailed planning
- Efficient monitoring and control
- Identify potential problem areas
- Proper use of resources
- Rescheduling
- Government requirement
- Easily understood
- Adaptive to computers
- Decision making tools
- Cost-time trade-off

F. PERT/CPM process

a. phase1 : formulation

- 1.analysis of the project
- 2.sequence the activities
- 3.estimate activity times and costs

b. phase2 : solution

- 1.construct the network
- 2.event analysis
- 3.activity analysis

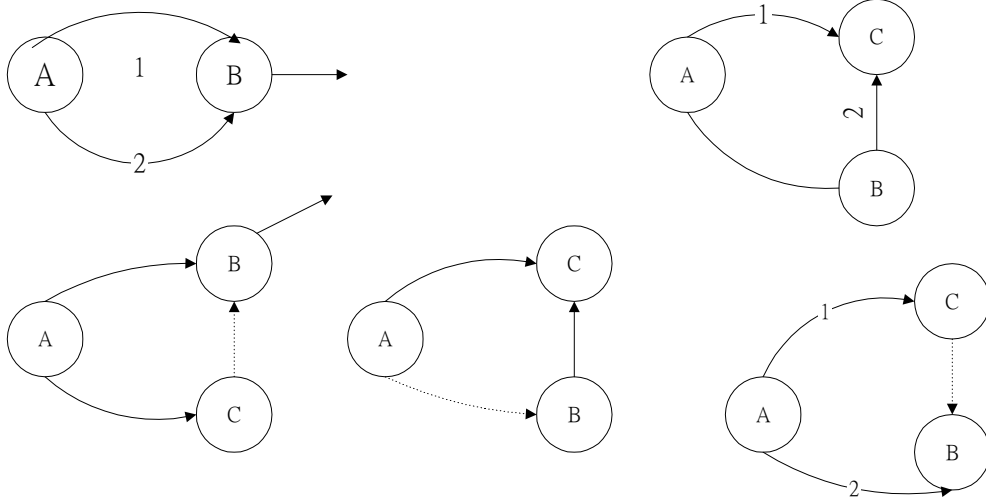
c. Analysis and control

- 1.Monitoring and control
- 2.Resource utilization

G. The network representation

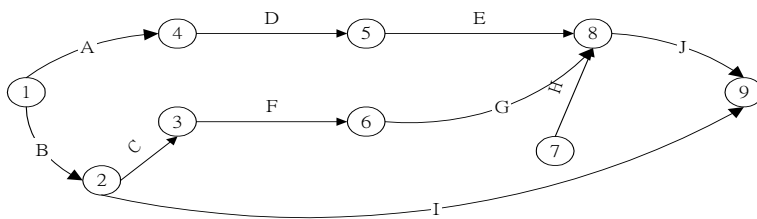
a. Activity (operation)

b. Node (event) : the beginning and ending of each activity



Ex :

Activity	Prerequisite	time
A	-	3
B	-	5
C	B	3
D	A,C	4
E	D	8
F	C	2
G	F	4
H	F	2
I	B	5
J	E,G,H	3



9.2 CPM

A. Find the “longest path” – “critical path”

— forward pass

ES_i : the Earliest start time of activity i

EF_i : the Earliest finish time of activity i

$$ES_j = \max[ES_i + t_i] = \max_i[EF_i]$$

$$EF_j = ES_j + t_j$$

— backward pass

LS_i : the Latest start time of activity i

LF_i : the Latest finish time of activity i

$$LF_i = \min[LF_j - t_j] = \min_j[LS_j]$$

$$LS_i = LF_i + t_i$$

— slack of free time

(1) Total Float (TF) : slack for on event

(2) Free Float (FF) : slack for a activity

$$TF_i = LS_i - ES_i = LF_i - EF_i$$

$$FF_i = ES_j - ES_i - t_i = ES_j - EF_i$$

Link	Activity	ES	EF	LS	LF	TF	FF
(1,4)	A	0	3	5	8	5	5
(1,2)	B	0	5	0	5	0	0
(2,3)	C	5	8	5	8	0	0
(4,5)	D	8	12	8	12	0	0
(5,7)	E	12	20	12	20	0	0
(3,6)	F	8	10	14	16	6	0
(6,7)	G	10	14	16	20	6	6
(8,6)	H	10	12	18	20	8	8
(2,9)	I	5	10	18	23	13	13
(7,9)	J	20	23	20	23	0	0

