Chapter 6 LP Application

6.1 Transportation Problem

A. Introduction

— To minimize the total cost of shipping products from supply point to demand points

— Each demand has min. requirement

— Each supply has capacity limitation

$$\min. \sum_i \sum_j C_{ij} X_{ij}$$

$$\text{s.t.} \quad \sum_j X_{ij} \le S_i \quad , \ \forall i$$

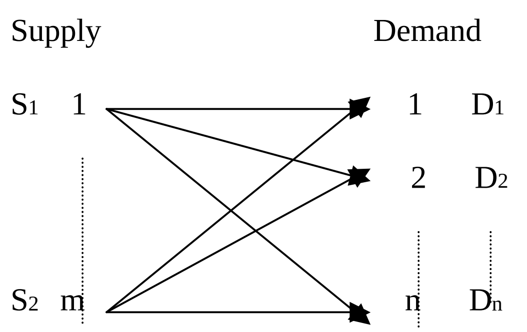
$$\sum_i X_{ij} \ge D_j \quad , \ \forall j$$

$$X_{ij} \ge 0$$

— Characteristics

(1) Supply and demand are integer

(2) Use plus/minus arithmetic operation



— Example

Origin	Destination				S_i
	1	2	3	4	
1	3	2	7	6	5000
2	7	5	2	3	6000
3	2	5	4	5	2500
	6000	4000	2000	1500	13500

— Special consideration

Total demand ≠ total supply

(1)Total demand < total supply → no modification

(2)Total demand > total supply → adding dummy origin

— Special routes

(1)Route capacities

Example $x_{ij} \geq 1000$ minimum flow

$x_{ij} \geq 1000$ capacity limitation

(2)Unacceptable routes : remove the associated decision variables

— Degeneracy

(1)Number of routes is less than n+m-1

(2)Assigned some routes with 0 shipments until n+m-1 routes is assigned

B. Special solution procedure

— Step1. Find the initial feasible solution

— Step2. Iteratively make improvement until optimal is reached

— Transportation tableau

		Destination				
		1	2	3	4	
Origin	1	C_{11}	C_{12}	C_{13}	C_{14}	S_1
	2	C_{21}	C_{22}	C_{23}	C_{24}	S_2
	3	C_{31}	C_{32}	C_{33}	C_{34}	S_3
Demand		D_1	D_2	D_3	D_4	

— Find the initial solution

(1)Minimum Cost Method

— Allocating as many as possible to minimum routes

— If there is tie to the one can ship the most unit

		Destination				
		1	2	3	4	
Origin	1	3 1000(5)	2 4000(1)	7	6	5000
	2	7 2500(6)	5	2 2000(3)	3 1500(4)	6000
	3	2 2500(2)	5	4	5	2500
Demand		6000	4000	2000	1500	

(2)Northwest Corner Rule

— Make initial assignment to the “upper left” corner

— Then to second-row

		Destination				
		1	2	3	4	
Origin	1	5000(1) [3]	[2]	[7]	[6]	5000
	2	1000(2) [7]	4000(3) [5]	1000(4) [2]	[3]	6000
	3	[2]	[5]	1000(5) [4]	1500(4) [5]	2500
Demand		6000	4000	2000	1500	

(3)Vogel Approximation Method

— Use relative cost comparison for allocation

— Opportunity cost : the different between two lowest cells in each row and column

		Destination				
		1	2	3	4	
Origin	1	1000(2) [3]	4000(1) [2]	[7]	[6]	5000
	2	2500(6) [7]	[5]	2000(4) [2]	1500(5) [3]	6000
	3	2500(3) [2]	[5]	[4]	[5]	2500
Demand		6000	4000	2000	1500	

— Improve initial solution

(1)Stepping-stone Method

— Step1 : select an empty cell

— Step2 : identify a closed path

— Step3 : compute the improvement

— Step4 : if △<0 (min) or △>0 (max) move maximum unit to empty cell otherwise do nothing

