Loading and Curling Stress Models for Concrete Pavement Design

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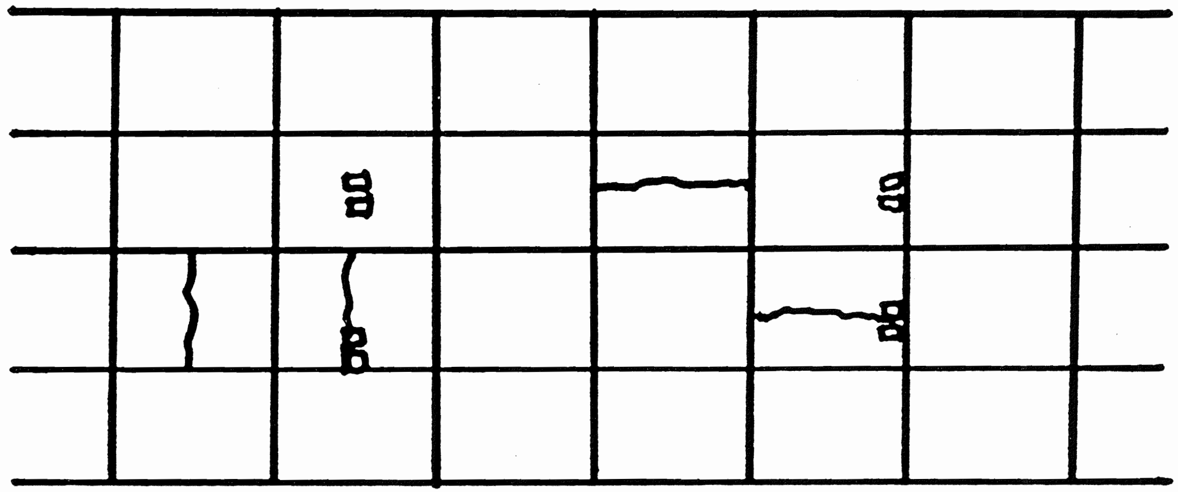
Objective

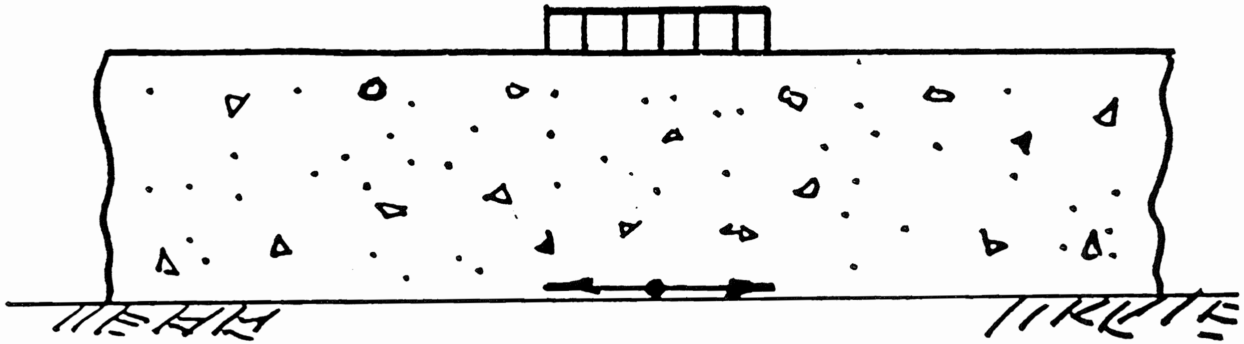
- Develop predictive models for load, curl, and load/curl that could be utilized in a spreadsheet or PC program for rapid calculation purposes.

- **May encourage use by practitioners.**

Edge Loads Cause Longitudinal And Transverse Cracks



Load and Temperature Curling Stresses



Load Stress - Located at Edge

Curl Stress - Positive or Negative Gradient

Combined Load and Curl Stress

Westergaard Edge Loading Solutions

$$\sigma_W = \frac{3(1+\mu)P}{\pi(3+\mu)h^2}\left[\log_e \frac{Eh^3}{100ka^4} + 1.84 - \frac{4\mu}{3} + \frac{1-\mu}{2} + 1.18(1+2\mu)\frac{a}{l}\right]$$

$$\delta_W = \frac{\sqrt{2+1.2\mu}\,P}{\sqrt{Eh^3k}}\left[1 - (0.76 + 0.4\mu)\frac{a}{l}\right]$$

$$l = \sqrt[4]{\frac{Eh^3}{12(1-\mu^2)k}}$$

- **Infinite slab length (L), Infinite slab width (W)**
- **Fully support slab**

Westergaard Thermal Curling Solutions

$$\sigma_y = \sigma_0 \{1 - \frac{2 \cos\lambda \cosh\lambda}{\sin 2\lambda + \sinh 2\lambda} [(\tan\lambda + \tanh\lambda) \cos\frac{y}{l\sqrt{2}} \cosh\frac{y}{l\sqrt{2}} + (\tan\lambda - \tanh\lambda) \sin\frac{y}{l\sqrt{2}} \sinh\frac{y}{l\sqrt{2}}] \}$$

$$\delta_y = - \delta_0 \frac{2 \cos\lambda \cosh\lambda}{\sin 2\lambda + \sinh 2\lambda} [(- \tan\lambda + \tanh\lambda) \cos\frac{y}{l\sqrt{2}} \cosh\frac{y}{l\sqrt{2}} + (\tan\lambda + \tanh\lambda) \sin\frac{y}{l\sqrt{2}} \sinh\frac{y}{l\sqrt{2}}]$$

$$\sigma_0 = \frac{E\alpha\Delta T}{2(1 - \mu)}, \quad \delta_0 = \frac{(1 + \mu) \alpha \Delta T l^2}{h}, \quad \lambda = \frac{W}{l\sqrt{8}}$$

- **Assume slab remains flat with thermal gradient**
- **No loss of support under the slab due to curling**

Use of Westergaard/Bradbury Models to Determine Combined Stress ???

