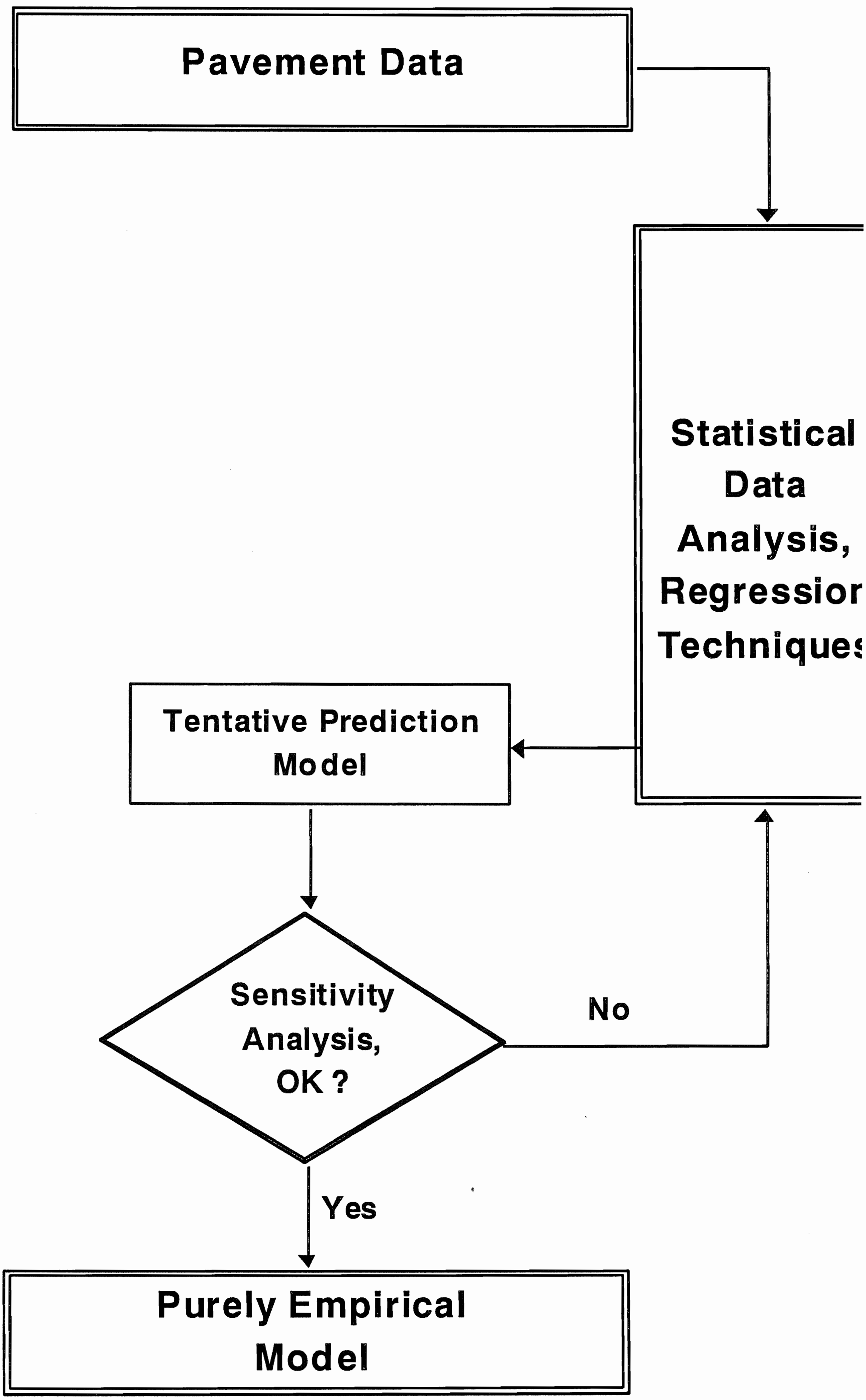
NEW PREDICTIVE MODELING TECHNIQUES FOR PAVEMENTS