— Step5 : if there is unevaluated cell go to step2 , otherwise stop

Empty Cell	Close Path	Unit Cost
X_{13}	$C_{13}-C_{23}+C_{21}-C_{11}$	7-2+7-3=9
X_{14}	$C_{14}-C_{24}+C_{21}-C_{11}$	6-3+7-3=7
X_{22}	$C_{22}-C_{21}+C_{11}-C_{12}$	5-7+3-2= -1
X_{32}	$C_{32}-C_{31}+C_{11}-C_{12}$	5-2+3-2=4
X_{33}	$C_{33}-C_{31}+C_{21}-C_{23}$	4-2+7-2=7
X_{34}	$C_{34}-C_{31}+C_{21}-C_{24}$	5-2+7-3=7

		Destination				
		1	2	3	4	
Origin	1	1000 [3]	4000 [2]	9 [7]	7 [6]	5000
	2	2500 [7]	-1 [5]	2000 [2]	1500 [3]	6000
	3	2500 [2]	4 [5]	7 [4]	7 [5]	2500
Demand		6000	4000	2000	1500	

Empty Cell	Close Path	Unit Cost
X_{13}	$C_{13}-C_{23}+C_{22}-C_{12}$	7-2+5-2=8
X_{14}	$C_{14}-C_{24}+C_{22}-C_{12}$	6-3+5-2=6
X_{21}	$C_{21}-C_{11}+C_{12}-C_{22}$	7-3+2-5= 1
X_{32}	$C_{32}-C_{31}+C_{11}-C_{12}$	5-2+3-2=4
X_{33}	$C_{33}-C_{31}+C_{11}-C_{12}+C_{22}-C_{23}$	4-2+3-2+5-2=6
X_{34}	$C_{34}-C_{31}+C_{11}-C_{21}+C_{22}-C_{24}$	5-2+3-2+5-3=7

		Destination				
		1	2	3	4	
Origin	1	3500 [3]	1500 [2]	8 [7]	6 [6]	5000
	2	1 [7]	2500 [5]	2000 [2]	1500 [3]	6000
	3	2500 [2]	4 [5]	6 [4]	6 [5]	2500
Demand		6000	4000	2000	1500	

(2)Modified Distribution Method (MODI)

Min $\sum\sum C_{ij} X_{ij}$

St. $\sum_j X_{ij} = S_i \quad \forall_i \quad (U_i)$

$\sum_i X_{ij} = D_j \quad \forall_j \quad (V_j)$

$X_{ij} \geq 0$

Dual

Max $\sum_i S_i U_i + \sum_j D_j V_j$

St. $U_i + V_i \leq C_{ij} \quad \forall_i \ , \ \forall_j$

$\rightarrow \quad X_{ij}(U_i + V_j - C_{ij}) = 0$

		Destination				
		1	2	3	4	
Origin	1	3500 [3]	1500 [2]	8 [7]	6 [6]	5000
	2	1 [7]	2500 [5]	2000 [2]	1500 [3]	6000
	3	2500 [2]	4 [5]	6 [4]	6 [5]	2500
Demand		6000	4000	2000	1500	

$X_{11} \neq 0$	$U_1 + V_1 = 3$	$U_1 = 0$	$V_1 = 3$
$X_{12} \neq 0$	$U_1 + V_2 = 2$		$V_2 = 2$
$X_{22} \neq 0$	$U_2 + V_2 = 5$	$U_2 = 3$	
$X_{23} \neq 0$	$U_2 + V_3 = 2$		$V_3 = -1$
$X_{24} \neq 0$	$U_2 + V_4 = 3$		$V_4 = 0$
$X_{31} \neq 0$	$U_3 + V_1 = 2$	$U_3 = -1$	

$d_{ij} = C_{ij} - U_i - V_j$

$d_{13} = 7 - 0 - (-1) = 8$

$d_{14} = 6 - 0 - 0 = 6$

$d_{21} = 7 - 3 - 3 = 1$

$d_{32} = 5 - (-1) - 2 = 4$

$d_{33} = 4 - (-1) - (-1) = 6$

$d_{34} = 5 - (-1) - 0 = 6$

— To guarantee the initial feasible solution has mth-1 basic routes , we add a shipment of 0 unit to a cell in the row or column which were last lined out before final assignment.