Loading (traffic)

Curling (thermal gradient)

Load + Curl $\neq$ Combined Stress

Finite Element Solutions

2-Dimension - More realistic modeling of combined load and curl stresses.

3-Dimension - Even more realistic modeling of combined load and curl stresses, especially for stiff unbonded base.

FE Problems - Complexity in use of FE programs, Potential for error, Long computer run times when many solutions needed.

Previous Attempts To Obtain Closed-Form Models For Combination Of Load And Curl Stress

Combined Stress = Load Stress + R * Curl Stress

Problem: R determined by multiple regression,
Inadequate accuracy

Current Approach

Utilize dimensional analysis and modern statistical modeling techniques to obtain solution that accurately reproduces FE model.

- Identify primary structural responses
  - wheel load only
  - temperature curling only
- Combined Load/Curl Stress =
  Load Stress + R * Curl Stress
- R = F(primary structural responses)

Primary Structural (Dimensionless) Variables For Load Stress

$$\sigma_{wes} = \frac{P}{h^2} * f_1\left(\frac{a}{l}, \mu\right)$$

$$\sigma_{wes}\frac{h^2}{P} = f_1\left(\frac{a}{l}\right)$$

$$\delta_{wes} = \frac{P}{\sqrt{Eh^3k}} * f_2\left(\frac{a}{l}, \mu\right)$$

$$\frac{\delta_{wes}\, k\, l^2}{P} = f_2\left(\frac{a}{l}\right)$$

Primary Structural (Dimensionless) Variables For Thermal Curling Stress

$$\sigma_{wesc} = E\alpha\Delta T * f_3\left(\frac{L}{l}, \frac{W}{l}, \mu\right)$$

$$\frac{\sigma_{wesc}}{E} = f_3\left(\alpha\Delta T, \frac{L}{l}, \frac{W}{l}\right)$$

$$\delta_{wesc} = \frac{\alpha\Delta T\, l^2}{h} * f_4\left(\frac{L}{l}, \frac{W}{l}, \mu\right)$$

$$\frac{\delta_{wesc}\, h}{l^2} = f_4\left(\alpha\Delta T, \frac{L}{l}, \frac{W}{l}\right)$$

Two Additional Dimensionless Parameters Identified

$$D_\gamma = \frac{\gamma h^2}{k l^2}$$

$$D_P = \frac{P h}{k l^4} = 12\left(1 - \mu^2\right)\frac{P}{E h^2}$$

D_γ and D_P are dimensionless parameters that represent the relative deflection stiffness due to the self-weight of the concrete slab, external wheel load, and the possible loss of subgrade support due to curling.

LOADING ONLY

Finite Slab Length Effect

$$R_L = \frac{\sigma_{fe}}{\sigma_{wes}} = F\left(\frac{a}{l}, \frac{L}{l}\right)$$

Finite Slab Width Effect

$$R_W = \frac{\sigma_{fe}}{\sigma_{wes}} = F\left(\frac{a}{l}, \frac{W}{l}\right)$$

LOADING ONLY (Continued)

Where:

- R_L = adjustment (multiplication) factor for finite slab length effect
- R_W = adjustment factor for finite slab width effect
- σ_{wes} = Westergaard's edge stress solution
- σ_{fe} = edge stress determined by the finite element model
- F = function to be derived from finite element outputs and statistical modeling techniques

THERMAL CURLING ONLY

$$R_c = \frac{\sigma_{fe}}{\sigma_{wesc}} = F\left(\alpha \Delta T, \frac{L}{l}, \frac{W}{l}, \frac{\gamma h^2}{k l^2}\right)$$

Where:

R_c = adjustment factor for thermal curling

σ_{wesc} = Westergaard/Bradbury's edge stress solution

σ_{fe} = edge stress determined by the finite element model

F = function to be derived from finite element outputs and statistical modeling techniques

LOADING AND THERMAL CURLING

$$\sigma_T = R_L * R_W * \sigma_{wes} + R_T * \sigma_{wesc}$$

$$R_T = F\left(\frac{a}{l}, \alpha\Delta T, \frac{L}{l}, \frac{W}{l}, \frac{\gamma h^2}{kl^2}, \frac{Ph}{kl^4}\right)$$

Where:

σ_T = total combined load and curl edge stress

$R_L{}^*R_W{}^*\sigma_{wes}$ = Westergaard edge loading stress adjusted for slab length and width effects

