Stability and Control

Outline of this Chapter

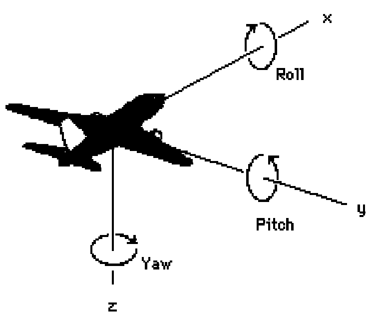
The chapter is divided into several sections. The first of these consist of an introduction to stability and control: basic concepts and definitions. The latter sections deal with more detailed stability and control requirements and tail design.

Stability and Control: Introduction

The methods in these notes allow us to compute the overall aircraft drag. With well-designed airfoils and wings, and a careful job of engine and fuselage integration, L/D's near 20 may be achieved. Yet some aircraft with predicted L/D's of 20 have actual L/D's of 0 as exemplified by any paper airplane contest. Many aircraft have been dismal failures even though their predicted performance is great. In fact, most spectacular failures have to do with stability and control rather than performance.

This section deals with some of the basic stability and control issues that must be addressed in order that the airplane is capable of flying at all. The section includes a general discussion on stability and control and some terminology. Basic requirements for static longitudinal stability, dynamic stability, and control effectiveness are described. Finally, methods for tail sizing and design are introduced.

The starting point for our analysis of aircraft stability and control is a fundamental result of dynamics: for rigid bodies motion consists of translations and rotations about the center of gravity (c.g.). The motion includes six degrees of freedom: forward and aft motion, vertical plunging, lateral translations, together with pitch, roll, and yaw.



Definitions

The following nomenclature is common for discussions of stability and control.

Forces and Moments			
Quantity	**Variable**	**Dimensionless Coefficient**	**Positive Direction**
Lift	L	$C_L = L/qS$	'Up' normal to freestream
Drag	D	$C_D = D/qS$	Downstream

Sideforce	Y	$C_Y = Y/qS$	Right, looking forward
Roll	l	$C_l = l / qSb$	Right wing down
Pitch	M	$C_m = M/qSc$	Nose up
Yaw	N	$C_n = N/qSb$	Nose right

Angles and Rates

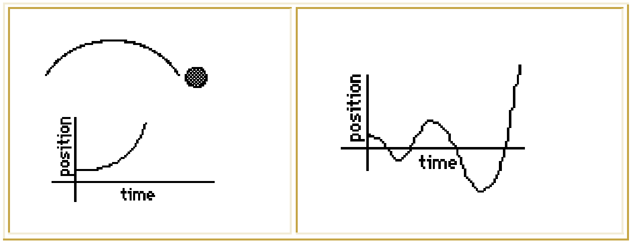
Quantity	Symbol	Positive Direction
Angle of attack	α	Nose up w.r.t. freestream
Angle of sideslip	β	Nose left
Pitch angle	Θ	Nose up
Yaw angle	Ψ	Nose right
Bank angle	Φ	Right wing down
Roll rate	p	Right wing down
Pitch rate	q	Nose up
Yaw rate	r	Nose Right

Aircraft velocities, forces, and moments are expressed in a body-fixed coordinate system. This has the advantage that moments of inertia and body-fixed coordinates do not change with angle of attack, but a conversion must be made from lift and drag to X force and Z force. The body axis system is the conventional one for aircraft dynamics work (x is forward, y is to the right when facing forward, and z is downward), but note that this differs from the conventions used in aerodynamics and wind tunnel testing in which x is aft and z is upward. Thus, drag acts in the negative x direction when the angle of attack is zero. The actual definition of the coordinate directions is up to the user, but generally, the fuselage reference line is used as the direction of the x axis. The rotation rates p, q, and r are measured about the x, y, and z axes respectively using the conventional right hand rule and velocity components u, v, and w are similarly oriented in these body axes.

Basic Concepts

Stability is the tendency of a system to return to its equilibrium condition after being disturbed from that point. Two types of stability or instability are important.

A static instability	A dynamic instability



An airplane must be a stable system with acceptable time constants. In general we want the dynamics to be acceptable, actually more than just stable -- we need appropriate damping and frequency. To assure this, a careful analysis of the dynamic response and controllability is required. The dynamic equations of motion are shown below, expressed in body axes. The top six equations are just forms of F=ma and M=I dΩ dt for each of the coordinate directions. The bottom three equations are kinematic expressions relating angular rates to the orientation angles Θ, Φ, Ψ, angles describing the airplane pitch, roll, and heading angles.

$$m\left[\frac{du}{dt} + qw - rv\right] = F_x - mg\sin\Theta$$

$$m\left[\frac{dv}{dt} + ru - pw\right] = F_y + mg\cos\Theta\sin\Phi$$

$$m\left[\frac{dw}{dt} + pv - qu\right] = F_z + mg\cos\Theta\cos\Phi$$

$$M_x = I_{xx}\frac{dp}{dt} - I_{xz}\frac{dr}{dt} + [I_{zz} - I_{yy}]qr - I_{xz}pq$$

$$M_y = I_{yy}\frac{dq}{dt} + [I_{xx} - I_{zz}]rp + I_{xz}[p^2 - r^2]$$

$$M_z = -I_{xz}\frac{dp}{dt} + I_{zz}\frac{dr}{dt} + [I_{yy} - I_{xx}]pq + I_{xz}qr$$

$$p = \frac{d\Phi}{dt} - \frac{d\Psi}{dt}\sin\Theta$$

$$q = \frac{d\Theta}{dt}\cos\Phi + \frac{d\Psi}{dt}\sin\Phi\cos\Theta$$

$$r = -\frac{d\Theta}{dt}\sin\Phi + \frac{d\Psi}{dt}\cos\Phi\cos\Theta$$

In general, we must solve these nonlinear, coupled, second order differential equations to describe the dynamics of the airplane. Many simplifying assumptions are often justified and make the analysis simpler.

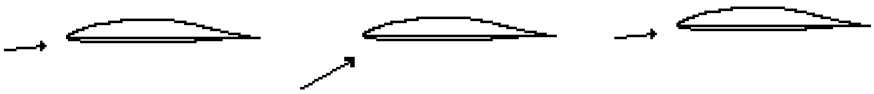
If we linearize the equations we find that there exist 5 interesting modes of dynamic motion. These are discussed further in the section on dynamic stability. But one of the useful results is that we usually obtain sets of nearly independent modes: those associated with symmetric, longitudinal motion, and those related to lateral motion. The modes are, of course, coupled for asymmetric aircraft such as oblique wings and the motion can be coupled by nonlinear effects such as pitching moment produced by large sideslip angles or alpha-dependent yawing moments that appear on fighters at high angles of attack, but for many cases the approximate decoupling is useful.