		Destination				
		1	2	3	4	
Origin	1	6	8 20(4)	3 10(2)	10 0(6)	30
	2	2 10(1)	9 5(5)	5	7	15
	3	7	8	6	4 15(3)	15
Demand		10	25	10	15	

<u>例題</u>

設某公司產銷某種產品，在三個地點（Ⅰ、Ⅱ、Ⅲ）分別設有工廠生產供應，其主要市場為A、B、C三個城市，並設有營業所，各個營業所需要之貨品皆可由此三個工廠供應，惟其供應成本，由於運輸距離不同、路況不一及當地情況特殊，使成本不一致，該公司各工廠之供給量、各營業所之需求量和各工廠運送至各營業所之每單位產品成本可列表如下：

運輸成本表

起點

	工廠 \ 營業所	A	B	C	供給量
起點	I	$ 14	$ 16	$ 19	$ 60
	II	$ 15	$ 18	$ 20	$ 85
	III	$ 12	$ 17	$ 12	$ 55
	需求量	$ 50	$ 80	$ 70	$ 200

(一)　最小成本法（Minimum cost method）

工廠＼營業所	A	B	C	供給量
I	14	16	19	60
II	15	18	20	85
III	12	17	12	55
需求量	50	80	70	200

(a)

工廠＼營業所	A	B	C	供給量
I	14	16	19	60
II	15	18	20	85
III	12 * 50	17	12	55
需求量	50	80	70	200

(b)

工廠＼營業所	A	B	C	供給量
I	14	16	19	60
II	15	18	20	85
III	12 50	17	12 * 5	5
需求量	0	80	70	200

(c)

工廠＼營業所	A	B	C	供給量
I	14	16 * 60	19	60
II	15	18	20	85
III	12 50	17	12 5	0
需求量	0	80	65	200

(d)

工廠＼營業所	A	B	C	供給量
I	14	16 60	19	0
II	15	18 * 20	20 65	85
III	12 50	17	12 5	0
需求量	0	20	65	200

(e)

工廠＼營業所	A	B	C	供給量
I	14	16 60	19	0
II	15	18 20	20 * 65	65
III	12 50	17	12 5	0
需求量	0	0	65	200

TC=12×50+16×60+18×20+20×65+12×5=3280

(二) 西北角法（Northwest corner method）

(a)

工廠＼營業所	A	B	C	供給量
I	14 50	16	19	60
II	15	18	20	85
III	12	17	12	55
需求量	50	80	70	200

(b)

工廠＼營業所	A	B	C	供給量
I	50	10		10
II				85
III				55
需求量	0	80	70	200

(c)

工廠＼營業所	A	B	C	供給量
I	50	10		0
II		70		15
III				55
需求量	0	70	70	200

(d)

工廠＼營業所	A	B	C	供給量
I	50	10		0
II		70	15	0
III				55
需求量	0	0	70	200

(e)

工廠＼營業所	A	B	C	供給量
I	14 50	16 10	19	0
II	15	18 70	20 15	0
III	12	17	12 55	55
需求量	0	0	55	200

TC=14×50+16×10+18×70+20×15+12×55=3080

(三) VAM法（Vogel’approximation method）

(a)

工廠＼營業所	A	B	C	橫列差額
I	14	16	19	16-14=2
II	15	18	20	18-15=3
III	12	17	12	12-12=0
縱行差額	14-12=2	17-16=1	19-12=7	

↑

工廠＼營業所	A	B	C	供給量
I				60
II				85
III			55*	55
需求量	50	80	70	200

(b)

工廠＼營業所	A	B	C	橫列差額	
I	14	16	19	16-14=2	
II	15*	18	20	18-15=3	←
~~III~~	~~12~~	~~17~~	~~12~~		
縱行差額	15-14=1	18-16=2	20-19=1		

工廠＼營業所	A	B	C	供給量
I				60
II	50*			85
III			55	0
需求量	50	80	15	200

(c)

工廠＼營業所	~~A~~	B	C	橫列差額	
I	~~14~~	16*	19	19-16=3	←
II	~~15~~	18	20	20-18=2	
~~III~~	~~12~~	~~17~~	~~12~~		
縱行差額		18-16=2	20-19=1		