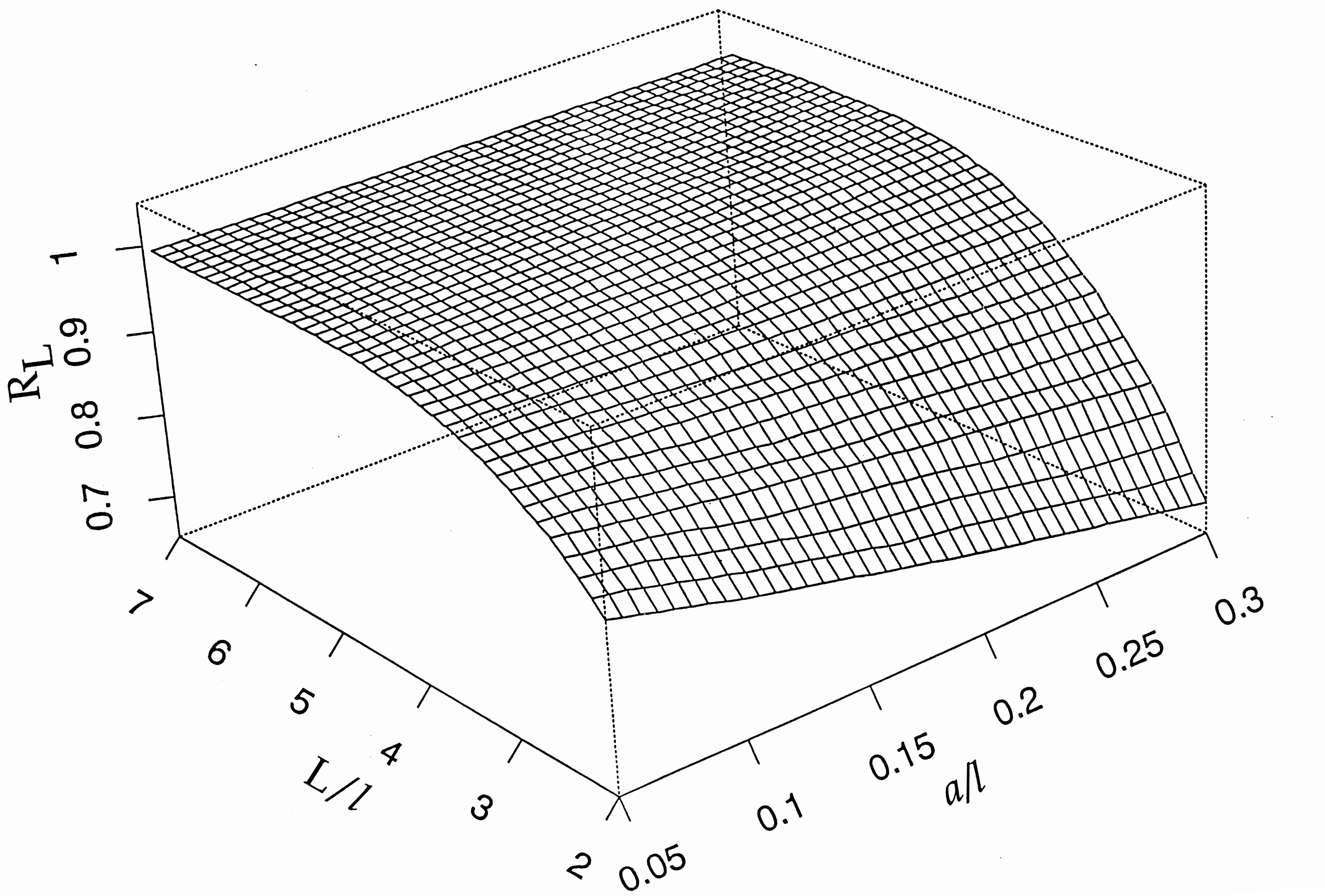
σ_{wesc} = Westergaard/Bradbury edge curling stress

R_T = adjustment factor for the effect of loading plus thermal curling

F = function to be derived from finite element outputs and statistical modeling techniques

DEVELOPMENT OF MODELS

- S-PLUS Statistical Package includes modern modeling techniques
- Used the "projection" algorithm to breakdown the multi-dimensional response surface into a sum of several smooth projected curves
- Used traditional linear and nonlinear regression techniques to obtain the parameter estimates of each projected curve and the overall regression statistics



R_L
0.7
0.8
0.9
1
L/l
2
3
4
5
6
7
a/l
0.05
0.1
0.15
0.2
0.25
0.3

Using this modeling approach, the following "projection" model was developed for R_L