Longitudinal Static Stability

Stability and Trim

In designing an airplane we would compute eigenvalues and vectors (modes and frequencies) and time histories, etc. But we don't need to do that at the beginning when we don't know the moments of inertia or unsteady aero terms very accurately. So we start with static stability.

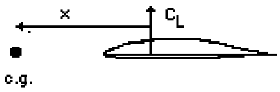
If we displace the wing or airplane from its equilibrium flight condition to a higher angle of attack and higher lift coefficient:



we would like it to return to the lower lift coefficient.

This requires that the pitching moment about the rotation point, C_m, become negative as we increase C_L:

$$\frac{\partial C_m}{\partial C_L} < 0$$

Note that: $C_m = C_{m_0} - \frac{x}{c} C_L$ 

where x is the distance from the system's aerodynamic center to the c.g..

So, $$\frac{\partial C_{m_{c.g.}}}{\partial C_L} = -\frac{x}{c} = -\text{static margin}$$

If x were 0, the system would be neutrally stable. x/c represents the margin of static stability and is thus called the static margin. Typical values for stable airplanes range from 5% to 40%. The airplane may therefore be made as stable as desired by moving the c.g. forward (by putting lead in the nose) or moving the wing back. One needs no tail for stability then, only the right position of the c.g..

A very stable airplane.

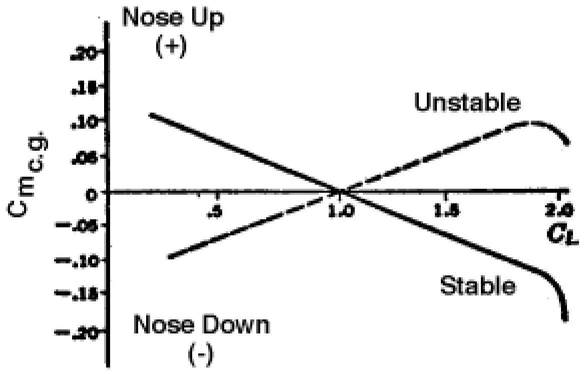
Although this configuration is stable, it will tend to nose down whenever any lift is produced. In addition to stability we require that the airplane be trimmed (in moment equilibrium) at the desired C_L.

This implies that: $$C_{m_{c.g.}} = C_{m_0} - \frac{x}{c} C_L = 0$$

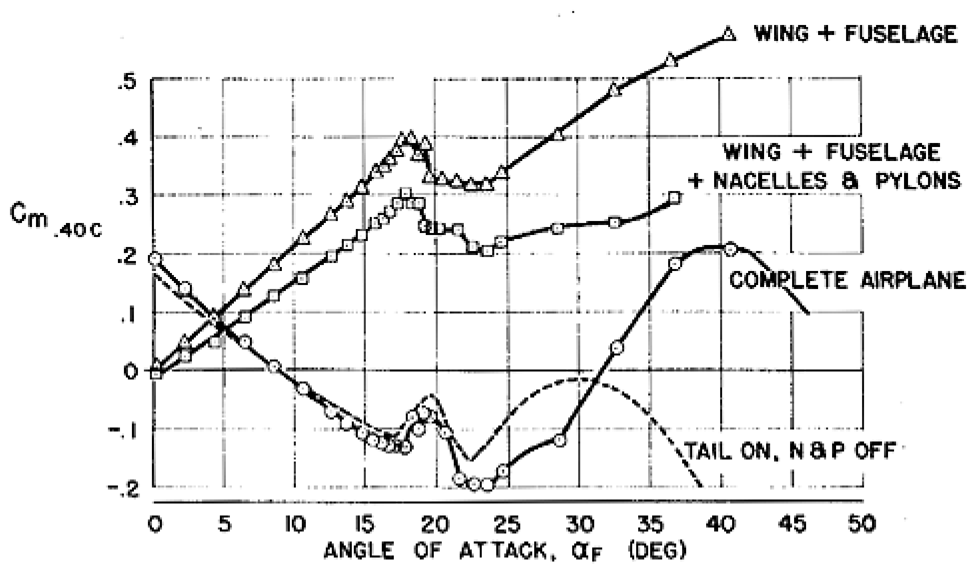
With a single wing, generating a sufficient C_m at zero lift to trim with a reasonable static margin and C_L is not so easy. (Most airfoils have negative values of C_{mo}.) Although tailless aircraft can generate sufficiently positive C_{mo} to trim, the more conventional solution is to add an additional lifting surface such as an aft-tail or canard. The following sections deal with some of the considerations in the design of each of these configurations.

Pitching Moment Curves

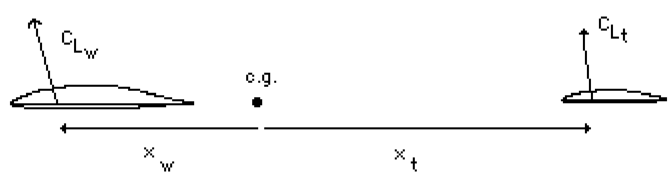
If we are given a plot of pitching moment vs. C_L or angle of attack, we can say a great deal about the airplane's characteristics.



For some aircraft, the actual variation of C_m with alpha is more complex. This is especially true at and beyond the stalling angle of attack. The figure below shows the pitching characteristics of an early design version of what became the DC-9. Note the contributions from the various components and the highly nonlinear post-stall characteristics.



Equations for Static Stability and Trim



The analysis of longitudinal stability and trim begins with expressions for the pitching moment about the airplane c.g..

$$C_{m_{c.g.}} = \frac{x_{c.g.}}{\bar{c}} C_{L_w} - \frac{l_h S_h}{\bar{c} S_w} C_{L_h} + C_{m_{a.c.w}} + C_{m_{c.g.body}}$$

Where:

$x_{c.g.}$ = distance from wing aerodynamic center back to the c.g. = x_w
c = reference chord
C_{Lw} = wing lift coefficient
l_h = distance from c.g. back to tail a.c. = x_t
S_h = horizontal tail reference area
S_w = wing reference area
C_{Lh} = tail lift coefficient
C_{macw} = wing pitching moment coefficient about wing a.c. = C_{mow}
$C_{mc.g.body}$ = pitching moment about c.g. of body, nacelles, and other components

The change in pitching moment with angle of attack, C_{ma}, is called the pitch stiffness. The change in pitching moment with C_L of the wing is given by:

$$\frac{\partial C_{m_{c.g.}}}{\partial \alpha} = \frac{x_{c.g.}}{\bar{c}} C_{L_{\alpha w}} - \frac{l_h S_h}{\bar{c} S_w} C_{L_{\alpha h}} + \frac{\partial C_{m_{c.g.body}}}{\partial \alpha}$$

