Chapter Fifteen
Market Demand

From Individual to Market Demand Functions

- Think of an economy containing $n$ consumers, denoted by $i = 1, \ldots, n$.
- Consumer $i$'s ordinary demand function for commodity $j$ is
  \[ x_{ij}(p_1, p_2, m^i) \]

When all consumers are price-takers, the market demand function for commodity $j$ is

\[ X_j(p_1, p_2, m_1, \ldots, m^n) = \sum_{i=1}^{n} x_{ij}(p_1, p_2, m^i). \]

- Aggregate demand depend on prices and the distribution of incomes.

If all consumers are identical then where $M = nm$ or a constant proportion of individual income.

\[ X_j(p_1, p_2, M) = n \times x_{ij}(p_1, p_2, m) \]
From Individual to Market Demand Functions

- The market demand curve is the “horizontal sum” of the individual consumers’ demand curves.
- E.g. suppose there are only two consumers; i = A, B.

\[
\begin{align*}
\text{The "horizontal sum" of the demand curves of individuals A and B.}
\end{align*}
\]

Elasticities

- Elasticity measures the “sensitivity” of one variable with respect to another.
- The elasticity of variable X with respect to variable Y is

\[
\varepsilon_{x,y} = \frac{\% \Delta x}{\% \Delta y}.
\]

Economic Applications of Elasticity

- Economists use elasticities to measure the sensitivity of
  - quantity demanded of commodity i with respect to the price of commodity i (own-price elasticity of demand)
  - demand for commodity i with respect to the price of commodity j (cross-price elasticity of demand).
Economic Applications of Elasticity
- demand for commodity i with respect to income (income elasticity of demand)
- quantity supplied of commodity i with respect to the price of commodity i (own-price elasticity of supply)

Economic Applications of Elasticity
- quantity supplied of commodity i with respect to the wage rate (elasticity of supply with respect to the price of labor)
- and many, many others.

Own-Price Elasticity of Demand
• Q: Why not use a demand curve’s slope to measure the sensitivity of quantity demanded to a change in a commodity’s own price?

Own-Price Elasticity of Demand
In which case is the quantity demanded $X_i^*$ more sensitive to changes to $p_i$? It is the same in both cases.
Own-Price Elasticity of Demand

- **Q:** Why not just use the slope of a demand curve to measure the sensitivity of quantity demanded to a change in a commodity’s own price?
- **A:** Because the value of sensitivity then depends upon the (arbitrary) units of measurement used for quantity demanded.

\[
\varepsilon_{x_1,p_1} = \frac{\% \Delta x_1}{\% \Delta p_1}
\]

is a ratio of percentages and so has no units of measurement. Hence own-price elasticity of demand is a sensitivity measure that is independent of units of measurement.

Arc and Point Elasticities

- An “average” own-price elasticity of demand for commodity i over an interval of values for \(p_i\) is an arc-elasticity, usually computed by a mid-point formula.
- Elasticity computed for a single value of \(p_i\) is a point elasticity.

Arc Own-Price Elasticity

What is the “average” own-price elasticity of demand for prices in an interval centered on \(p_i^*\)?

\[
\varepsilon_{x_i^*,p_i} = \frac{\% \Delta x_i^*}{\% \Delta p_i} = \frac{100 \times \frac{2h}{p_i^*} \% \Delta x_i^*}{100 \times \frac{(X_i^{**}-X_i^{***})}{(X_i^{**}+X_i^{***})} / 2}
\]
Arc Own-Price Elasticity

$$\varepsilon_{X_i^*,p_i} = \frac{\% \Delta X_i^*}{\% \Delta p_i} = \frac{100 \times 2h}{p_i}$$

$$\% \Delta X_i^* = 100 \times \frac{(X_i''-X_i''')}{(X_i''+X_i''')/2}$$

So

$$\varepsilon_{X_i^*,p_i} = \frac{\% \Delta X_i^*}{\% \Delta p_i} = \frac{p_i'}{(X_i''+X_i''')/2} \times \frac{(X_i''-X_i''')}{2h}.$$

is the arc own-price elasticity of demand.

Point Own-Price Elasticity

What is the own-price elasticity of demand in a very small interval of prices centered on $$p_i'$$?

As $$h \to 0$$,

$$\varepsilon_{X_i^*,p_i} = \frac{\% \Delta X_i^*}{\% \Delta p_i} = \frac{p_i'}{(X_i''+X_i''')/2} \times \frac{(X_i''-X_i''')}{2h}.$$
Point Own-Price Elasticity

What is the own-price elasticity of demand in a very small interval of prices centered on \( p_i' \)?

\[ \varepsilon_{X_i', p_i} = \frac{p_i'}{X_i'} \times \frac{dX_i'}{dp_i} \]

is the elasticity at the point \((X_i', p_i')\).

E.g. Suppose \( p_i = a - bX_i \).
Then \( X_i = (a - p_i)/b \) and
\[ dX_i = \frac{1}{b} \] Therefore,
\[ \varepsilon_{X_i', p_i} = \frac{p_i}{(a - p_i)/b} \times \left( \frac{-1}{b} \right) = -\frac{p_i}{a} \]
**Point Own-Price Elasticity**

\[ p_i = a - bX_i^* \]

\[ \varepsilon_{X_i^*,p_i} = \frac{-p_i}{a} \]

\[ \varepsilon = \frac{-p_i}{a} \]

- \( \varepsilon = \infty \) when \( p = a \) and \( \varepsilon = -1 \)
- \( \varepsilon = 0 \) at \( a/2 \)
- \( \varepsilon = -1 \) everywhere along the demand curve.