B. Observations

— $TF_i = 0 \Rightarrow FF_i = 0$

— $FF_i = 0 \neq TF_i = 0$

— if $TF_i \neq 0$ noncritical activity

— if $FF_i \leq TF_i$ (noncritical event) i can be delayed as much as FF_i

9.3 PERT (for uncertain activity time)

A. Assumption

- Time estimate for each activity is based on three difference aspects
- Based on Beta distribution assumption
 - (1) a: optimistic time (minimum time)
 - (2) b: pessimistic time (maximum time)
 - (3) m: most probable time (normal time)

B. Average Time (expected time)

- based on Beta probability distribution

$$(1) E = \frac{\frac{(a+b)}{2} + 2m}{3} = \frac{a + 4m + b}{6}$$

$$(2) \sigma^2 = \left(\frac{b-a}{6}\right)^2$$

- treat the expected time as a fixed time proceed with CPM
- if there are more than one critical paths identify the one with the largest variance

C. Compute the $E[U_i]$ and $Var[U_i]$

- U_i = the earliest occurrence time of event
- assume the activities are statistically independently

$$E[U_i] = ES_i = \sum E_k$$

$$Var[U_i] = \sum \sigma_k^2$$

- where k = the activities along the longest path leading to i
- if we specify a certain project completion time (ST_i), we can compute the probability of meeting this dead line

$$P(U_i \leq ST_i) = P\left[\frac{U_i - E(U_i)}{\sqrt{Var(U_i)}} \leq \frac{ST_i - E(U_i)}{\sqrt{Var(U_i)}}\right] = P[Z \leq Z^*]$$

Example:

Activity	a	m	b	E	S^2	S
A	1	3	5	3	4/9	2/3
B	3	4.5	9	5	1	1
C	2	3	4	3	1/9	1/3
D	2	4	6	4	4/9	2/3
E	4	7	16	8	4	2
F	1	1.5	5	2	4/9	2/3
G	2.5	3.5	7.3	4	25/36	5/6
H	1	2	3	2	1/9	1/3
I	4	5	6	5	1/9	1/3
J	1.5	3	4.5	3	1/4	1/2

$$E = \frac{a + 4m + b}{6} = \frac{1 + 4 \cdot 3 + 5}{6} = 3$$

$$\sigma^2 = \left(\frac{b - a}{6} \right)^2 = \left(\frac{2}{3} \right)^2$$

$$E(U_i) = 5 + 3 + 4 + 8 + 3 = 23$$

$$\sigma^2(U_i) = 1 + 1/9 + 4/9 + 4 + 1/4 = 5.836$$

$$S(U_i) = 2.41$$

$$ST_i = 23 \quad P \left[Z \leq \frac{ST_i - E(U_i)}{S(U_i)} \right] = P(Z \leq 0) = 0.5$$