Note that: $\frac{\partial C_{m_{c.g.}}}{\partial \alpha} = 0$ when $\frac{x_{c.g.}}{\bar{c}} = \frac{l_h S_h}{\bar{c} S_w} \frac{C_{L_{\alpha h}}}{C_{L_{\alpha w}}} - \frac{1}{C_{L_{\alpha w}}} \frac{\partial C_{m_{c.g.body}}}{\partial \alpha}$

The position of the c.g. which makes $dC_m/dC_L = 0$ is called the neutral point. The distance from the neutral point to the actual c.g. position is then:

$$\frac{\Delta c.g.}{\bar{c}} = \frac{x_{c.g.}}{\bar{c}} - \frac{l_h S_h}{\bar{c} S_w} \frac{C_{L_{\alpha h}}}{C_{L_{\alpha w}}} + \frac{1}{C_{L_{\alpha w}}} \frac{\partial C_{m_{c.g.body}}}{\partial \alpha}$$

This distance (in units of the reference chord) is called the static margin. We can see from the previous equation that:

$$\text{static margin} = -\frac{\Delta \mathrm{c.g.}}{\bar{\mathrm{c}}} \approx -\frac{\partial \mathrm{C}_{\mathrm{m}_{c.g.}}}{\partial \mathrm{C}_{\mathrm{L_w}}}$$

(A note to interested readers: This is approximate because the static margin is really the derivative of $C_{mc.g.}$ with respect to C_{LA}, the lift coefficient of the entire airplane. Try doing this correctly. The algebra is just a bit more difficult but you will find expressions similar to those above. In most cases, the answers are very nearly the same.)

We consider the expression for static margin in more detail:

$$\text{static margin} = -\frac{x_{c.g.}}{\bar{c}} + \frac{l_h S_h}{\bar{c} S_w} \frac{C_{L_{\alpha h}}}{C_{L_{\alpha w}}} - \frac{1}{C_{L_{\alpha w}}} \frac{\partial C_{m_{c.g.body}}}{\partial \alpha}$$

The tail lift curve slope, C_{Lah}, is affected by the presence of the wing and the fuselage. In particular, the wing and fuselage produce downwash on the tail and the fuselage boundary layer and contraction reduce the local velocity of flow over the tail. Thus we write:

$$C_{L_{\alpha h}} = C_{L_{\alpha h 0}} (1 - \frac{\partial \epsilon}{\partial \alpha}) \eta_h$$

where: C_{Lah0} is the isolated tail lift curve slope.

The isolated wing and tail lift curve slopes may be determined from experiments, simple codes such as the wing analysis program in these notes, or even from analytical expressions such as the DATCOM formula:

$$C_{L_\alpha} \approx \frac{2\pi AR}{2 + \sqrt{(AR/\eta)^2 (1 + \tan^2 \Lambda - M^2) + 4}}$$

where the oft-used constant h accounts for the difference between the theoretical section lift curve slope of 2p and the actual value. A typical value is 0.97.

In the expression for pitching moment, h_h is called the tail efficiency and accounts for reduced velocity at the tail due to the fuselage. It may be assumed to be 0.9 for low tails and 1.0 for T-Tails.

The value of the downwash at the tail is affected by fuselage geometry, flap angle wing planform, and tail position. It is best determined by measurement in a wind tunnel, but lacking that, lifting surface computer programs do an acceptable job. For advanced design purposes it is often possible to approximate the downwash at the tail by the downwash far behind an elliptically-loaded wing:

$$\epsilon \approx \frac{2 C_{L_w}}{\pi \ AR_w} \quad \text{So,} \ \frac{\partial \epsilon}{\partial \alpha} \approx \frac{2 C_{L_{\alpha w}}}{\pi \ AR_w}$$

We have now most of the pieces required to predict the airplane stability. The last, and important, factor is the fuselage contribution. The fuselage produces a pitching moment about the c.g. which depends on the angle of attack. It is influenced by the fuselage shape and interference of the wing on the local flow. Additionally, the fuselage affects the flow over the wing. Thus, the destabilizing effect of the fuselage depends on: Lf, the fuselage length, wf, the fuselage width, the wing sweep, aspect ratio, and location on the fuselage.

Gilruth (NACA TR711) developed an empirically-based method for estimating the effect of the fuselage:

$$\frac{\partial C_{m_{fuse}}}{\partial C_L} = \frac{K_f w_f^2 L_f}{S_w \bar{c} C_{L_{\alpha w}}}$$

where:
C_{Law} is the wing lift curve slope per radian
L_f is the fuselage length
w_f is the maximum width of the fuselage
K_f is an empirical factor discussed in NACA TR711 and developed from an extensive test of wing-fuselage combinations in NACA TR540.

K_f is found to depend strongly on the position of the quarter chord of the wing root on the fuselage. In this form of the equation, the wing lift curve slope is expressed in rad^{-1} and K_f is given below. (Note that this is not the same as the method described in Perkins and Hage.) The data shown below were taken from TR540 and Aerodynamics of the Airplane by Schlichting and Truckenbrodt:

Position of 1/4 root chord on body as fraction of body length	K_f
.1	.115
.2	.172
.3	.344
.4	.487
.5	.688
.6	.888
.7	1.146

Finally, nacelles and pylons produce a change in static margin. On their own nacelles and pylons produce a small destabilizing moment when mounted on the wing and a small stabilizing moment when mounted on the aft fuselage.

With these methods for estimating the various terms in the expression for pitching moment, we can satisfy the stability and trim conditions. Trim can be achieved by setting the incidence of the tail surface (which adjusts its C_L) to make $C_m = 0$:

$$C_m = C_{m_{a.c.}} + C_{L_w} \frac{x_w}{\bar{c}} - C_{L_h} \frac{S_h}{S_w} \frac{l_h}{\bar{c}} + \text{fuselage effects} = 0$$

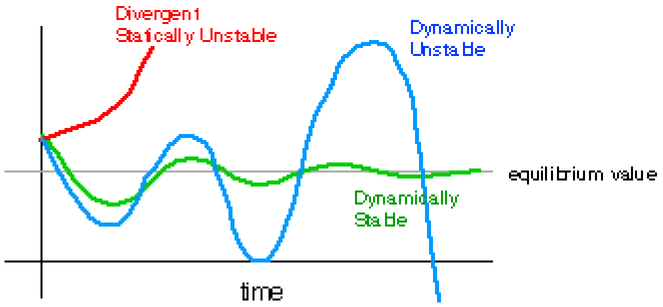
Stability can simultaneously be assured by appropriate location of the c.g.:

$$\frac{\partial C_m}{\partial C_{L_w}} = \frac{x_w}{\bar{c}} - \frac{C_{L_{\alpha_h}} S_h}{C_{L_{\alpha_w}} S_w} \frac{l_h}{\bar{c}} + \text{fuselage effects} \approx -\text{static margin}$$