Ying-Haur Lee
Tamkang University
and
Michael I. Darter
University of Illinois

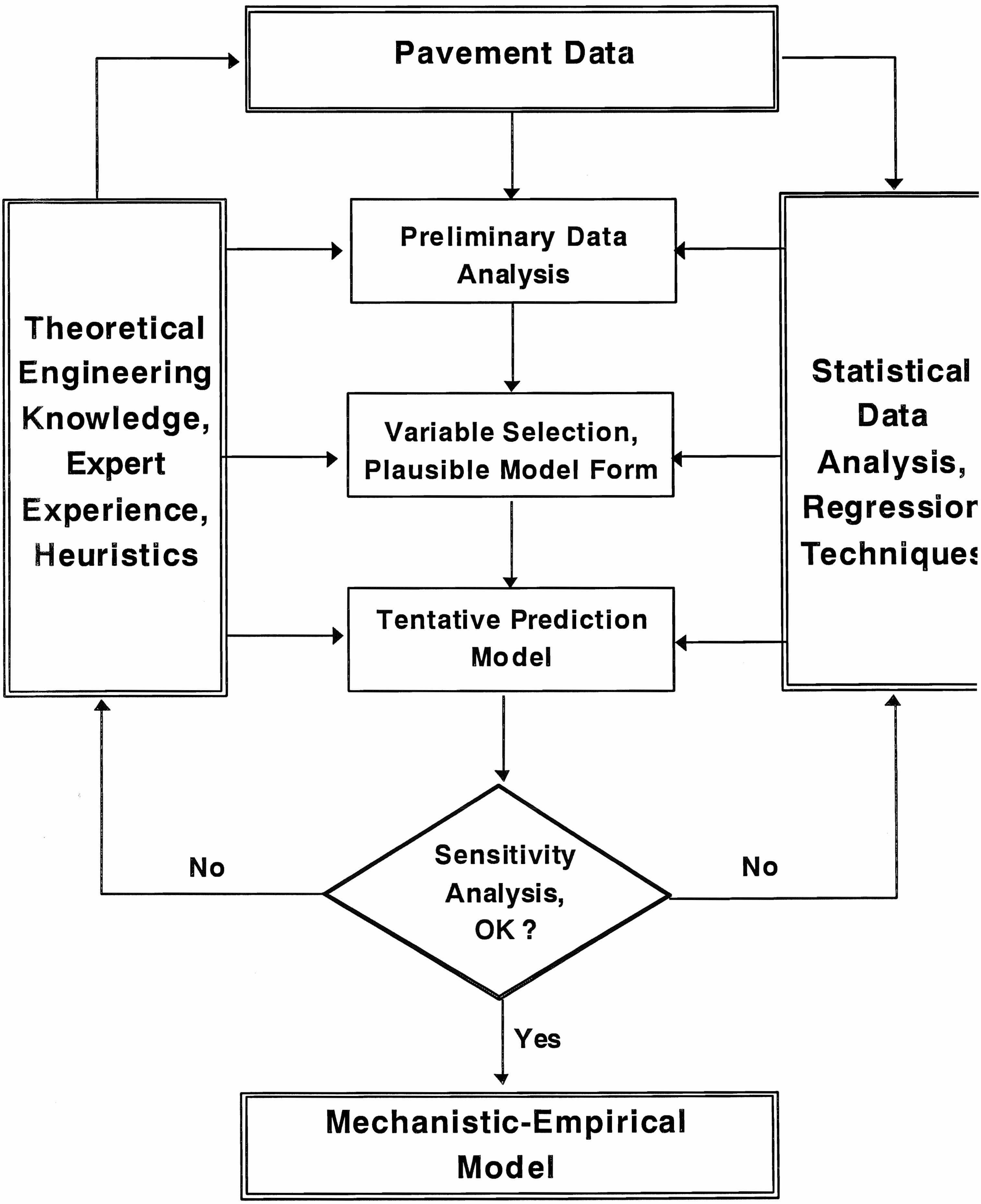
PROBLEM STATEMENT

- Various applications of pavement prediction models (new pavement design, pavement evaluation and rehabilitation plan, pavement management programming, etc.)
- Accuracy of prediction is very inconsistent and often very poor
- Existing models often fail to satisfy some statistical assumptions and engineering boundary conditions
- Lack of guidelines for model development