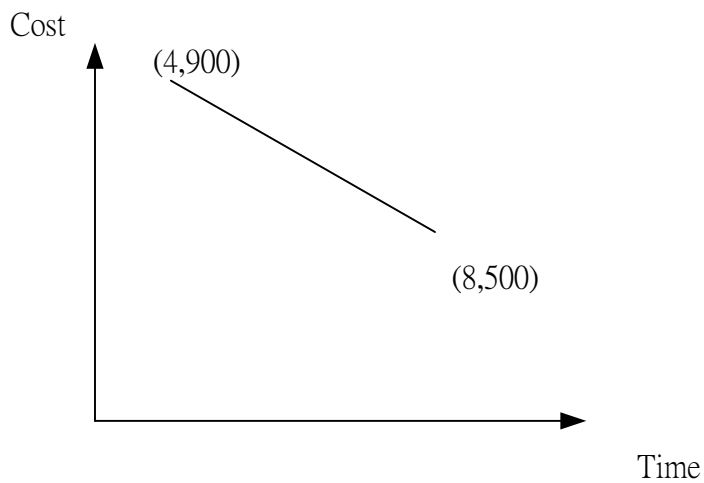
$$ST_i = 25 \quad P \left[Z \leq \frac{25 - 23}{2.41} \right] = P(Z \leq 0.83) = 0.7963$$

9.4 Considering Time-Cost Trade-Offs

- A. The duration of activity can be shortened by adding resource to that activity
- B. The need to identify the least cost to crash activity time to meet the desired date
- C. Notation

- T_j = normal time for activity j
- T_j' = time for activity j under maximum crashing
- C_j = cost for activity j under maximum crashing
- K_j = crash cost for activity j

$$K_j = \frac{C_j' - C_j}{T_j - T_j'}$$



$$K_j = \frac{C_j' - C_j}{T_j - T_j'} = \frac{900 - 500}{8 - 4} = 100$$

D. Observation

- other path may be critical
- check the revised network

E. LP model for crashing consideration

— Variables

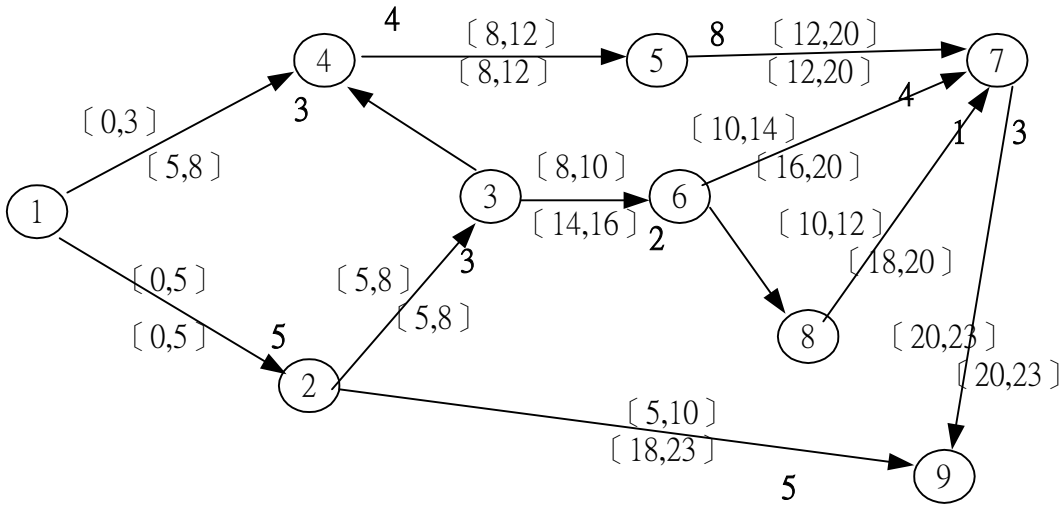
- X_i = the occurrence of event i
- Y_i = amount of crash time used for activity j

— objective function

$$\text{— } \min \sum_j K_j Y_j$$

— constraints

- (1) describe the network
- (2) limiting the activity crash time
- (3) meeting the project complete time