工廠＼營業所	A	B	C	供給量
I		60*		60
II	50			35
III			55	0
需求量	0	80	15	200

(d)

工廠＼營業所	A	B	C	橫列差額	
I	14	16	19		
II	15	18	20	20-18=2	←
III	12	17	12		
縱行差額					

工廠＼營業所	A	B	C	供給量
I		60		0
II	50	20*		35
III			55	0
需求量	0	20	15	200

(e)

工廠＼營業所	A	B	C	橫列差額
I	14	16	19	
II	15	18	20*	
III	12	17	12	
縱行差額				

工廠＼營業所	A	B	C	供給量
I	14	16 60	19	0
II	15 50	18 20	20 15	15
III	12	17	12 55	0
需求量	0	0	15	200

TC=15×50+16×60+18×20+20×15+12×55=3030

- 踏腳石法（Stepping-stone method）

→以西北角法得到的起始解為例

工廠＼營業所	A	B	C	供給量
Ⅰ	14 50	16 10	19	60
Ⅱ	15	18 70	20 15	85
Ⅲ	12	17	12 55	55
需求量	50	80	70	200

Empty cell→ⅠC、ⅡA、ⅢA、ⅢB

Empty cell	路徑	隱值
ⅡA	ⅡA（＋）→ⅠA（－）→ⅠB（＋）→ⅡB（－）	15－14＋16－18＝－1

工廠＼營業所	A	B	C	供給量
Ⅰ	14 50 －	16 ＋ 10	19	60
Ⅱ	15 ＋	18 － 70	20 15	85
Ⅲ	12	17	12 55	55
需求量	50	80	70	200

Empty cell	路徑	隱值
ⅠC	ⅠC（＋）→ⅠB（－）→ⅡB（＋）→ⅡC（－）	19－16＋18－20＝1

工廠 \ 營業所	A	B	C	供給量
I	14 50	16 10 －	19 ＋	60
II	15	18 70 ＋	20 － 15	85
III	12	17	12 55	55
需求量	50	80	70	200

Empty cell	路徑	隱值
ⅢA	ⅢA(＋)→ⅠA(－)→ⅠB(＋)→ⅡB(－)→ⅡC(＋)→ⅢC(－)	12-14+16-18+20-12=4

工廠 \ 營業所	A	B	C	供給量
I	14 50 －	16 ＋ 10	19	60
II	15	18 － 70	20 ＋ 15	85
III	12 ＋	17	12 － 55	55
需求量	50	80	70	200

Empty cell	路徑	隱值
IIIB	IIIB（＋）→IIB（－）→IIC（＋）→IIIC（－）	17－18＋20－12＝7

工廠 \ 營業所	A	B	C	供給量
I	14 50	16 10	19	60
II	15	18 70 －	20 ＋ 15	85
III	12	17 ＋	12 － 55	55
需求量	50	80	70	200

工廠 \ 營業所	A	B	C	供給量
I	14 50	16 10	19 **+1**	60
II	15 **－1**	18 70	20 15	85
III	12 ＋**4**	17 ＋**7**	12 55	55
需求量	50	80	70	200

工廠 \ 營業所	A	B
I	14 － 50	16 ＋ 10
II	15 ＋	18 － 70

⇒

工廠 \ 營業所	A	B
I	0	60
II	50	20

工廠 \ 營業所	A	B	C	供給量
Ⅰ	14	16 / 60	19	60
Ⅱ	15 / 50	18 / 20	20 / 15	85
Ⅲ	12	17	12 / 55	55
需求量	50	80	70	200

TC=16×60+15×50+18×20+20×15+12×55=3030

Empty cell→ⅠA、ⅠC、ⅢA、ⅢB

Empty cell	路徑	隱值
ⅠA	ⅠA（＋）→ⅠB（－）→ⅡB（＋）→ⅡA（－）	14－16＋18－15＝1
ⅠC	ⅠC（＋）→ⅡC（－）→ⅡB（＋）→ⅠB（－）	19－20＋18－16＝1
ⅢA	ⅢA（＋）→ⅡA（－）→ⅡB（＋）→ⅠB（－）→ⅠC（＋）→ⅢC（－）	12－15＋18－16＋19－12＝6
ⅢB	ⅢB（＋）→ⅡB（－）→ⅡC（＋）→ⅢC（－）	17－18＋20－12＝7

● 修正係數法（MODI）

→以西北角法得到的起始解為例

	v_j	v_1	v_2	v_3	
u_i	工廠 \ 營業所	A	B	C	供給量
u_1	Ⅰ	14 / 50	16 / 10	19	60
u_2	Ⅱ	15	18 / 70	20 / 15	85
u_3	Ⅲ	12	17	12 / 55	55
	需求量	50	80	70	200

$$c_{11} = u_1 + v_1 = 14$$
$$c_{12} = u_1 + v_2 = 16$$
$$c_{22} = u_2 + v_2 = 18$$
$$c_{23} = u_2 + v_3 = 20$$
$$c_{33} = u_3 + v_3 = 12$$

設 $u_1 = 0$ 得 $v_1 = 14$

$v_2 = 16$

得 $u_2 = 2$

得 $v_3 = 18$

得 $u_3 = -6$

$$d_{ij} = c_{ij} - u_i - v_j$$

$$d_{\mathrm{I\,C}} = d_{13} = c_{13} - u_1 - v_3 = 19 - 0 - 18 = 1$$
$$d_{\mathrm{II\,A}} = d_{21} = c_{21} - u_2 - v_1 = 15 - 2 - 14 = -1$$
$$d_{\mathrm{III\,A}} = d_{31} = c_{31} - u_3 - v_1 = 12 - (-6) - 14 = 4$$
$$d_{\mathrm{III\,B}} = d_{32} = c_{32} - u_3 - v_2 = 17 - (-6) - 16 = 7$$

	v_j	v_1=14	v_2=16	v_3=18	
u_i	工廠 \ 營業所	A	B	C	供給量
u_1=0	I	14 / 50	16 / 10	19 / 19-18-0=1	60
u_2=2	II	15 / 15-14-2=－1*	18 / 70	20 / 15	85
u_3=-6	III	12 / 12-14+6=14	17 / 17-16+6=7	12 / 55	55
	需求量	50	80	70	200

工廠＼營業所	A	B	C	供給量
I	14 － 50	16 ＋ 10	19 ＋1	60
II	15 ＋ －1	18 － 70	20 15	85
III	12 ＋4	17 ＋7	12 55	55
需求量	50	80	70	200

路徑：ⅡA（+）→ⅠA（－）→ⅠB（+）→ⅡB（－）

$$d_{\text{II A}} = 15 - 14 + 16 - 18 = -1$$

工廠＼營業所	A	B	C	供給量
I	14	16 60	19	60
II	15 50	18 20	20 15	85
III	12	17	12 55	55
需求量	50	80	70	200

$c_{12} = u_1 + v_2 = 16$

$c_{22} = u_2 + v_2 = 18$

$c_{21} = u_2 + v_1 = 15$

$c_{23} = u_2 + v_3 = 20$

$c_{33} = u_3 + v_3 = 12$

設 $u_1 = 0$ 得 $v_2 = 16$

得 $u_2 = 2$

$v_1 = 13$

得 $v_3 = 18$

得 $u_3 = -6$

	v_j	v_1=13	v_2=16	v_3=18	
u_i	工廠 \ 營業所	A	B	C	供給量
u_1=0	I	14 / +1	16 / 60	19 / +1	60
u_2=2	II	15 / 50	18 / 20	20 / 15	85
u_3=-6	III	12 / +5	17 / +7	12 / 55	55
	需求量	50	80	70	200