PURELY EMPIRICAL APPROACH



MECHANISTIC-EMPIRICAL APPROACH



OBJECTIVES

- Investigate the advantages and disadvantages of the current modeling procedures and techniques
- Introduce modern regression techniques
- Propose a systematic statistical and engineering approach for model development
- Demonstrate the proposed modeling procedures

TRADITIONAL REGRESSION TECHNIQUES

- **Multiple Linear Regression:**

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \dots + \beta_{p-1} x_{p-1} + \varepsilon$$

- **Nonlinear Regression:**

$$y = F\left(\beta_0, \beta_1, \dots, \beta_{p-1}, x_1, x_2, \dots, x_k\right) + \varepsilon$$

Both minimizing the sum of squared residuals:

$$RSS(\hat{\beta}) = \sum_{i=1}^{n} \left\{ r_i^2(\hat{\beta}) \right\}$$

MULTIPLE LINEAR REGRESSION

- Preliminary or explanatory analysis of linear relationships of a group of important variables
- Stepwise and all-subset regressions used for automatic variable selection
- Very sensitive to the presence of outliers and influential data points
- Regression diagnostics based on delete-one statistics are often masked by some groups of influential observations

NONLINEAR REGRESSION

- Can handle a complicated nonlinear model
- Model specifications: assuming a descriptive model form and guessing initial parameter estimates (specifying bounds if necessary)
- Very sensitive to the presence of outliers and influential data points
- Often fail to satisfy convergence criterion and some statistical assumptions
- Parameter estimates often insignificant or toward wrong direction in physical interpretations

MODERN REGRESSION TECHNIQUE

- Projection Pursuit Regression (PPREG, "**Projection**") Algorithm:
  - capable of modeling variable interactions (Friedman and Stuetzle, 1981)
  - attempting to model the response surface as a sum of nonparametric functions of projections of the explanatory variables through the use of local smoothing techniques

"PROJECTION" (PPREG) ALGORITHM

$$y = \bar{y} + \sum_{m=1}^{M_0} \beta_m \phi_m(a_m^T x) + \varepsilon$$

$$E[\phi_m(a_m^T x)] = 0, \quad E[\phi_m^2(a_m^T x)] = 1$$

Minimizing the mean squared residuals:

$$E[r^2] = E\left[y - \bar{y} - \sum_{m=1}^{M_0} \beta_m \phi_m(a_m^T x)\right]^2$$

A CASE STUDY: EDGE STRESS DUE TO LOADING AND FINITE SLAB LENGTH EFFECT

- Determine the maximum bending stress at the longitudinal edge of the slab
- Finite element model can not be easily implemented as a part of a design procedure
- To illustrate the advantages of introducing mechanistic variables and selecting proper functional forms in model development

THREE DIFFERENT APPROACHES

- Use arbitrary but "best" linear combinations of individual variables (Darter, 1977)

- Introduce as many mechanistic variables as possible and also find "best" linear combinations of them (Salsilli, 1991)

- Introduce as many mechanistic variables as possible and also find the best functional forms using the **"Projection" (PPREG)** algorithm