$$R_L = 0.940 + 0.0799\,\Phi_1(A1)$$

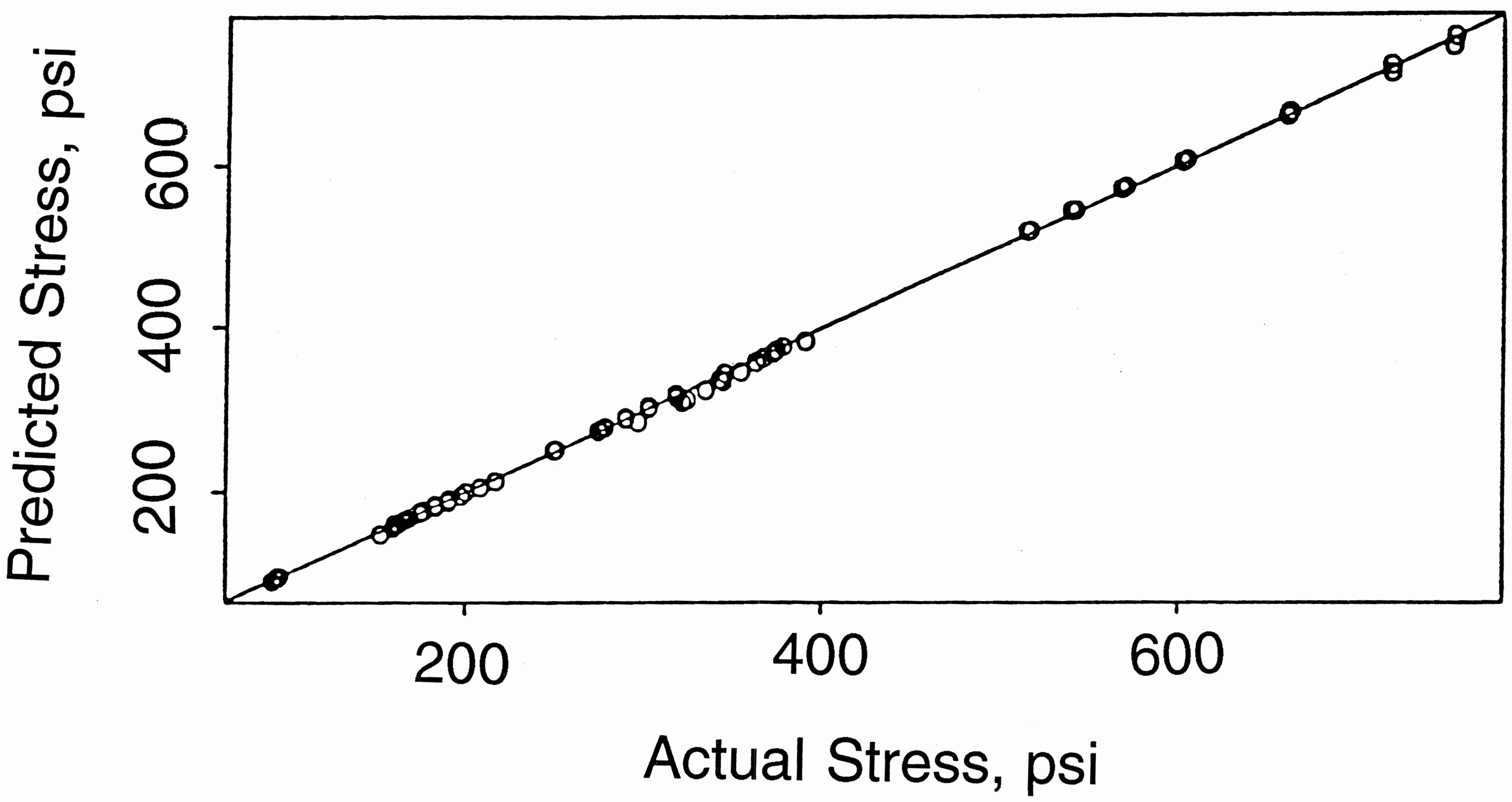
$$\Phi_1(A1) = -4.031 + \frac{1}{0.203 + 0.0345\,A1^{-3.304}}$$

$$A1 = -0.944\frac{a}{l} + 0.331\frac{L}{l}$$

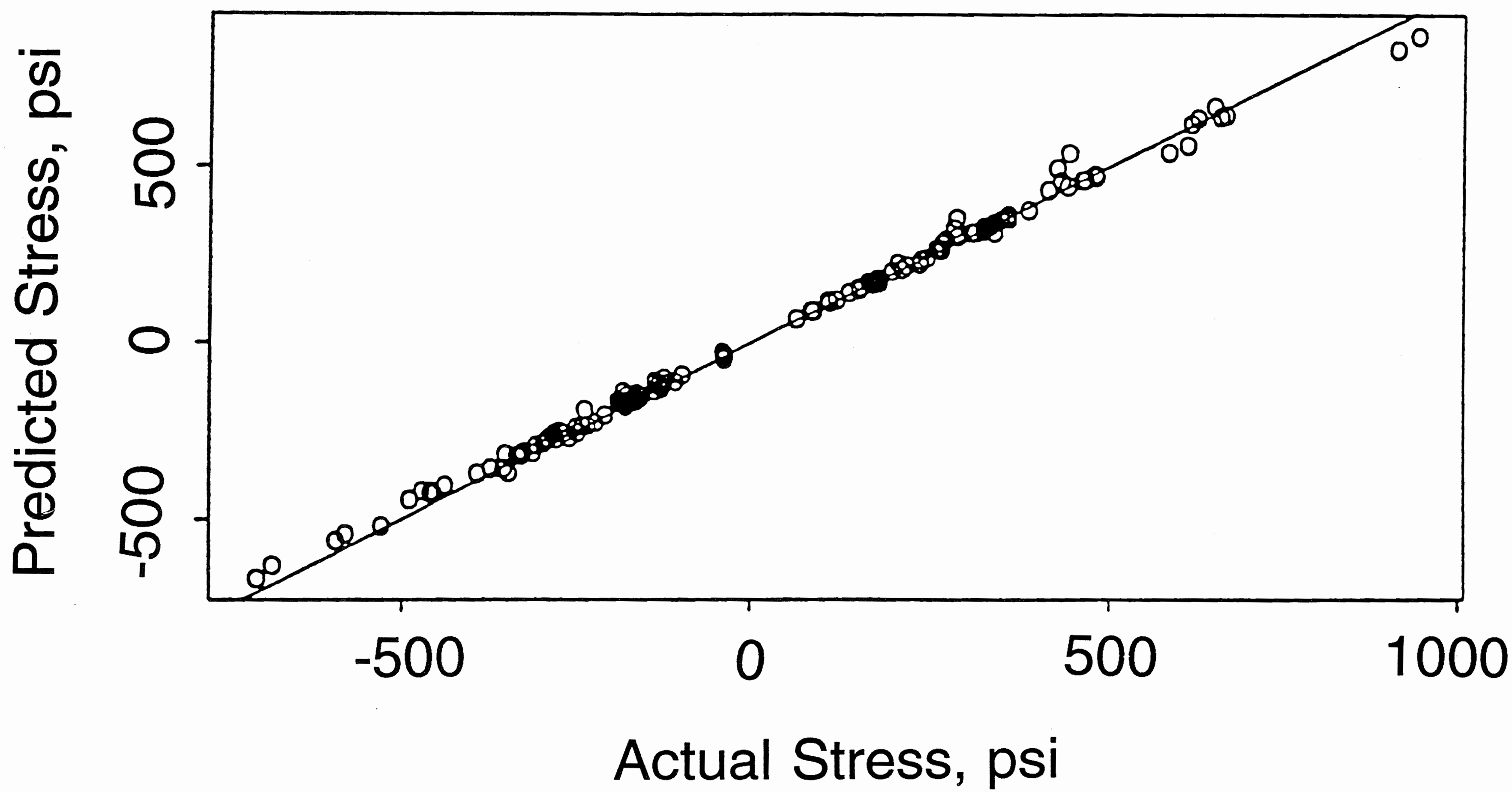
Statistics: $N = 36$, $R^2 = 0.994$, $SEE = 0.0063$,
$CV = 0.67\%$