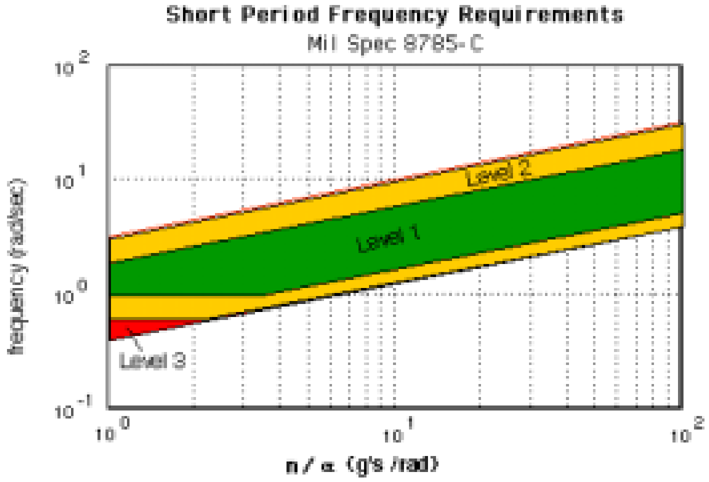
Thus, given a stability constraint and a trim requirement, we can determine where the c.g. must be located and can adjust the tail lift to trim. We then know the lifts on each interfering surface and can compute the combined drag of the system.

Dynamic Stability

The evaluation of static stability provides some measure of the airplane dynamics, but only a rather crude one. Of greater relevance, especially for lateral motion, is the dynamic response of the aircraft. As seen below, it is possible for an airplane to be statically stable, yet dynamically unstable, resulting in unacceptable characteristics.



Just what constitutes acceptable characteristics is often not obvious, and several attempts have been made to quantify pilot opinion on acceptable handling qualities. Subjective flying qualities evaluations such as Cooper-Harper ratings are used to distinguish between "good-flying" and difficult-to-fly aircraft. New aircraft designs can be simulated to determine whether they are acceptable. Such real-time, pilot-in-the-loop simulations are expensive and require a great deal of information about the aircraft. Earlier in the design process, flying qualities estimate may be made on the basis of various dynamic characteristics. One can correlate pilot ratings to the frequencies and damping ratios of certain types of motion as in done in the U.S. Military Specifications governing airplane flying qualities. The figure below shows how the short-frequency longitudinal motion of an airplane and the load factor per radian of angle of attack are used to establish a flying qualities estimate. In Mil Spec 8785C, level 1 handling is considered "clearly adequate" while level 3 suggests that the airplane can be safely controlled, but that the pilot workload is excessive or the mission effectiveness is inadequate.



Rather than solve the relevant equations of motion, we describe here some of the simplified results obtained when this is done using linearized equations of motion.

When the motions are small and the aerodynamics can be assumed linear, many useful, simple results can be derived from the 6 degree-of-freedom equations of motion. The first simplification is the decoupling between symmetric, longitudinal motion, and lateral motion. (This requires that the airplane be left/right symmetric, a situation that is often very closely achieved.) Other decoupling is also observed, with 5 decoupled modes required to describe the general motion. The stability of each of these modes is often used to describe the airplane dynamic stability.

Modes are often described by their characteristic frequency and damping ratio. If the motion is of the form: $x = A\, e^{(n + i\,\omega)\,t}$, then the period, T, is given by: $T = 2\pi / \omega$, while the time to double or halve the amplitude of a disturbance is: t_{double} or $t_{half} = 0.693 / |n|$. Other parameters that are often used to describe these modes are the undamped circular frequency: $\omega_n = (\omega^2 + n^2)^{1/2}$ and the damping ratio, $\zeta = -n / \omega_n$.

Longitudinal Stability

When the aircraft is not perturbed about the roll or yaw axis, only the longitudinal modes are required to describe the motion. These modes usually are divided into two distinct types of motion.

Short-Period

The first, short period, motion involves rapid changes to the angle of attack and pitch attitude at roughly constant airspeed. This mode is usually highly damped; its frequency and damping are very important in the assessment of aircraft handling. For a 747, the frequency of the short-period mode is about 7 seconds, while the time to halve the amplitude of a disturbance is only 1.86 seconds. The short period frequency is strongly related to the airplane's static margin, in the simple case of straight line motion, the frequency is proportional to the square root of $C_{m\alpha} / C_L$.

Phugoid

The long-perioid of phugoid mode involves a trade between kinetic and potential energy. In this mode, the aircraft, at nearly constant angle of attack, climbs and slows, then dives, losing altitude while picking up speed. The motion is usually of such a long period (about 93 seconds for a 747) that it need not be highly damped for piloted aircraft. This mode was studied (and named) by Lanchester in 1908. He showed that if one assumed constant angle of attack and thrust=drag, the period of the phugoid could be written as: T = π V2 U/g = 0.138 U. That is, the period is independent of the airplane characteristics and altitude, and depends only on the trimmed airspeed. With similarly strong assumtions, it can be shown that the damping varies as z = 1 / (V2 L/D).

Lateral Dynamics

Three dynamic modes describe the lateral motion of aircraft. These include the relatively uninteresting roll subsidence mode, the Dutch-roll mode, and the spiral mode.

The roll mode consists of almost pure rolling motion and is generally a non-oscillatory motion showing how rolling motion is damped.

Of somewhat greater interest is the spiral mode. Like the phugoid motion, the spiral mode is usually very slow and often not of critical importance for piloted aircraft. A 747 has a nonoscillatory spiral mode that damps to half amplitude in 95 seconds under typical conditions, while many airplanes have unstable spiral modes that require pilot input from time to time to maintain heading.

The Dutch-roll mode is a coupled roll and yaw motion that is often not sufficiently damped for good handling. Transport aircraft often require active yaw dampers to suppress this motion.

High directional stability ($C_{n\beta}$) tends to stabilize the Dutch-roll mode while reducung the stability of the spiral mode. Conversely large effective dihedral (rolling moment due to sideslip, $C_{l\beta}$) stabilizes the spiral mode while destabilizing the Dutch-roll motion. Because sweep produces effective dihedral and because low wing airplanes often have excessive dihedral to improve ground clearance, Dutch-roll motions are often poorly damped on swept-wing aircraft.

Longitudinal Control Requirements

Control power is usually critical in sizing the tail.