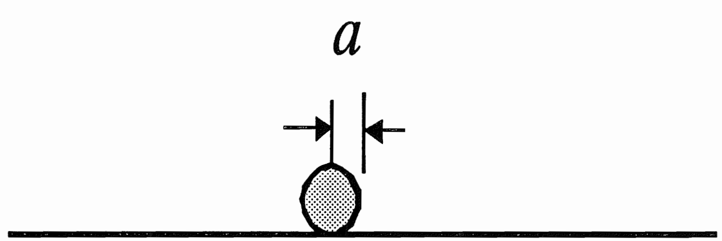
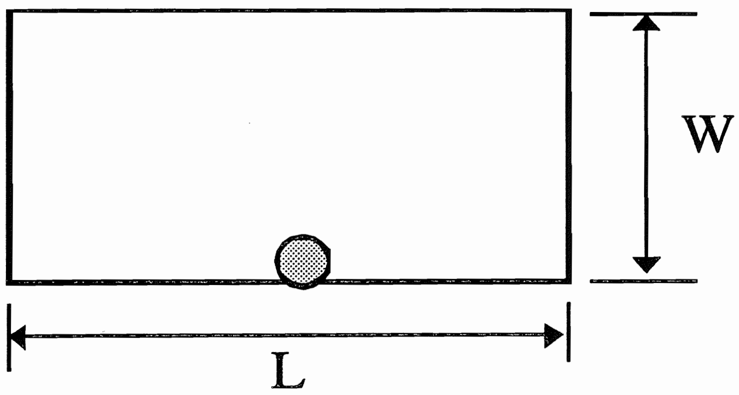
ARBITRARY LINEAR COMBINATIONS OF VARIABLES

- Perform a large factorial of finite element runs:
  1. Slab length, L = 15, 20, 25, 30 ft
  2. Slab thickness, h = 8, 10, 14 in.
  3. Foundation support, k = 50, 200, 500 pci

  (E = 5 Mpsi, W = 12 ft, loaded area = 12 x 15 in^2)

- Resulting model for edge stress prediction:

$$\sigma = \frac{P}{18h^2}\left(17.4 - 0.05\frac{h^3}{k} + 7.4\log\frac{h^3}{k}\right)$$

Concrete Pavement	Mechanistic Variables
Stress, Deflection	$\sigma h^2/P$, $\delta k l^2/P$
	a/l
	a/l, L/l, W/l

EDGE STRESS DUE TO LOADING

$$R = \frac{\sigma_i}{\sigma_w} = f\left(\frac{a}{l}, \frac{L}{l}\right)$$

$$l = \sqrt[4]{\frac{Eh^3}{12(1-\mu^2)k}}$$

R = adjustment (multiplication) factor for the finite slab length effect

σ_w = Westergaard's edge stress solution

σ_i = edge stress determined by the finite element model

INTRODUCING MECHANISTIC VARIABLES

- A small factorial of finite element runs:
  a/l : 0.05, 0.1, 0.2, 0.3
  L/l : 3.0, 4.0, 5.0, 7.0