$$\min K_A Y_A + K_B Y_B + K_C Y_C + K_D Y_D + K_E Y_E + K_F Y_F + K_G Y_G + K_H Y_H + K_I Y_I + K_J Y_J$$

$$X_2 \geq T_B - Y_B + X_1$$

$$\text{st } X_3 \geq T_C - Y_C + X_2$$

$$X_4 \geq T_A - Y_A + X_1$$

$$X_6 \geq T_F - Y_F + X_3$$

$$X_7 \geq T_E - Y_E + X_5$$

$$X_7 \geq T_G - Y_G + X_6$$

$$X_7 \geq T_H - Y_H + X_8$$

$$X_9 \geq T_J - Y_J + X_7$$

$$X_9 \geq T_I - Y_I + X_2$$

$$X_4 \geq X_3$$

$$X_6 \geq X_8$$

$$X_5 \geq T_0 - Y_0 + X_4$$

Activity	Prerequisite	time
A	-	3
B	-	5
C	B	3
D	A,C	4
E	D	8
F	C	2
G	F	4
H	F	2
I	B	5
J	E,G,H	3

$$Y_i \leq T_i \quad \forall_i$$

$$Y_A \leq 2 \quad Y_0 \leq 1$$

$$X_9 \leq 20$$

9.5 PERT/COST

A. Concentrate on time aspect of a project

B. Budget cost

- identify cost
- project when cost occurs

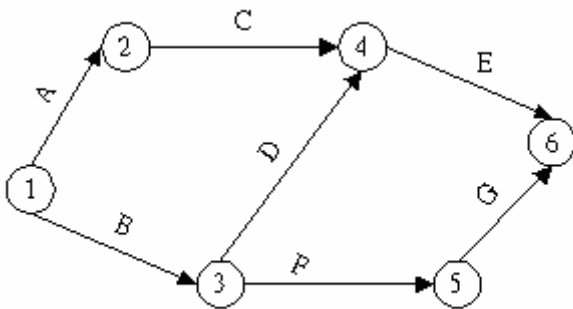
C. Assumption

- cost occurs at a constant rate

D. Procedure

- forecast the project cost based on the earliest and latest start time
- divide the cost uniformly into unit-time cost the shadow area between two cost curves shows the feasible budget

Ex.



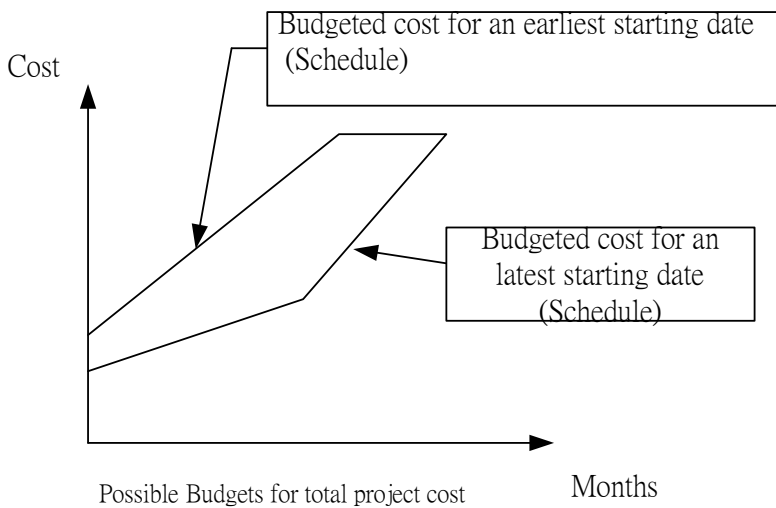
for earliest start

Budget cost for an earliest starting pate schedule

Activity	Month							
	1	2	3	4	5	6	7	8
A	5	5						
B	10	10	10					
C			3					
D				2	2	2		
E				10	10			
F							5	5
G						8		
Monthly cost	15	15	13	12	12	10	5	5
Total cost	15	30	43	55	67	77	82	87

Budget cost for an latest starting pate schedule

Activity	Month							
	1	2	3	4	5	6	7	8
A				5	5			
B	10	10	10					
C						3		
D				2	2	2		
E						10	10	
F							5	5
G								8
Monthly cost	10	10	10	7	7	15	15	23
Total cost	10	20	30	37	44	49	64	87