Some very large airplane designs are cruise trim critical. The tail is sized to be buffet free or below drag divergence at dive Mach number. Drag divergence is used as a measurement of likelihood of elevator control reversal. Drag divergence is accompanied by strong shocks on the suction side of the stabilizer. Deflecting the elevator to diminish lift in this condition can improve the flow behind the shock, increasing lift instead of reducing it and causing a control reversal. Typically the tail would be designed to be below drag divergence at dive Mach number and at its mid center of gravity cruise lift coefficient, a lift coefficient of 0.2 to 0.3. For actively-controlled airplanes in cruise, the tail may carry almost no load at mid CG, positive load at aft CG, and negative load at forward CG. In this case the tail is probably designed to be divergence free at dive Mach number and at its worst cruise lift coefficient.

Control requirements at low speed are usually critical. One requirement that determines the elevator sizing is a go around maneuver. The airplane begins in approach trim, flaps down, stabilizer set for 1g flight, no elevator. By deflecting the elevator only, the pilot should be able to get a pitch acceleration of 5 deg/s^2, minimum. On new aircraft with no stretch history, the elevator would be designed to provide 10 deg/s^2 pitch acceleration. 8 deg/s^2 is desirable.

Nosewheel liftoff may be a critical constraint, especially on advanced aircraft because of a trend toward moving the center of gravity aft relative to the aerodynamic center. In this maneuver, the aircraft is trimmed for climbout at V2 + 10 knots, which is about 1.3 Vstall. The elevator should generate enough moment to crack the nosewheel off the ground and provide 3 deg/s^2 pitch acceleration. In designing the tail, one would shoot for 6 deg/s^2 pitch acceleration.

The approach trim constraint is often critical. This constraint involves a 1g level acceleration from approach speed, 1.3 Vstall, to maximum flaps extended speed, VFE, which is typically 1.8 Vstall. The aircraft begins in approach trim and must be reach VFE using only the elevator, not the stabilizer, to retrim. In approaching VFE, the angle of attack decreases and must be accompanied by deflecting the elevator down. For trim at 1.3 Vstall, however, the stabilizer is deflected up to generate download. At VFE, the stabilizer and elevator end up working against each other. At this condition, the tail must be 2 deg below stall.

Icing affects estimation of maximum section lift. With evaporative anti-icing systems the properties of the clean section can be used. For aircraft without ice protection, the tail should be oversized by as much as 30%.

At VFE, it is common for the wing flap to be stalled. Because of the low angle of attack, there is no flow through the wing slat. Flow separates on the lower surface of the slat, and this disturbance impinges on the flap causing it to stall.

Takeoff normally does not stall the tail. The elevator typically has a limited throw. This usually keeps the tail within 2 deg of its stall angle of attack. Maximum stabilizer deflections of about 12 deg and a maximum elevator deflection around 25 deg are typical of transport aircraft.

Pitching moments from landing gear are usually small and act opposite to one's intuition. The gear struts block the flaps and reduce their nose down pitching moment. The gear also causes a slight increase in lift.

Structural sizing for fins are often set by a tail stop maneuver. Pilot applies a maximum rudder input, limited by either a pedal stop or a mechanical stop in the fin. The airplane sideslips and is carried by its inertia beyond its equilibrium sideslip angle. From the maximum equilibrium sideslip, the pilot releases the pedals causing the airplane to swing back and oscillate around zero sideslip. The maximum fin loads encountered during this maneuver are used to size the fin structure. For this reason, some companies use rudder throw limiters that provide full deflection, typically +/-30 deg, up to 160 knots, then decrease maximum deflection inversely proportional with dynamic pressure.

Lateral Control Requirements

For older and current aircraft up through the very large aircraft designs, stability requirements such as Dutch roll were an issue in sizing the vertical tail. In these aircraft, despite the presence of active control systems, the design philosophy was that the aircraft should be flyable with all electrons dead. An alternate philosophy is to examine how much reliance is placed on the control system and estimate the number of failures expected based on statistical data on failure rates. Control systems would then be designed with sufficient redundancy to achieve two orders of magnitude more reliability than some desired level.

This alternate philosophy that trusts active control may be used by some companies for future advanced aircraft design work; it will probably be used in any HSCT design. Some basic control will still be available even without active control in that pitch trim and rudder will still be mechanically activated. In the future, vertical tails will not be sized for Dutch roll, so long as the control system has sufficient authority to stabilize the airplane.

There is a limit to the instability that can be tolerated; the control system cannot be saturated. For this purpose, the rudder should be designed to return aircraft from a 10?sideslip disturbance at any altitude. For reliability, rudders may be split into upper and lower halves, with independent signals and actuators plus redundant processors.

The critical control sizing constraint is often VMCG, minimum controlled ground speed. In this condition, flight is straight and unaccelerated laterally. Nose gear reaction is zero. Aerodynamic moments must balance engine thrust with one engine out and creating windmilling drag, and the other engine at max thrust plus a thrust bump for a "hot" engine. If the moment balance is done about the aircraft center of gravity, main gear reactions caused by rudder sideforce must be considered. If the main gear reactions were ignored, rudder force would be underestimated by 15% to 20%. Alternately, the moment balance can be done about the main gear center, which lies in line with the gear and halfway between them. Engine thrust imbalance should be controllable with full rudder deflection.

VMCG is relatively independent of flap setting or aircraft weight because it is primarily a matter of balancing engine thrust imbalance with the rudder. Flaps may affect rudder performance sometimes because of aerodynamic interaction. Aircraft weight does not enter the moment balance because, when moments are taken about the main gear, there are no ground moment reactions and there are no inertial forces because there is no lateral acceleration. The engine thrust imbalance is constant because full thrust is always used for takeoff, regardless of aircraft weight. To determine a required VMCG speed, one would examine an aircraft in its lightest commercial weight. This would be the weight with a minimum passenger load to break even on a particular range, say a 30% passenger load. At low takeoff weights, more flaps will be used as a result of optimizing flap deflection for best lift to drag in second segment climb. The light weight and large flap deflection should reduce speeds for second segment climb and rotation. In establishing the balanced field length for this condition, VMCG should be set at the speed where second segment climb or rotation becomes critical. For aircraft such as the DC9 or DC10 this speed is about 110 knots. For heavier aircraft, VMCG is higher, 120 knots.

VMCA, minimum control airspeed, is usually not critical because dynamic pressure is higher, making the rudder more effective, the thrust imbalance is smaller, because of thrust lapse, plus the airplane is allowed to sideslip to trim. The VMCG condition is at zero sideslip; rudders may be double hinged to enable large lift coefficients to be achieved on the fin at this condition.

While VMCG is critical for 2 engine airplanes, on 4 engine airplanes VMCL2 may be critical. In this landing condition, 2 engines are out on same side of the airplane while the other two are at max takeoff thrust. The rudder is more effective since this is done at approach speed, 1.3 Vstall.