Limits: $2 \le L/l \le 7$, $0.05 \le a/l \le 0.3$

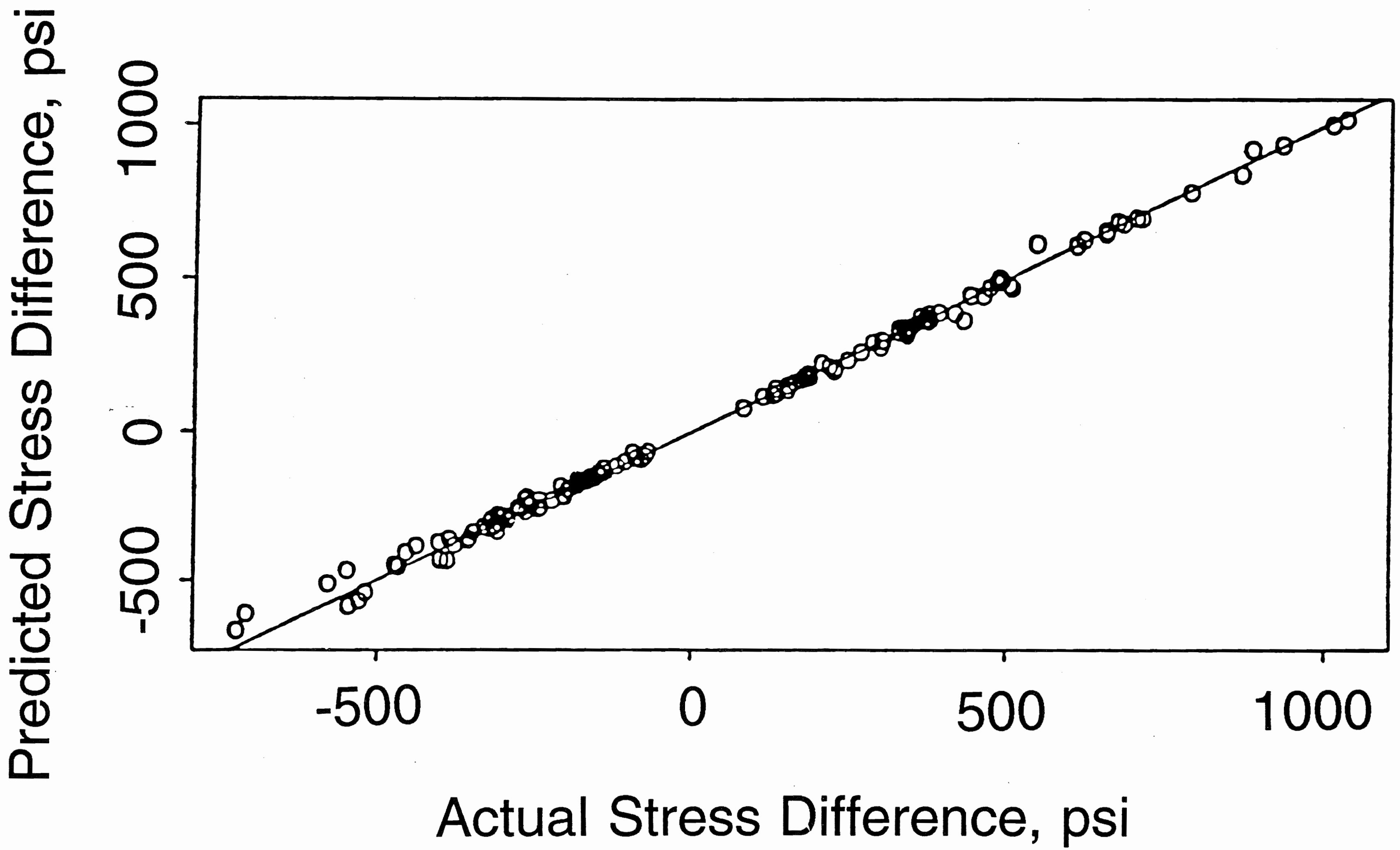
Validation of the Proposed Models for Loading Only