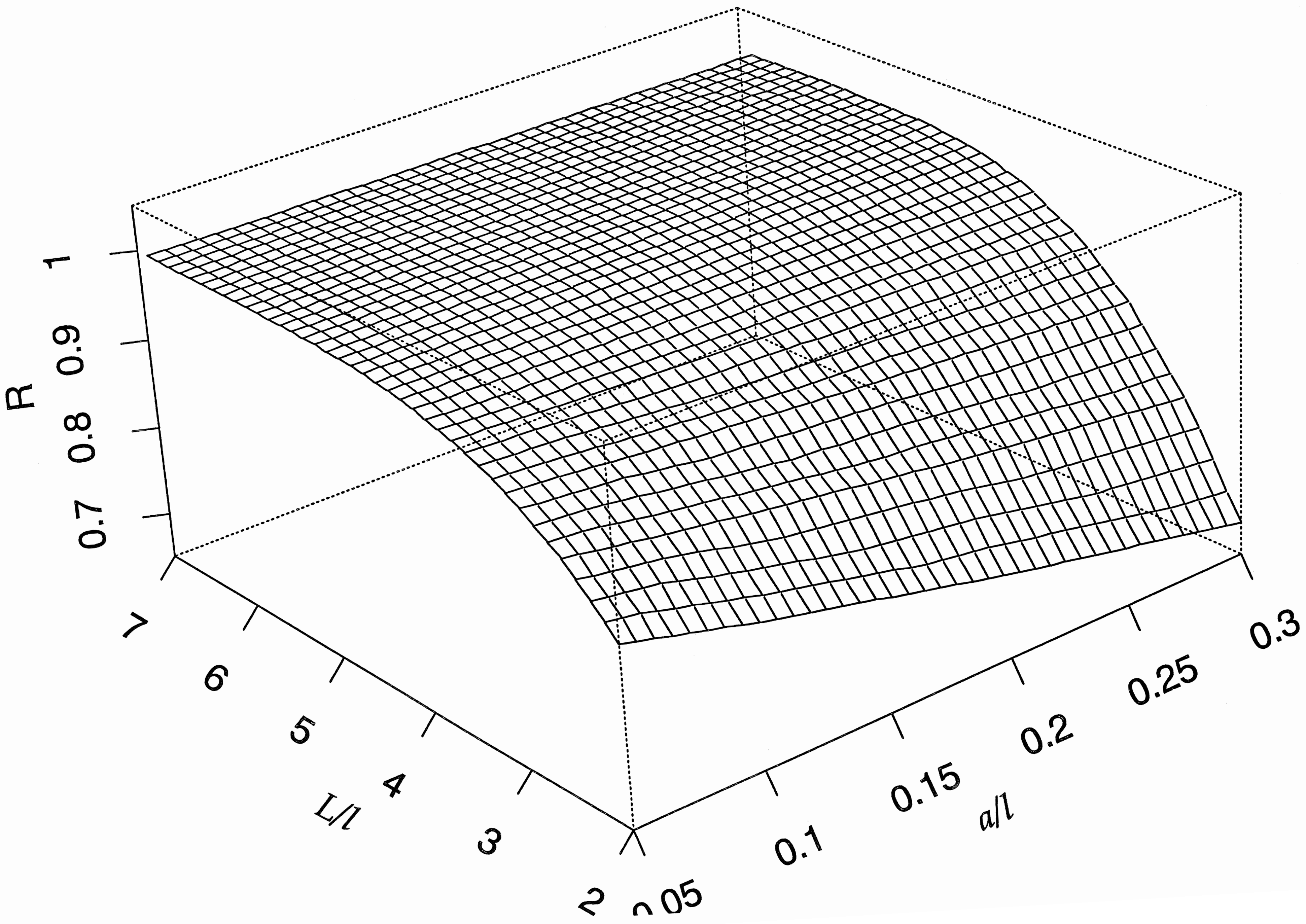
- Resulting model (Model #2):

$$R = 0.58 - 0.53\left(\frac{a}{l}\right) + 0.18\left(\frac{L}{l}\right) - 0.02\left(\frac{L}{l}\right)^2 + 0.11\left(\frac{a}{l}\right)\left(\frac{L}{l}\right)$$

Limits: $3 \le L/l \le 5$, $0.05 \le a/l \le 0.3$
N = 12, $R^2 = 0.996$, SEE = 0.0028, CV = 0.29%

PROPER FUNCTIONAL FORMS

- A small factorial of finite element runs:
  a/l : 0.05, 0.1, 0.2, 0.3
  L/l : 2.0, 2.5, 3.0, 3.5, 4.0, 4.5, 5.0, 6.0, 7.0
- Use of the "Projection" (PPREG) algorithm to select proper functional forms
- Discussion of "prediction" within and "extrapolation" beyond the specified ranges