One airborne condition that might size the rudder is a crosswind landing decrab. This condition is at 1.3 Vstall with a 35 knot crosswind. The rudder is used to control an aerodynamic sideslip of 13?to 15? Increasing the vertical tail area does not help here because it increases the resistance to sideslip. If this condition is critical the proportion of rudder to vertical tail area should be adjusted.

Tail Design and Sizing

Tail Design Introduction

Tail surfaces are used to both stabilize the aircraft and provide control moments needed for maneuver and trim. Because these surfaces add wetted area and structural weight they are often sized to be as small as possible. Although in some cases this is not optimal, the tail is general sized based on the required control power as described in other sections of this chapter. However, before this analysis can be undertaken, several configuration decisions are needed. This section discusses some of the considerations involved in tail configuration selection.

A large variety of tail shapes have been employed on aircraft over the past century. These include configurations often denoted by the letters whose shapes they resemble in front view: T, V, H, + , Y, inverted V. The selection of the particular configuration involves complex system-level considerations, but here are a few of the reasons these geometries have been used.

The conventional configuration with a low horizontal tail is a natural choice since roots of both horizontal and vertical surfaces are conveniently attached directly to the fuselage. In this design, the effectiveness of the vertical tail is large because interference with the fuselage and horizontal tail increase its effective aspect ratio. Large areas of the tails are affected by the converging fuselage flow, however, which can reduce the local dynamic pressure.

A T-tail is often chosen to move the horizontal tail away from engine exhaust and to reduce aerodynamic interference. The vertical tail is quite effective, being 'end-plated' on one side by the fuselage and on the other by the horizontal tail. By mounting the horizontal tail at the end of a swept vertical, the tail length of the horizontal can be increased. This is especially important for short-coupled designs such as business jets. The disadvantages of this arrangement include higher vertical fin loads, potential flutter difficulties, and problems associated with deep-stall.

One can mount the horizontal tail part-way up the vertical surface to obtain a cruciform tail. In this arrangement the vertical tail does not benefit from the endplating effects obtained either with conventional or T-tails, however, the structural issues with T-tails are mostly avoided and the configuration may be necessary to avoid certain undesirable interference effects, particularly near stall.

V-tails combine functions of horizontal and vertical tails. They are sometimes chosen because of their increased ground clearance, reduced number of surface intersections, or novel look, but require mixing of rudder and elevator controls and often exhibit reduced control authority in combined yaw and pitch maneuvers.

H-tails use the vertical surfaces as endplates for the horizontal tail, increasing its effective aspect ratio. The vertical surfaces can be made less tall since they enjoy some of the induced drag savings associated with biplanes. H-tails are sometimes used on propeller aircraft to reduce the yawing moment associated with propeller slipstream impingment on the vertical tail. More complex control linkages and reduced ground clearance discourage their more widespread use.

Y-shaped tails have been used on aircraft such as the LearFan, when the downward projecting vertical surface can serve to protect a pusher propeller from ground strikes or can reduce the 1-per-rev interference that would be more severe with a conventional arrangement and a 2 or 4-bladed prop. Inverted V-tails have some of the same features and problems with ground clearance, while producing a favorable rolling moments with yaw control input.

Specific design guidelines:

The tail surfaces should have lower thickness and/or higher sweep than the wing (about 5° usually) to prevent strong shocks on the tail in normal cruise. If the wing is very highly swept, the horizontal tail sweep is not increased this much because of the effect on lift curve slope. Tail t/c values are often lower than that of the wing since t/c of the tail has a less significant effect on weight. Typical values are in the range of 8% to 10%.

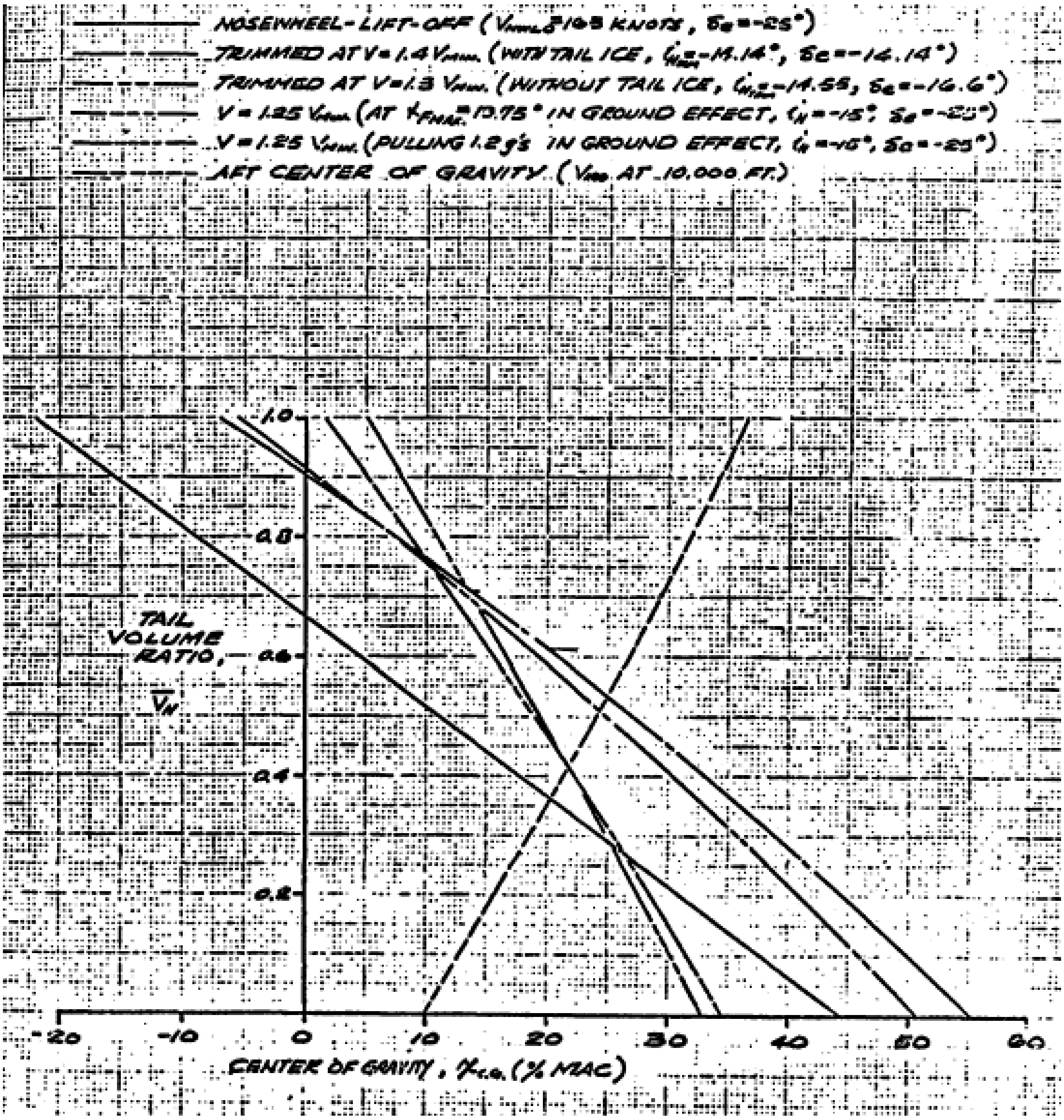
Typical aspect ratios are about 4 to 5. T-Tails are sometimes higher (5-5.5), especially to avoid aft-engine/pylon wake effects.

ARv is about 1.2 to 1.8 with lower values for T-Tails. The aspect ratio is the square of the vertical tail span (height) divided by the vertical tail area, b_v^2 / S_v.

Taper ratios of about .4 to .6 are typical for tail surfaces, since lower taper ratios would lead to unacceptably small Reynolds numbers. T-Tail vertical surface taper ratios are in the range of 0.85 to 1.0 to provide adequate chord for attachment of the horizontal tail and associated control linkages.

Tail Sizing

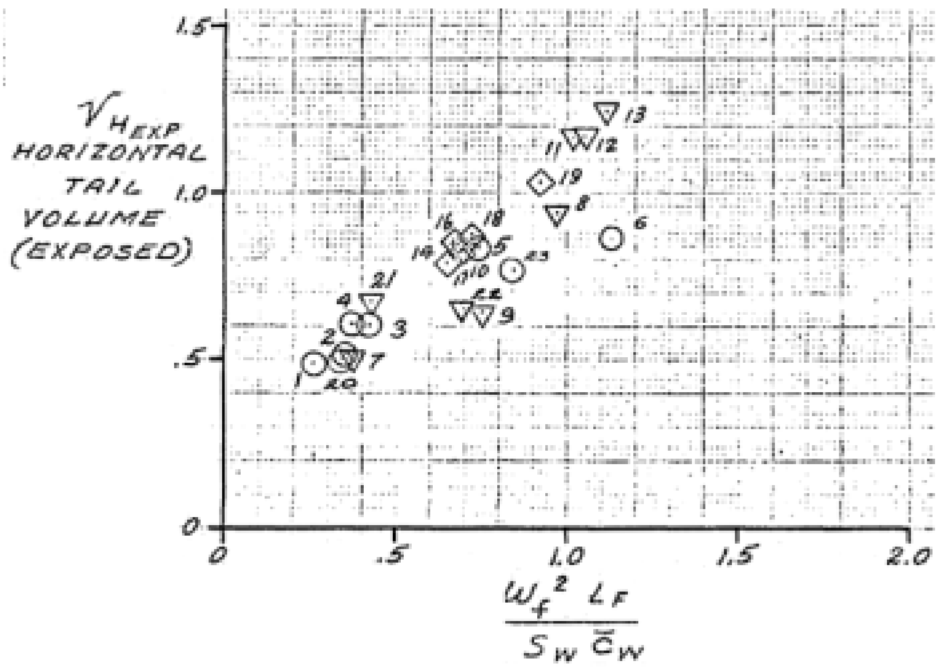
Horizontal tails are generally used to provide trim and control over a range of conditions. Typical conditions over which tail control power may be critical and which sometimes determine the required tail size include: take-off rotation (with or without ice), approach trim and nose-down acceleration near stall. Many tail surfaces are normally loaded downward in cruise. For some commercial aircraft the tail download can be as much as 5% of the aircraft weight. As stability requirements are relaxed with the application of active controls, the size of the tail surface and/or the magnitude of tail download can be reduced. Actual tail sizing involves a number of constraints that are often summarized on a plot called a scissors curve. An example is shown below.



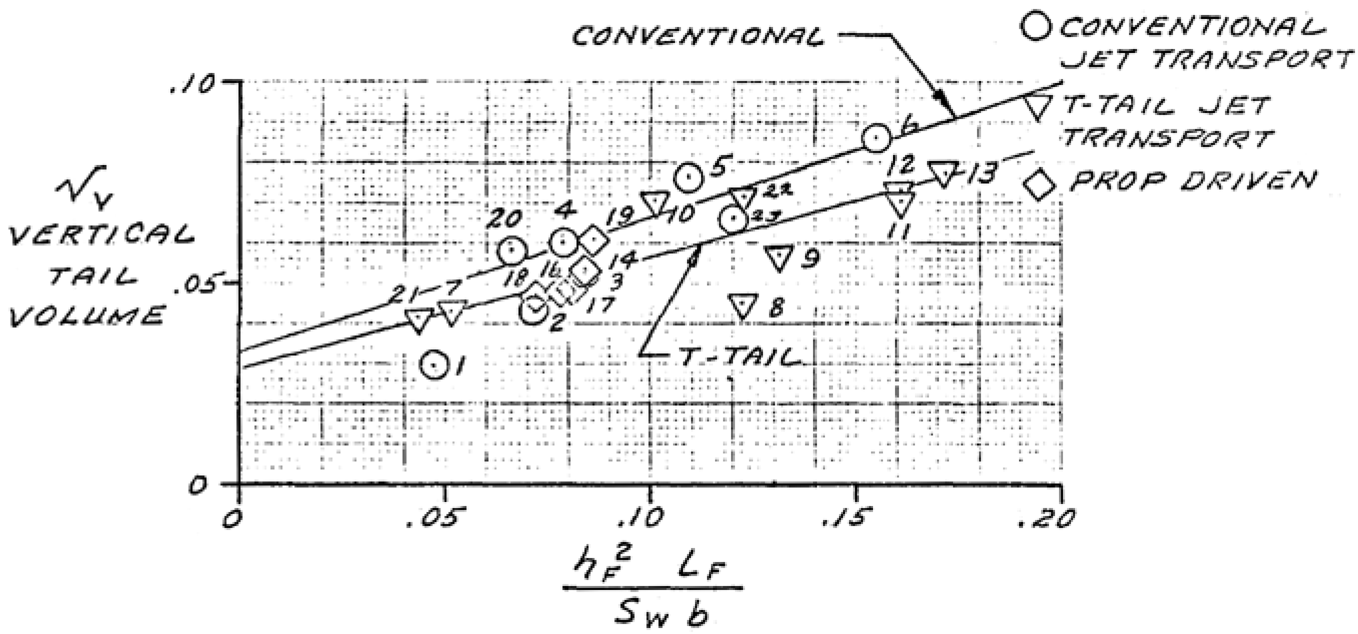
Scissors curve used for sizing tail based on considerations of stability and control.