Validation of the Proposed Models for Curling Only



Validation of the Proposed Loading and Curling Models



Numerical Example

Pavement slab with the following characteristics:

E = 5.5 Mpsi, K = 250 pci

L = 10 ft, W = 12 ft, h = 12 in.

$\gamma = 0.087$ pci, $\mu = 0.15$, $\alpha = 5.5 \times 10^{-6} / {}^{o}F$

Loading: Single wheel load of 9,000 lbs with a loaded rectangle of the size of 10x10 in^2 is applied.

Thermal Gradient: Linear temperature differential of +20 ^{o}F (day-time condition) through the slab thickness.

Determine: Critical edge stresses due to loading and loading plus curling.

Numerical Example (Continued)

Solution:

The equivalent radius of the load area is a = 5.64 in. and the radius of relative stiffness of the slab-subgrade system is l = 31.20 in.

The dominating mechanistic variables are:

a/l = 0.18, L/l = 3.83, W/l = 4.60

$\alpha\Delta T$ = + 11.0E-05,

D_{γ} = 2.27E-05, D_P = 29.99E-05

Westergaard's solutions:

σ_{wes} = 346 psi for loading only

σ_{wesc} = 118 psi for curling only.

Numerical Example (Continued)

Adjustment factors for finite slab length and width for loading only:

$R_L = 0.968$, $R_W = 1.000$

Loading only edge stress calculated by the proposed models:

$\sigma_L = \sigma_{wes} * R_L * R_W = 346 * 0.968 * 1.000 = 335$ psi

Actual FE load only edge stress = 344 psi

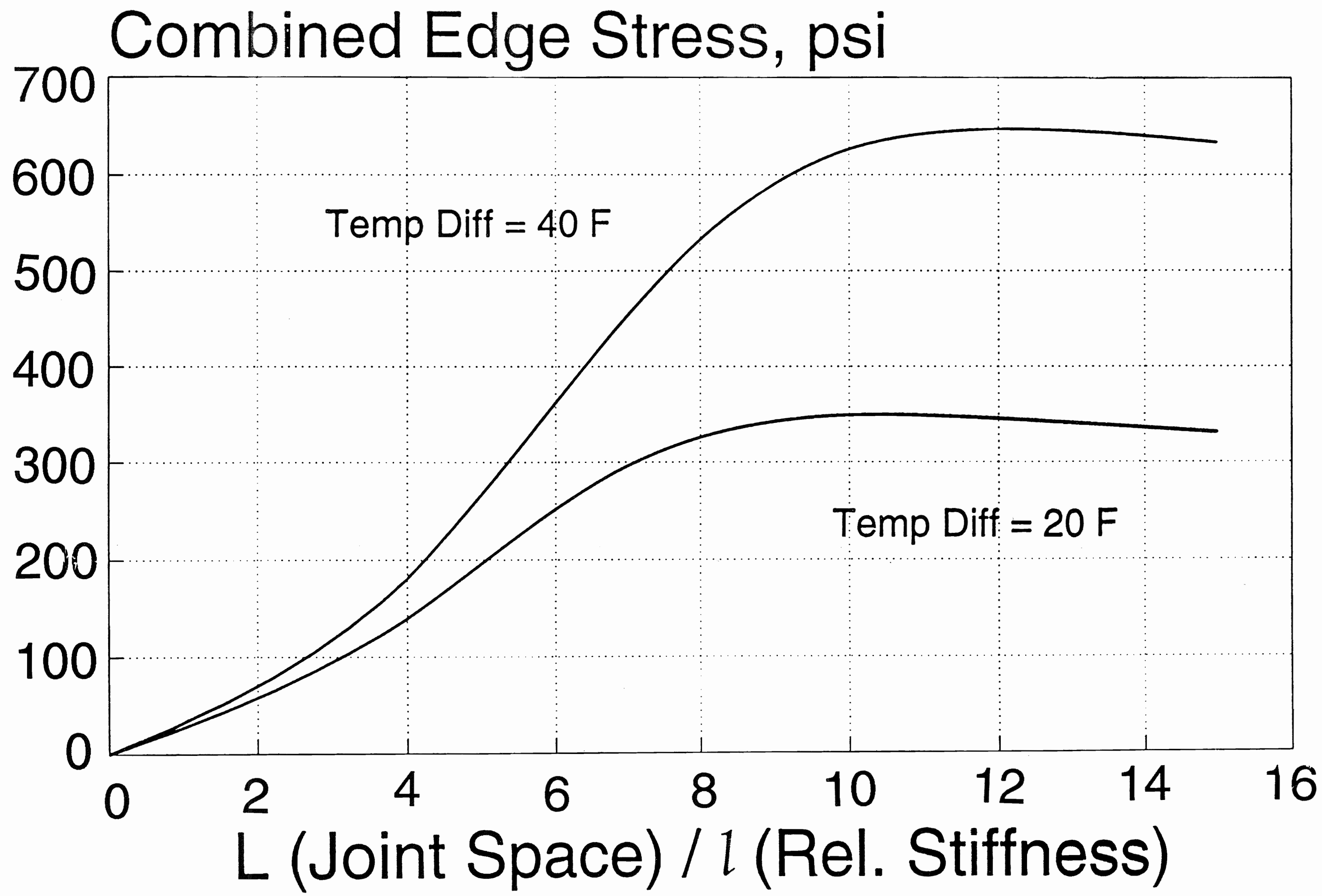
Adjustment factor for loading plus curling:

$R_T = 0.732$

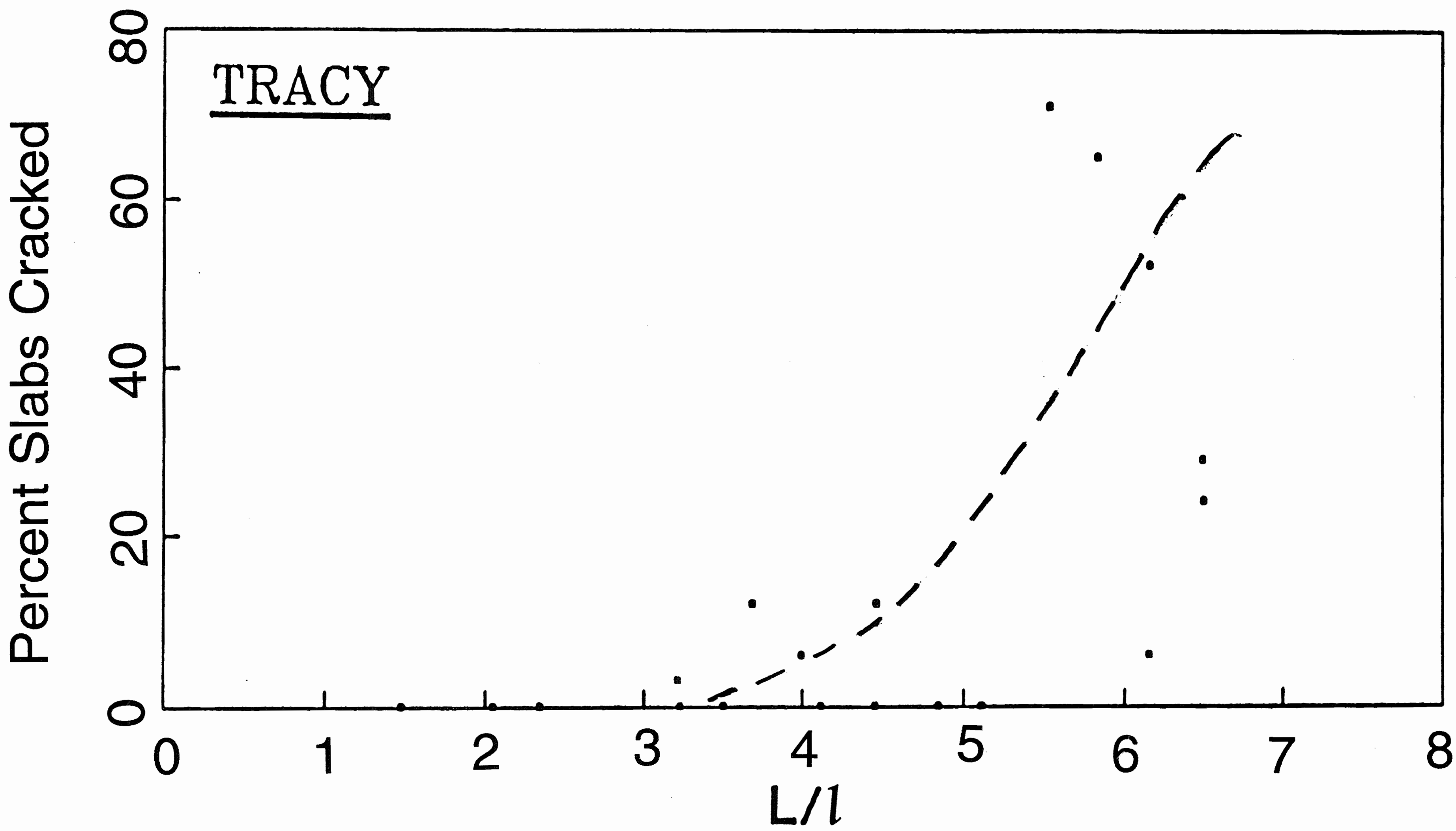
Predicted combined load and curl edge stress determined by the proposed model is:

$\sigma_T = \sigma_L + R_T * \sigma_{wesc} = 335 + 0.732 * 118 = 421$ psi

FE total combined edge stress = 436 psi.


Combined Edge Stress, psi
700
600
500
400
300
200
100
0
Temp Diff = 40 F
Temp Diff = 20 F
0
2
4
6
8
10
12
14
16
L (Joint Space) / l (Rel. Stiffness)

Percent Slab Cracked vs. L/*l* for California 1 (Tracy) Sections



Conclusions

- Closed-form predictive models were developed for edge stresses (load and curl) using dimensional analysis and modern modeling techniques.

- Previous research using dimensional analysis included the major independent variables:
  - normalized load radius (a/l),
  - normalized slab length (L/l),
  - normalized slab width (W/l), and
  - dimensionless product ($\alpha\Delta T$) of a temperature differential and thermal expansion coefficient.

  However, the actual structural response to a temperature influence could not be adequately described using only these four parameters.

Conclusions (Continued)

- Two additional dimensionless parameters (D_{γ} and D_P) representing the relative deflection stiffness due to the self-weight of the concrete slab, the applied load, and loss of support from curling were identified.

- A new modeling approach was used which makes use of projection pursuit regression algorithm and linear and nonlinear regression techniques to develop the proposed predictive models.

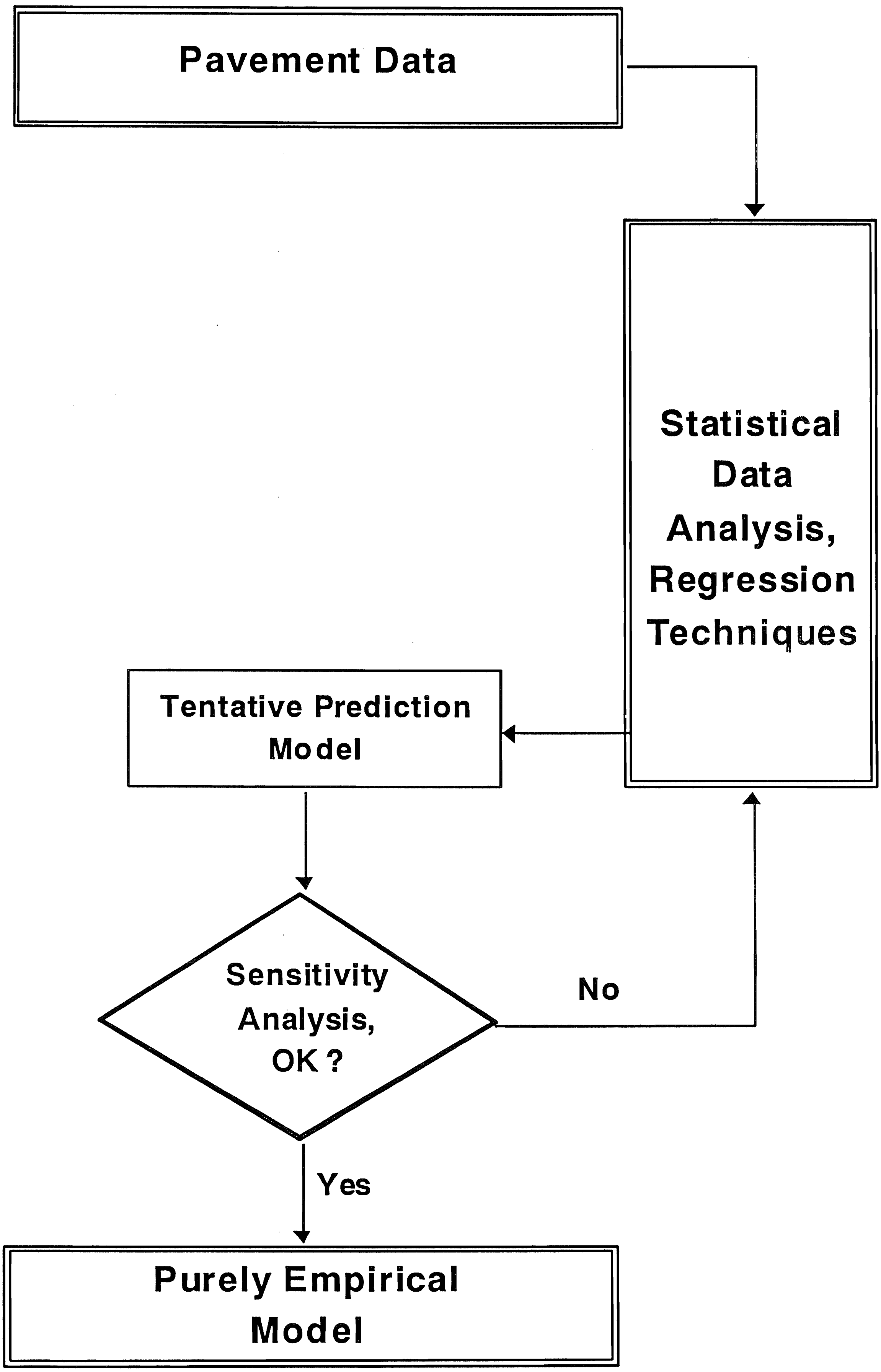
- The new models use only the dominating structural variables as opposed to earlier attempts using arbitrary linear combinations of input parameters with very few insights to the actual relationships among the variables.

Conclusions (Continued)

- Consequently, closed-form mechanistic design models that have been carefully validated are ready for implementation on a spreadsheet or computer program. These stress models turned out to be very accurate representations of the 2-D finite element model.

- The new models were also properly formulated to satisfy applicable engineering boundary conditions. They are also simple, easy to comprehend, dimensionally correct, and may be extrapolated to wider ranges of other input parameters.

PURELY EMPIRICAL APPROACH



MECHANISTIC-EMPIRICAL APPROACH