9.6 Project Control

A. Purpose

- a. Identify the actual of activities
- b. Periodically information collections
 - Activity cost update
 - Percent completion to date

B. Objective

- Identify cost overrun and underruns take necessary correction actions
- Notations
 - V_i = value of work completed for activity i
 - P_i = percentage completed for activity i
 - B_i = budget cost activity i

$$— V_i = \frac{(P_i / 100)}{B_i}$$

— Cost overruns

— AC_i = actual cost to date of activity i

— D_i = difference between actual cost and value of work completed for activity i

$$— D_i = AC_i - V_i$$

$$V_i = \text{completion\%} * B_i$$

Activity	AC_i	completion%	B_i	V_i	D_i
A	12000	100	10000	10000	2000
B	3000	100	30000	30000	0
C	1000	50	3000	1500	-500
D	2000	33	6000	2000	0
E	10000	25	20000	5000	5000
F	0	0	5000	0	0
G	0	0	4000	0	0
	55000			485000	6500

6500/48500=13.4% **SERIOUS PROBLEM**

— Identify cost over runs A,E

— A is completed and E is only 25% then adjust activity E

— Drawback

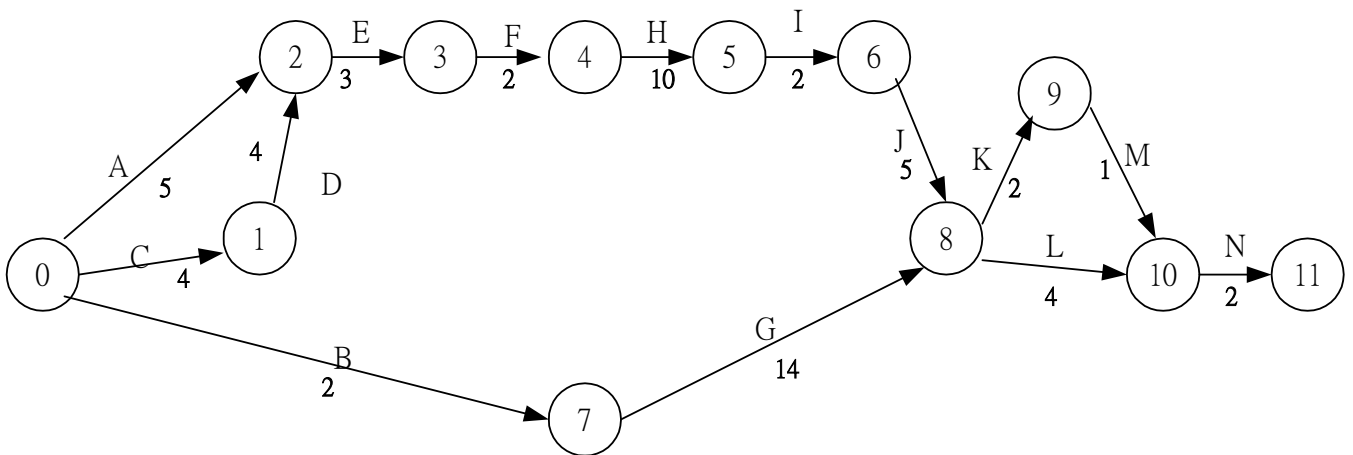
— need tremendous clerical efforts

— cost allocation problem

— need different cost accounting system

例. 競標

工程代號	工作情況	工作天數	前置作業
A	事先準備	5	
B	通知財產所有人	2	
C	選擇有關競標人	4	
D	與競標人聯絡	4	C
E	發出競標通知	3	A, D
F	取得競標者保薦書	2	E
G	取得財產所有權狀	14	B
H	等競標人準備競標	10	F
I	取得競標	2	H
J	評估競標	5	I
K	準備工作分配	2	G, J
L	與自來水公司取得聯絡	4	G, J
M	通知已入選的競標者	1	K
N	商洽簽約核定事宜	2	L, M
	完成合約	0	N