R
1
0.9
0.8
0.7
7
6
5
4
3
2
L/l
0.05
0.1
0.15
0.2
0.25
0.3
a/l

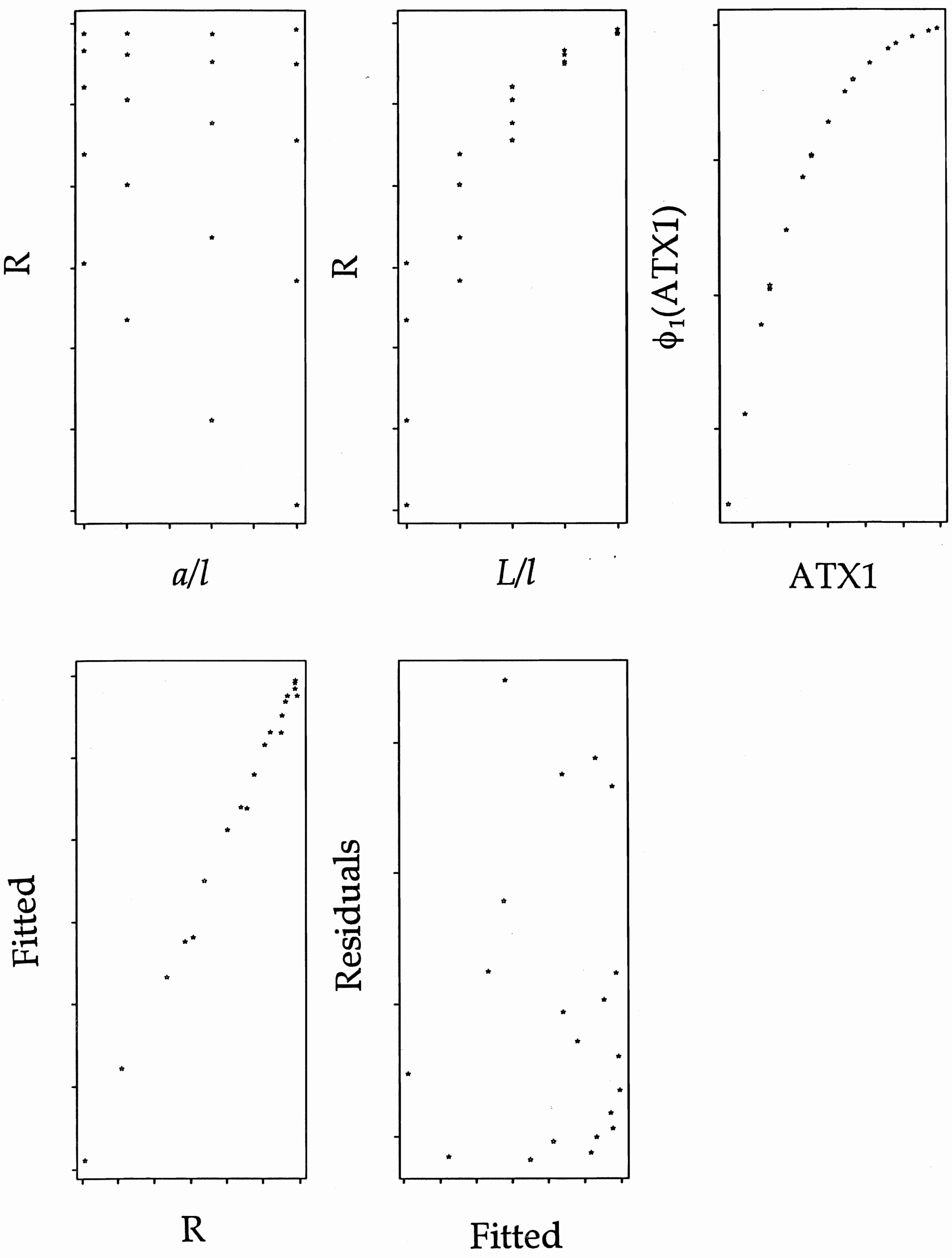
USE OF THE "PROJECTION" ALGORITHM

- The 3-dimensional response surface is broken down into a sum of several smooth projected curves, which are graphically representable in two dimensions.

- Plausible functional forms and applicable boundary conditions may then be easily identified and specified.

- Traditional linear and nonlinear regressions are then utilized to model each projected curve individually.

One-Term "Projection" Model



RESULTING ONE-TERM "PROJECTION" MODEL

Model #3:

$$R = 0.967 + 0.033\,\Phi_1(ATX1)$$

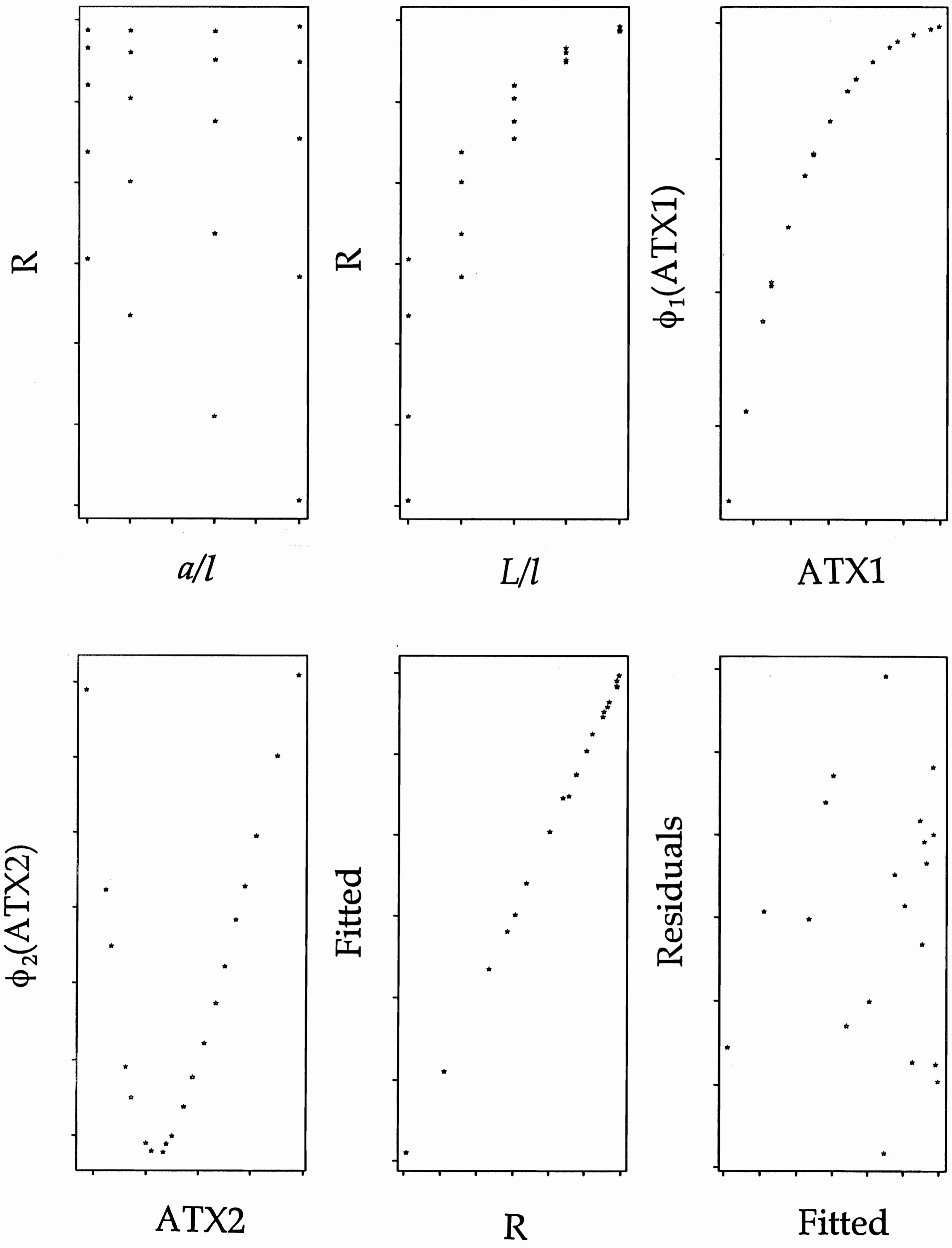
$$\Phi_1(ATX1) = -5.587 + \frac{1}{0.147 + 0.263\,ATX1^{-5.17}}$$