$d_{ij} = c_{ij} - u_i - v_j$

$$d_{\mathrm{I\,A}} = d_{11} = c_{11} - u_1 - v_1 = 14 - 0 - 13 = 1$$
$$d_{\mathrm{IIIA}} = d_{31} = c_{31} - u_3 - v_1 = 19 - 0 - 18 = 1$$
$$d_{\mathrm{IIIA}} = d_{31} = c_{31} - u_3 - v_1 = 12 - (-6) - 13 = 5$$
$$d_{\mathrm{IIIB}} = d_{32} = c_{32} - u_3 - v_2 = 17 - (-6) - 16 = 7$$

if $\sum Si = 210 \quad \sum Dj = 200$

minimum cost=balance or non-balance→same answers

if $\sum Si = 210 \quad \sum Dj = 220$

must add dummy supplier with

Sd=$\sum Dj$ - $\sum Si$ =10

6.2 Assignment Problem

A. Introduction

— Resource are being assigned to the activating on one-to-one base

— Decision variable

$$X_{ij} = \begin{cases} 1\text{ , if resource i is assigned to activity j} \\ 0\text{ , otherwise} \end{cases}$$

— Model

$$\text{Min } \sum_i \sum_j C_{ij} X_{ij}$$

St. $\sum_j X_{ij} = 1 \quad , \quad \forall_i$

$$\sum_i X_{ij} = 1 \quad , \quad \forall_j$$

$$X_{ij} = 0,1 \qquad \forall_i \quad , \forall_j$$

Project Leader	Client		
	1	2	3
1	10	15	9
2	9	18	5
3	6	14	3

Min $10X_{11} + 15X_{12} + 9X_{13} + 9X_{21} + 18X_{22} + 5X_{23} + 6X_{31} + 14X_{32} + 3X_{33}$

St. $X_{11} + X_{12} + X_{13} = 1$

$$X_{21} + X_{22} + X_{23} = 1$$

$$X_{31} + X_{32} + X_{33} = 1$$

$$X_{11} + X_{21} + X_{31} = 1$$

$$X_{12} + X_{22} + X_{32} = 1$$

$$X_{13} + X_{23} + X_{33} = 1$$

B. Solution method

— Use LP method

— Use transportation problem method

— Use Hungarian method

$$\sum_i \sum_j (C_{ij} - P_i - Q_j) X_{ij}$$

$$= \sum \sum C_{ij} X_{ij} - \sum_i P_{ij} \sum_j X_{ij} - \sum_j Q_j \sum_i X_{ij}$$

$$= 2 - \sum_i P_i - \sum_j Q_j$$

$$= 2 - C$$

C. Hungarian Method Procedure

— Step1 : subtract the smallest element from each row

— Step2 : subtract the smallest element from each column

— Step3 : find the minimum line to cover all zero cell stop

— Step4 : if number of lines equal to number of rows

stop otherwise go to step5

— Step5 : subtract the smallest unlined element from every unlined element , adding this element to every element at the intersection of two lines , go to step3

	1	2	3	mix			1	2	3
1	10	15	9	9		1	1	6	0
2	9	18	5	5		2	4	13	0
3	6	14	3	3		3	3	11	0
						Mix	1	6	0

	1	2	3			1	2	3
1	0	0	0	→	1	0	0	2
2	3	7	0		2	1	5	0
3	2	5	0		3	0	3	0

D. Special Considerations

— supply $\neq$ demand : add dummy demand or supply set $C_{ij} = 0$

— unacceptable assignment : $C_{ij} = M$

— maximization

(1)Converting to opportunity losses

(2)Subtracting every element in each column from the largest element in that column

Ex. Iteration1

Profit	a	b	c	d	e	loss	a	b	c	d	e
1	5	2	9	6	4	1	4	7	0	3	5
2	4	3	10	7	9	2	6	7	0	3	1
3	9	7	5	3	6	3	0	2	4	6	3
4	10	8	4	5	7	4	0	2	6	5	3
5	0	0	0	0	0	5	0	0	0	0	0

Iteration2 Iteration3

loss	a	b	C	d	e	loss	a	b	c	d	e
1	4	6	0	2	4	1	4	5	0	1	4
2	6	6	0	2	0	2	6	5	0	1	0
3	0	1	4	5	2	3	0	0	4	4	2
4	0	1	6	4	2	4	0	0	6	3	2
5	1	0	1	0	0	5	2	0	2	0	1

