Home Work 12(Due 12/2)—You must show all your work.

Name:

Student ID No.:

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Calculus
Home Work 12

1. Graph the function \( f(t) = (t - 1)^2 \) on \([0, 3]\) and find its average value over the given interval.

Answer:

2. Evaluate the integrals.
   
   (a) \[ \int_{0}^{\pi/3} 2 \sec^2 x \, dx. \]
   
   (b) \[ \int_{\pi/2}^{0} \frac{1 + \cos(2t)}{2} \, dt. \]
   
   (c) \[ \int_{-4}^{4} |x| \, dx. \]

Answer:

   (a)
3. Find the derivatives. \( \frac{d}{dt} \int_{0}^{t^4} \sqrt{u} \, du \).
Answer:

4. Find the derivatives. \( \frac{d}{dx} \int_{0}^{\sin(x)} \frac{dt}{\sqrt{1-t^2}} \), \(-\frac{\pi}{2} < x < \frac{\pi}{2}\).
Answer:
5. Find the area of the region between the curve \( y = x^3 - 3x^2 + 2x, \ 0 \leq x \leq 2 \) and the \( x \)-axis.

Answer:

6. Express the solutions of the initial value problem in terms of integrals.

\[
\frac{dy}{dx} = \sec x. \quad y(2) = 3.
\]

Answer:

7. Suppose that \( f \) has a positive derivative for all values of \( x \) and that \( f(1) = 0 \). Which of the following statements must be true of the function \( g(x) = \int_0^x f(t) \, dt \)?

Give reasons for your answers.

(a) \( g \) has a local maximum at \( x = 1 \).

(b) \( g \) has a local minimum at \( x = 1 \).

(c) The graph of \( g \) has an inflection point at \( x = 1 \).

(d) The graph of \( \frac{dg}{dx} \) crosses the \( x \)-axis at \( x = 1 \).
8. Evaluate the integrals.
(a) \( \int \csc^2(2\theta) \cot(2\theta) \, d\theta \).
(b) \( \int 3y \sqrt{7 - 3y^2} \, dy \).
(c) \( \int \sec^2(3x + 2) \, dx \).
(d) \( \int \frac{\sin(2t + 1)}{\cos^2(2t + 1)} \, dt \).
(e) \( \int \frac{1}{t^2} \cos \left( \frac{1}{t} - 1 \right) \, dt \).
(f) \( \int x^3 \sqrt{x^2 + 1} \, dx \).

Answer:
9. Evaluate the integral \( \int \frac{18 \tan^2 x \sec^2 x}{(2 + \tan^3 x)^2} \, dx \) using the following substitutions.
   (a) \( u = \tan^3 x \), followed by \( v = 2 + u \).
   (b) \( u = 2 + \tan^3 x \).

Answer:

(a) 

(b) 

10. Use the Substitution Formula to evaluate the integrals.
   (a) \( \int_0^1 \frac{5r}{(4 + r^2)^2} \, dr \).
   (b) \( \int_0^{\sqrt{3}} \frac{4x}{\sqrt{x^2 + 1}} \, dx \).
   (c) \( \int_0^{2\pi} \frac{\cos z}{\sqrt{4 + 3 \sin z}} \, dx \).
   (d) \( \int_0^1 (4y - y^2 + 4y^3 + 1)^{-2/3} (12y^2 - 2y + 4) \, dy \).

Answer:
11. Find the total areas of the shaded regions.

Answer: