9.1 Introduction

A. Each project is consist of numerous jobs or tasks performed by a variety of departments or individuals

B. Project management
   - Identify project goals or objectives
   - Work breakdown structure
   - Establish activities relationship
   - Estimate activities times
   - Estimate project completion time
   - Post completion evaluation
   - Resource allocations

C. Gantt chart
   - Disadvantage of Gantt chart
     - Unable to reflect the interrelation among activities
     - No revelation of the delay of activity on other activities

D. Disadvantage of Gantt chart
   a. PERT (Program Evaluation and Review Technique) : Probabilistic model (stochastical model)
   b. CPM (Critical Path Model) : deterministic model

E. Nations in PERT
   a. Activity : effort need resource and time
   b. Event : milestones or checkpoints
   c. Project : a collection of activity
   d. Network : a logical and chronological set of activities and events
   e. Critical activity
   f. Path
g. Critical path
   — Advantage of PERT/CPM
      — Commitments and communications
      — Detailed planning
      — Efficient monitoring and control
      — Identify potential problem areas
      — Proper use of resources
      — Rescheduling
      — Government requirement
      — Easily understood
      — Adaptive to computers
      — Decision making tools
      — Cost-time trade-off

F. PERT/CPM process
   a. phase1 : formulation
      1. analysis of the project
      2. sequence the activities
      3. estimate activity times and costs
   b. phase2 : solution
      1. construct the network
      2. event analysis
      3. activity analysis
   c. Analysis and control
      1. Monitoring and control
      2. Resource utilization

G. The network representation
   a. Activity (operation)
   b. Node (event) : the beginning and ending of each activity
Ex:

<table>
<thead>
<tr>
<th>Activity</th>
<th>Prerequisite</th>
<th>time</th>
</tr>
</thead>
<tbody>
<tr>
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<td>B</td>
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<td>C</td>
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<td>D</td>
<td>A,C</td>
<td>4</td>
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<td>E</td>
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<td>8</td>
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<td>F</td>
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<td>5</td>
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<tr>
<td>J</td>
<td>E,G,H</td>
<td>3</td>
</tr>
</tbody>
</table>
### 9.2 CPM

A. Find the “longest path” – “critical path”

- forward pass

\[
ES_i : \text{the Earliest start time of activity } i
\]

\[
EF_i : \text{the Earliest finish time of activity } i
\]

\[
ES_i = \max[ES_i + t_i] = \max_i[EF_i]
\]

\[
EF_j = ES_j + t_j
\]

- backward pass

\[
LS_i : \text{the Latest start time of activity } i
\]

\[
LF_i : \text{the Latest finish time of activity } i
\]

\[
LF_i = \min[LF_i - t_i] = \min_j[LS_j]
\]

\[
LS_i = LF_i + t_i
\]

- slack of free time

(1)Total Float (TF) : slack for on event

(2)Free Float (FF) : slack for a activity

\[
TF_i = LS_i - ES_i = LF_i - EF_i
\]

\[
FF_i = ES_j - ES_i - t_i = ES_j - EF_i
\]

<table>
<thead>
<tr>
<th>Link</th>
<th>Activity</th>
<th>ES</th>
<th>EF</th>
<th>LS</th>
<th>LF</th>
<th>TF</th>
<th>FF</th>
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<tbody>
<tr>
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<td>20</td>
<td>23</td>
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</tr>
</tbody>
</table>

B. Observations

- \( TF_i = 0 \Rightarrow FF_i = 0 \)

- \( FF_i = 0 \neq TF_i = 0 \)

- if \( TF_i \neq 0 \) noncritical activity

- if \( FF_i \leq TF_i \) (noncritical event) \( i \) can be delayed as much as \( FF_i \)
9.3 PERT (for uncertain activity time)

A. Assumption

— Time estimate for each activity is based on three difference aspects
— Based on Beta distribution assumption
  (1) a: optimistic time (minimum time)
  (2) b: pessimistic time (maximum time)
  (3) m: most probable time (normal time)

B. Average Time (expected time)

— based on Beta probability distribution
  \[
  E = \frac{(a + b) + 2m}{3} = \frac{a + 4m + b}{6}
  \]
  \[
  \sigma^2 = \left(\frac{b - a}{6}\right)^2
  \]
— treat the expected time as a fixed time proceed with CPM
— if there are more than one critical paths identify the one with the largest variance

C. Compute the \( E[U_i] \) and \( Var[U_i] \)

— \( U_i = \) the earliest occurrence time of event
— assume the activities are statistically independently
  \[
  E[U_i] = ES_i = \sum E_k
  \]
  \[
  Var[U_i] = \sum \sigma^2_k
  \]
— where \( k = \) the activities along the longest path leading to \( i \)
— if we specify a certain project completion time \( (ST_i) \), we can computes the probability of meeting this dead line

\[
P(U_i \leq ST_i) = P\left[ \frac{U_i - E(U_i)}{\sqrt{Var(U_i)}} \leq \frac{ST_i - E(U_i)}{\sqrt{Var(U_i)}} \right] = P[Z \leq Z^*]\]
9.4 Considering Time-Cost Trade-Offs