**Point Own-Price Elasticity**

\[ p_i = a - bX_i^* \]

\[ \varepsilon_{X_i^*,p_i} = \frac{-p_i}{a} \]

\[ \varepsilon = \frac{-p_i}{a} \]

- \( \varepsilon = \infty \) when \( p = a \) and \( \varepsilon = -1 \)
- \( \varepsilon = 0 \) at \( a/2 \)
- \( \varepsilon = -1 \) (own-price unit elastic)
- \( \varepsilon = \infty \) (own-price elastic)
- \( \varepsilon = 0 \) (own-price inelastic)

**E.g.** Then \( X_i^* = kp_i^a \). Then

\[ \frac{dX_i^*}{dp_i} = ap_i^{a-1} \]

so

\[ \varepsilon_{X_i^*,p_i} = \frac{p_i}{kp_i^a} \times kp_i^{a-1} = a \]

\[ \frac{p_i}{p_i^a} = a \]
## Revenue and Own-Price Elasticity of Demand

<table>
<thead>
<tr>
<th>If raising a commodity’s price causes little decrease in quantity demanded, then sellers’ revenues rise.</th>
<th>If raising a commodity’s price causes a large decrease in quantity demanded, then sellers’ revenues fall.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hence own-price inelastic demand causes sellers’ revenues to rise as price rises.</td>
<td>Hence own-price elastic demand causes sellers’ revenues to fall as price rises.</td>
</tr>
</tbody>
</table>

### Revenue and Own-Price Elasticity of Demand

Sellers’ revenue is 

\[ R(p) = p \times X^*(p). \]

So 

\[ \frac{dR}{dp} = X^*(p) + p \frac{dX^*}{dp}. \]

\[ = X^*(p) \left[ 1 + \frac{p}{X^*(p)} \frac{dX^*}{dp} \right] \]

\[ = X^*(p)[1 + \varepsilon]. \]

### Revenue and Own-Price Elasticity of Demand

\[
\frac{dR}{dp} = X^*(p) [1 + \varepsilon]
\]

so if 

\[ \varepsilon = -1 \]

then 

\[ \frac{dR}{dp} = 0 \]

and a change to price does not alter sellers’ revenue.
Revenue and Own-Price Elasticity of Demand

\[ \frac{dR}{dp} = X^*(p)(1 + \varepsilon) \]

but if \(-1 < \varepsilon \leq 0\) then \(\frac{dR}{dp} > 0\)

and a price increase raises sellers’ revenue.

And if \(\varepsilon < -1\) then \(\frac{dR}{dp} < 0\)

and a price increase reduces sellers’ revenue.

In summary:
- Own-price inelastic demand; \(-1 < \varepsilon \leq 0\) price rise causes rise in sellers’ revenue.
- Own-price unit elastic demand; \(\varepsilon = -1\) price rise causes no change in sellers’ revenue.
- Own-price elastic demand; \(\varepsilon < -1\) price rise causes fall in sellers’ revenue.

Marginal Revenue and Own-Price Elasticity of Demand

- A seller’s marginal revenue is the rate at which revenue changes with the number of units sold by the seller.

\[ MR(q) = \frac{dR(q)}{dq} \]
Marginal Revenue and Own-Price Elasticity of Demand

\[ p(q) \] denotes the seller’s inverse demand function; i.e. the price at which the seller can sell \( q \) units. Then

\[
R(q) = p(q) \times q
\]

so

\[
MR(q) = \frac{dR(q)}{dq} = \frac{dp(q)}{dq} \times q + p(q)
\]

\[
= p(q) \left[ 1 + \frac{q \times dp(q)}{p(q) \times dq} \right].
\]

Marginal Revenue and Own-Price Elasticity of Demand

\[
MR(q) = p(q) \left[ 1 + \frac{q \times dp(q)}{p(q) \times dq} \right].
\]

and

\[
\varepsilon = \frac{dq}{dp} \times \frac{p}{q}
\]

so

\[
MR(q) = p(q) \left[ 1 + \frac{1}{\varepsilon} \right].
\]

Marginal Revenue and Own-Price Elasticity of Demand

\[
MR(q) = p(q) \left[ 1 + \frac{1}{\varepsilon} \right]
\]

says that the rate at which a seller’s revenue changes with the number of units it sells depends on the sensitivity of quantity demanded to price; i.e., upon the of the own-price elasticity of demand.

Marginal Revenue and Own-Price Elasticity of Demand

\[
MR(q) = p(q) \left[ 1 + \frac{1}{\varepsilon} \right]
\]

If \( \varepsilon = -1 \) then \( MR(q) = 0 \).
If \( -1 < \varepsilon \leq 0 \) then \( MR(q) < 0 \).
If \( \varepsilon < -1 \) then \( MR(q) > 0 \).
Marginal Revenue and Own-Price Elasticity of Demand

If $\epsilon = -1$ then $MR(q) = 0$. Selling one more unit does not change the seller’s revenue.

If $-1 < \epsilon \leq 0$ then $MR(q) < 0$. Selling one more unit reduces the seller’s revenue.

If $\epsilon < -1$ then $MR(q) > 0$. Selling one more unit raises the seller’s revenue.

Marginal Revenue and Own-Price Elasticity of Demand

An example with linear inverse demand.

$p(q) = a - bq$.

Then $R(q) = p(q)q = (a - bq)q$
and $MR(q) = a - 2bq$. 

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[Graph showing the relationship between price, quantity, marginal revenue, and own-price elasticity of demand.]