例題

某公司有5位員工可從事4個職位，每位員工擔任各項工作的訓練費用不一，如下表所示，公司應如何指派方能使訓練費用最低？

成本 / 職位 / 員工	1	2	3	4
A	17	14	17	19
B	11	15	7	13
C	10	16	12	18
D	9	12	18	18
E	12	15	10	15

解：先增加一行，使行列相等。

	1	2	3	4	D_u	
A	17	14	17	19	0	−0
B	11	15	7	13	0	−0
C	10	16	12	18	0	−0
D	9	12	18	18	0	−0
E	12	15	10	15	0	−0
	−9	−12	−7	−13	−0	

	1	2	3	4	5
A	8^{-1}	2^{-1}	10^{-1}	6^{-1}	0
B	2	3	0	0	0 $^{+1}$
C	**1** $^{-1}$	4^{-1}	5^{-1}	5^{-1}	0
D	0	0	11	5	0 $^{+1}$
E	3^{-1}	3^{-1}	3^{-1}	2^{-1}	0

	1	2	3	4	5
A	7	1	9^{-1}	5^{-1}	0
B	2^{+1}	3^{+1}	0	0	1^{+1}
C	0	3	4^{-1}	4^{-1}	0
D	0	0	11^{-1}	5^{-1}	1
E	2	2	2^{-1}	**1** $^{-1}$	0

	1	0	3	4	5	
A	7	3	8	4	0	1
B	3	2	0	0	2	2
C	1	0	11	5	2	2
D	0	1	10	4	1	2
E	2	2	1	0	0	2

	1	2	3	4	5
A	7	1	8	4	0 *
B	3	4	0	0	2
C	0	3	3	3	0
D	0	0	10	4	1
E	2	2	1	0	0

員工	指派	職位	費用
A	$\rightarrow$	D_u	0
B	$\rightarrow$	3	7
C	$\rightarrow$	1	10
D	$\rightarrow$	2	12
E	$\rightarrow$	4	15
		合計	$ 44

6.3 Transshipment Problem

A. Introduction

— intermediate node (transshipment nodes) are added to account for location where goods are temporary stored

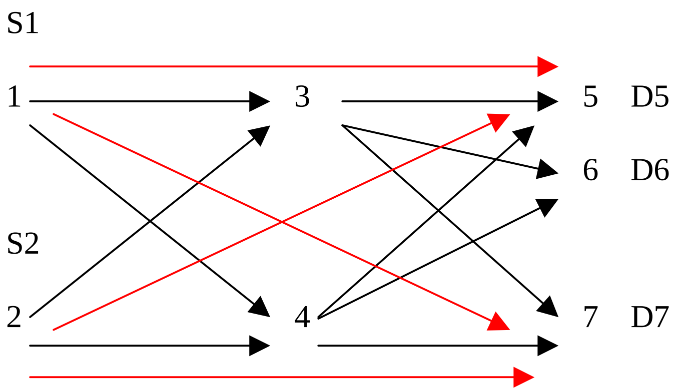
— general form

$$\text{Min.} \sum_i \sum_j C_{ij} X_{ij}$$

$$\text{St.} \quad \sum_j Xij = S_i \quad , \quad \forall_i = \text{supply point}$$

$$\sum_j X_{aj} - \sum_i X_{ia} = 0 \quad , \quad \forall_a = \text{intermediate point}$$

$$\sum_i Xij = D_i \quad , \quad \forall_j = \text{demand point}$$



B. Solution

— By converting to transportation problem

(1)Set all point as demand point and supply point

(2)Set cost for unacceptable route is M

(3) Set cost for a point to itself as 0

— Example

	I_3	I_4			D_5	D_6	D_7
S_1	2	3	400	I_3	2	6	3
S_2	3	1	300	I_4	4	4	6
	400	300			200	150	350

	S_1	S_2	I_3	I_4	D_5	D_6	D_7	
S_1	0	M	2	3	M	M	M	1100
S_2	M	0	3	1	M	M	M	1000
I_3	M	M	0	M	2	6	3	700
I_4	M	M	M	0	4	4	6	700
D_5	M	M	M	M	0	M	M	700
D_6	M	M	M	M	M	0	M	700
D_7	M	M	M	M	M	M	0	700
	700	700	700	700	900	850	1050	