A. The duration of activity can be shorten by adding resource to that activity
B. The need to identify the least cost to crash activity time to meet the desired date
C. Notation

\[ T_j = \text{normal time for activity } j \]
\[ T'_j = \text{time for activity } j \text{ under maximum crashing} \]
\[ C_j = \text{cost for activity } j \text{ under maximum crashing} \]
\[ K_j = \text{crash cost for activity } j \]

\[ K_j = \frac{C'_j - C_j}{T_j - T'_j} \]
D. Observation
   — other path may be critical
   — check the revised network

E. LP model for crashing consideration
   — Variables
     — \( X_j \) = the occurrence of event \( i \)
     — \( Y_j \) = amount of crash time used for activity \( j \)
   — objective function
     — \( \min \sum_j K_j Y_j \)
   — constraints
     (1) describe the network
     (2) limiting the activity crash time
     (3) meeting the project complete time

\[ K_j = \frac{C'_j - C_j}{T'_j - T_j} = \frac{900 - 500}{8 - 4} = 100 \]
min \( K_A Y_A + K_B Y_B + K_C Y_C + K_D Y_D + K_E Y_E + K_F Y_F + K_G Y_G + K_H Y_H + K_I Y_I + K_J Y_J \)

\[
X_2 \geq T_B - Y_B + X_1 \\
X_3 \geq T_C - Y_C + X_2 \\
X_4 \geq T_A - Y_A + X_1 \\
X_6 \geq T_F - Y_F + X_3 \\
X_7 \geq T_E - Y_E + X_5 \\
X_7 \geq T_G - Y_G + X_6 \\
X_7 \geq T_H - Y_H + X_8 \\
X_9 \geq T_J - Y_J + X_7 \\
X_9 \geq T_I - Y_I + X_2 \\
X_4 \geq X_3 \\
X_6 \geq X_8 \\
X_5 \geq T_0 - Y_0 + X_4
\]

<table>
<thead>
<tr>
<th>Activity</th>
<th>Prerequired</th>
<th>time</th>
</tr>
</thead>
<tbody>
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<td>J</td>
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</table>

\( Y_i \leq T_i \quad \forall_j \)
\( Y_A \leq 2 \quad Y_0 \leq 1 \)
\( X_9 \leq 20 \)
9.5 PERT/COST

A. Concentrate on time aspect of a project

B. Budget cost
   — identify cost
   — project when cost occurs

C. Assumption
   — cost occurs at a constant rate

D. Procedure
   — forecast the project cost based on the earliest and latest start time
   — divide the cost uniformly into unit-time cost the shadow area between two cost curves shows the feasible budget

Ex.

<table>
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<th>Month</th>
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<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
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<td>77</td>
<td>82</td>
<td>87</td>
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</table>
9.6 Project Control

A. Purpose
   a. Identify the actual of activities
   b. Periodically information collections
      — Activity cost update
      — Percent completion to date

B. Objective
   — Identify cost overrun and underruns take necessary correction actions
   — Notations
      — \( V_i \) = value of work completed for activity \( i \)
      — \( P_i \) = percentage completed for activity \( i \)
      — \( B_i \) = budget cost activity \( i \)
— $V_i = \frac{(P_i/100)}{B_i}$

— Cost overruns

— $AC_i =$ actual cost to date of activity $i$

— $D_i =$ difference between actual cost and value of work completed for activity $i$

— $D_i = AC_i - V_i$

$V_i =$ completion%* Bi

<table>
<thead>
<tr>
<th>Activity</th>
<th>$AC_i$</th>
<th>completion%</th>
<th>Bi</th>
<th>$V_i$</th>
<th>$D_i$</th>
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<td>30000</td>
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<td>6500</td>
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</tbody>
</table>

6500/48500=13.4% SERIOUS PROBLEM

— Identify cost over runs A,E

— A is completed and E is only 25%then adjust activity E

— Drawback

— need tremendous clerical efforts

— cost allocation problem

— need different cost accounting system
例. 競標

<table>
<thead>
<tr>
<th>工程代號</th>
<th>工作情況</th>
<th>工作天數</th>
<th>前置作業</th>
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</tr>
<tr>
<td>C</td>
<td>選擇有關競標人</td>
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\[
ES_i = \max \{ES_k + D_{k,i}\}
\]

\[
EF_{ij} = ES_j + D_{i,j}
\]

\[
LF_i = \min \{LF_j - D_{i,j}\}
\]

\[
LS_i = LF_i - D_{i,i}
\]

\[
TF_i = LS_i - ES_i = LF_i - EF_i
\]

\[
FF_i = ES_j - ES_i - t_i = ES_j - EF_i
\]
### 9.7 Resource Utilization

**A. purpose**

- resource smoothing
  - allocate resource for smooth usage
- resource leveling
- reduce peak resource requirements
- resource allocation
  - for a desired resource level, find the shortest possible project schedule

**B. Trial and error heuristic approach**

- Step 1: resource accumulation by the earliest start schedule
- Step 2: draw the Gantt chart
- Step 3: start from the last noncritical activity not considered move it in total float period
- Step 4: if no more noncritical activity to be considered stop, otherwise go to step 3