Statistical Method

For the purposes of early conceptual design it is useful to estimate the required size of tail surfaces very simply. This can be done on the basis of comparison with other aircraft.



Correlation of aircraft horizontal tail volume.



Correlation of aircraft vertical tail volume as a function of fuselage maximum height and length.

The above correlations are based on old airplane designs (as are most statistical methods). Some reduction in tail volumes are possible with stability augmentation. In any case, this tail sizing method is only used to establish a starting point for further analysis. The airplanes included above are:

1 Comet
2 DC-8-50
3 DC-8-61
4 B-720
5 B-747
6 B-737-200
7 C-141
8 BAC-111
9 DH-121
10 B-727
11 DC-9-10
12 DC-9-30
13 DC-9-40
14 DC-7C
15 DC-4
16 DC-6
17 DC-6B
18 DC-7
19 C-133
20 C-990
21 VC-10
22 C-5
23 DC-10-10

The correlation is based on a fuselage destabilizing parameter:

h_f is the fuselage height
w_f is the fuselage width
L_f is the fuselage length
S_w, c_w, and b are the wing area, MAC, and span.

and provides a rough estimate for the required horizontal tail volume ($V_h = l_h S_h / c_w S_w$) and vertical tail volume ($V_v = l_v S_v / b S_w$). Recall that l_h and l_v are the distances from the c.g. to the a.c. of the horizontal and vertical tails

Rational Method

The following procedure may be used to compute the required tail size for a given stability level as a function of c.g. position. It assumes that the critical airplane control requirement is nosewheel rotation, although this is just one of many possible constraints.

For c.g. positions ranging from the leading edge of the M.A.C. to about 60% of the M.A.C. compute and plot the required tail volume coefficient,

$$V_h = \frac{l_h S_h}{S_w \bar{c}}$$

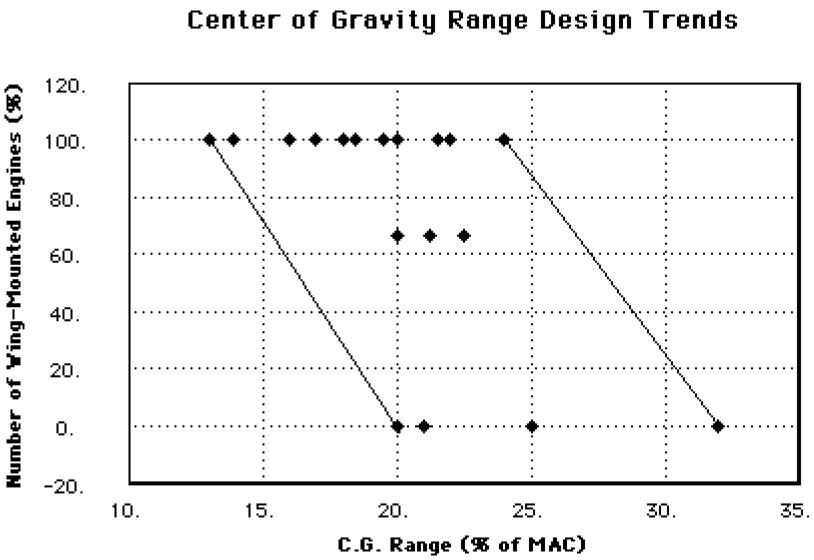
for the desired level of static stability. The minimum static margin would typically be about .10 but it must be increased because bending of the wing and the fuselage at high speeds reduces the rigid airplane stability. (Assume a change in sm of about -.10 for swept wing transport aircraft. sm changes due to aeroelasticity can usually be neglected in preliminary design of general aviation aircraft.) In addition, the desired static margin may be increased by about .10 for T-tail airplanes to improve high angle of attack stability.

In order to compute the required tail volume, you will need to find the distance from the c.g. to the wing a.c.. The position of the wing a.c. may be computed using the program Wing that was used in a previous assignment. The lift curve slope of the isolated tail and wing may also be computed using this program.

Control Power

The second requirement for the horizontal tail is that it provides sufficient control power. It must not only be possible to trim the airplane in cruise but also in more critical conditions. Typical critical conditions include: Rotation and nosewheel lift-off on take-off at forward c.g., trim without tail stall at maximum flap extension speed, and trim at forward c.g. with landing flaps at C_{Lmax}.

For this exercise we will consider only the problem of take-off rotation. We assume that the tail incidence and elevator angle settings are such that the horizontal tail can achieve a certain maximum lift coefficient C_{LHmax} (in the downward direction). The force required from the tail to rotate the airplane depends on the wing and body pitching moments to some extent but largely on the weight moment about the rear wheels.



At aft c.g. the force is smallest, but a certain amount is required since the c.g. must lie in front of the rear wheels to prevent the airplane from tipping over on its tail. Actually, the requirement is not so much to avoid tipping backward but rather providing sufficient weight on the nosewheel to permit acceptable traction for steering. This is satisfied with about 8% of the weight on the forward wheels. With this load on the forward wheels, the moment about the rear wheels due to the forward position of the c.g. is at least: $|M| = .08\, l_g\, W$ where l_g is the distance from the main gear to the nose gear.

The pitching moment coefficient at take-off is then:

$$C_{m_{wheels}} = \frac{.08\, \Delta l_g\, W}{q\, S_w\, \bar{c}} + C_{m_{aero}}$$

We will ignore the aerodynamic term for now, although a detailed study would include this. For rotation, then, the load on the tail must be:

$$C_{L_h} = -\, C_{m_{wheels}} \left(\frac{S_w\, \bar{c}}{S_h\, l_h}\right) = -\, C_{m_{wheels}} \frac{1}{V_h}$$

The minimum tail volume required can then be calculated with the assumed C_{LHmax}. (For airplanes with variable incidence stabilizers and elevators $C_{LHmax} = 1.0$ will be an acceptable estimate.)

At forward c.g. positions, a larger tail is required since the moment about the rear wheels is:
$M = M_{aft\text{-}c.g.} + W\, \Delta c.g.$

(Note that δc.g. is the c.g. range. It is not the static margin, discussed earlier.)

So, $$C_{m_{fwd\,c.g.}} = C_{m_{aft\,c.g.}} + C_{L_{TO}} \frac{\Delta c.g.}{\bar{c}}$$

The required tail volume may be determined from this analysis at the forward c.g. position. It may be interesting to compare your results with the statistical method shown in the previous section. Also note that we have previously estimated the main gear position at 50% of the MAC. If we desire 8% of the load on the nose gear at aft c.g. this means that the main gear must be located .08 l_g behind the aft c.g.