$$ES_i = \max_k \{ ES_k + D_{k,i} \}$$

$$EF_{ij} = ES_j + D_{i,j}$$

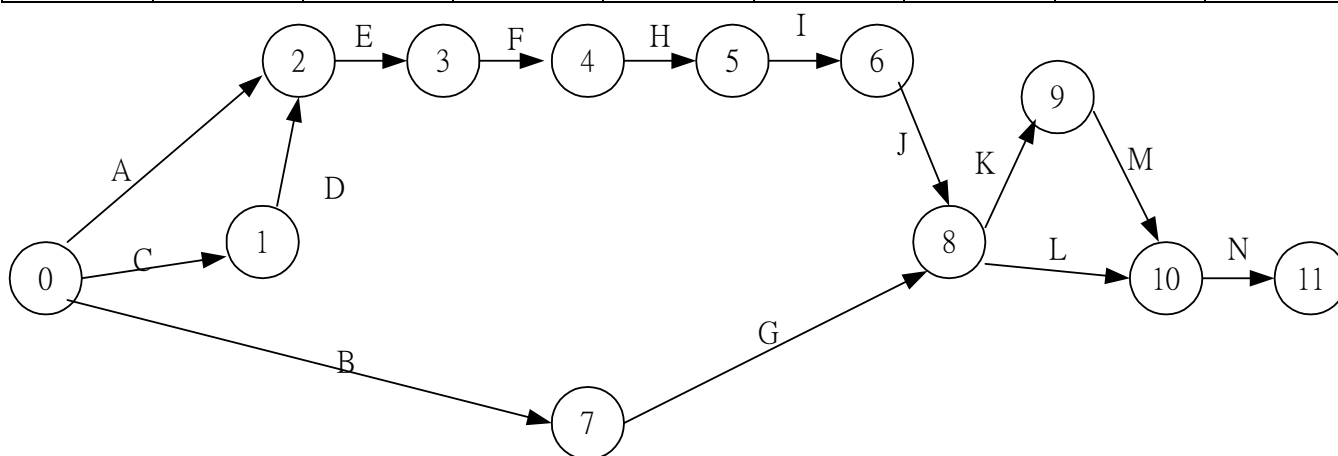
$$LF_i = \min [LF_j - D_{i,j}]$$

$$LS_i = LF_i - D_{i,i}$$

$$TF_i = LS_i - ES_i = LF_i - EF_i$$

$$FF_i = ES_j - ES_i - t_i = ES_j - EF_i$$

作業代號		工作天數	ES_i	EF_{ij}	LS_{ij}	LF_j	TF	FF
A	0-2	5	0	5	3	8	3	3
B	0-7	2	0	2	14	16	14	0
C	0-1	4	0	4	0	4	0	0
D	1-2	4	4	8	4	8	0	0
E	2-3	3	8	11	8	11	0	0
F	3-4	2	11	13	11	13	0	0
G	7-8	14	2	16	16	30	14	14
H	4-5	10	13	23	13	23	0	0
I	5-6	2	23	25	23	25	0	0
J	6-8	5	25	30	25	30	0	0
K	8-9	2	30	32	31	33	1	0
L	8-10	4	30	34	30	34	0	0
M	9-10	1	32	33	33	34	1	1
N	10-11	2	34	36	34	36	0	0



要徑：C-D-E-F-H-I-J-L-N (0-1-2-3-4-5-6-8-10-11)

9.7 Resource Utilization

A. purpose

—resource smoothing

—allocate resource for smooth usage

—resource leveling

—reduce peak resource requirements

—resource allocation

—for a desired resource level, find the shortest possible project schedule

B. Trial and error heuristic approach

—Step1 : resource accumulation by the earliest start schedule

—Step2 : draw the Gantt chart

—Step3 : start from the last noncritical activity not considered move it in total float period

—Strp4 : if no more noncritical activity to be considered stop, otherwise go to step3