$$ATX1 = -0.895\frac{a}{l} + 0.447\frac{L}{l}$$

Limits: $3 \le L/l \le 5$, $0.05 \le a/l \le 0.3$

N = 20, $R^2 = 0.994$, SEE = 0.0027, CV = 0.28%

Two-Term "Projection" Model



RESULTING TWO-TERM "PROJECTION" MODEL

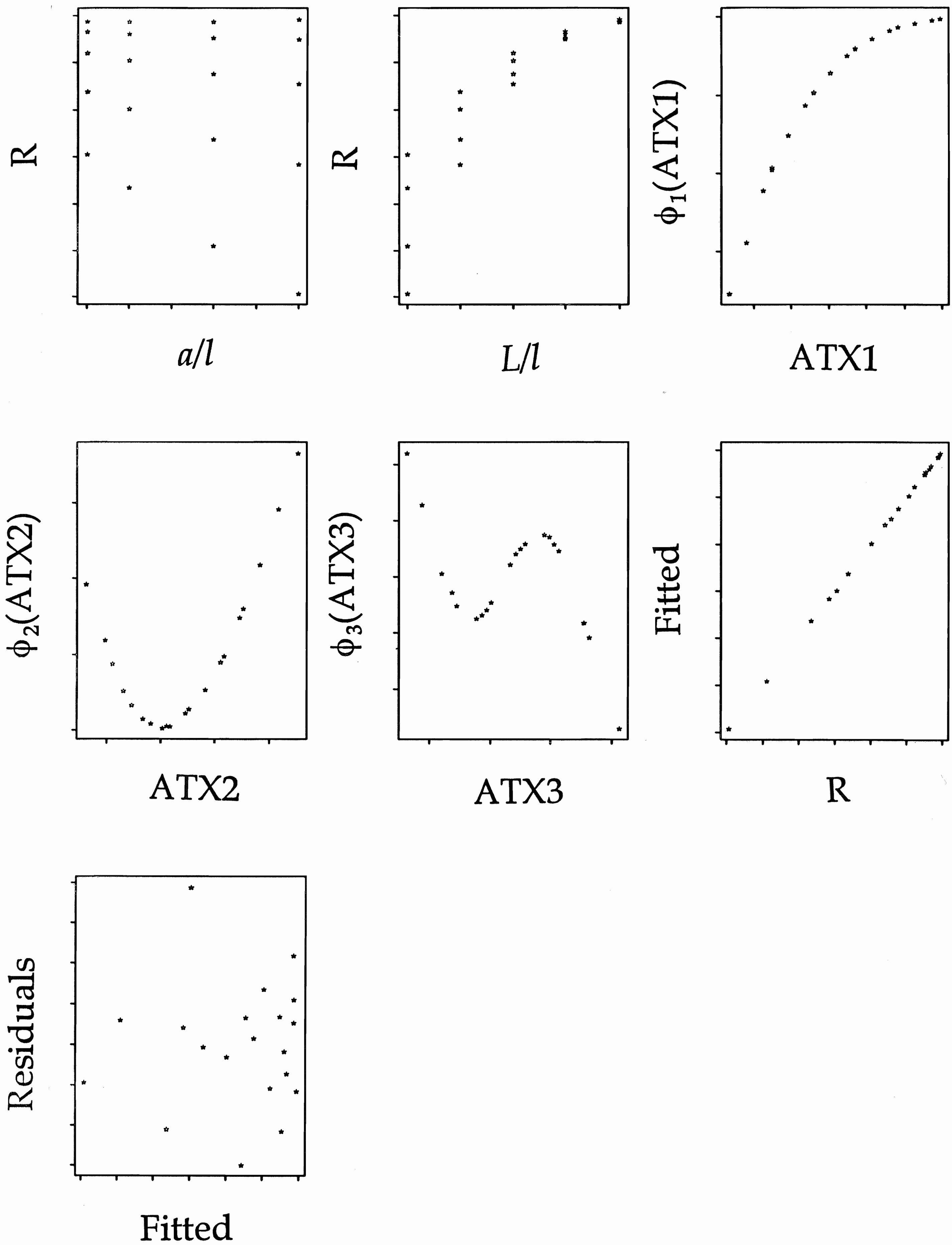
$$R = 0.967 + 0.033\,\Phi_1(ATX1) + 0.0021\,\Phi_2(ATX2)$$

$$ATX1 = -\,0.895\frac{a}{l} + 0.447\frac{L}{l}$$

$$ATX2 = 0.997\frac{a}{l} + 0.0779\frac{L}{l}$$

Note: The second projected term contributes little to the prediction of R (i.e. $\beta_2 = 0.0021$ vs. $\beta_1 = 0.033$).

Three-Term "Projection" Model



PREDICTION AND EXTRAPOLATION

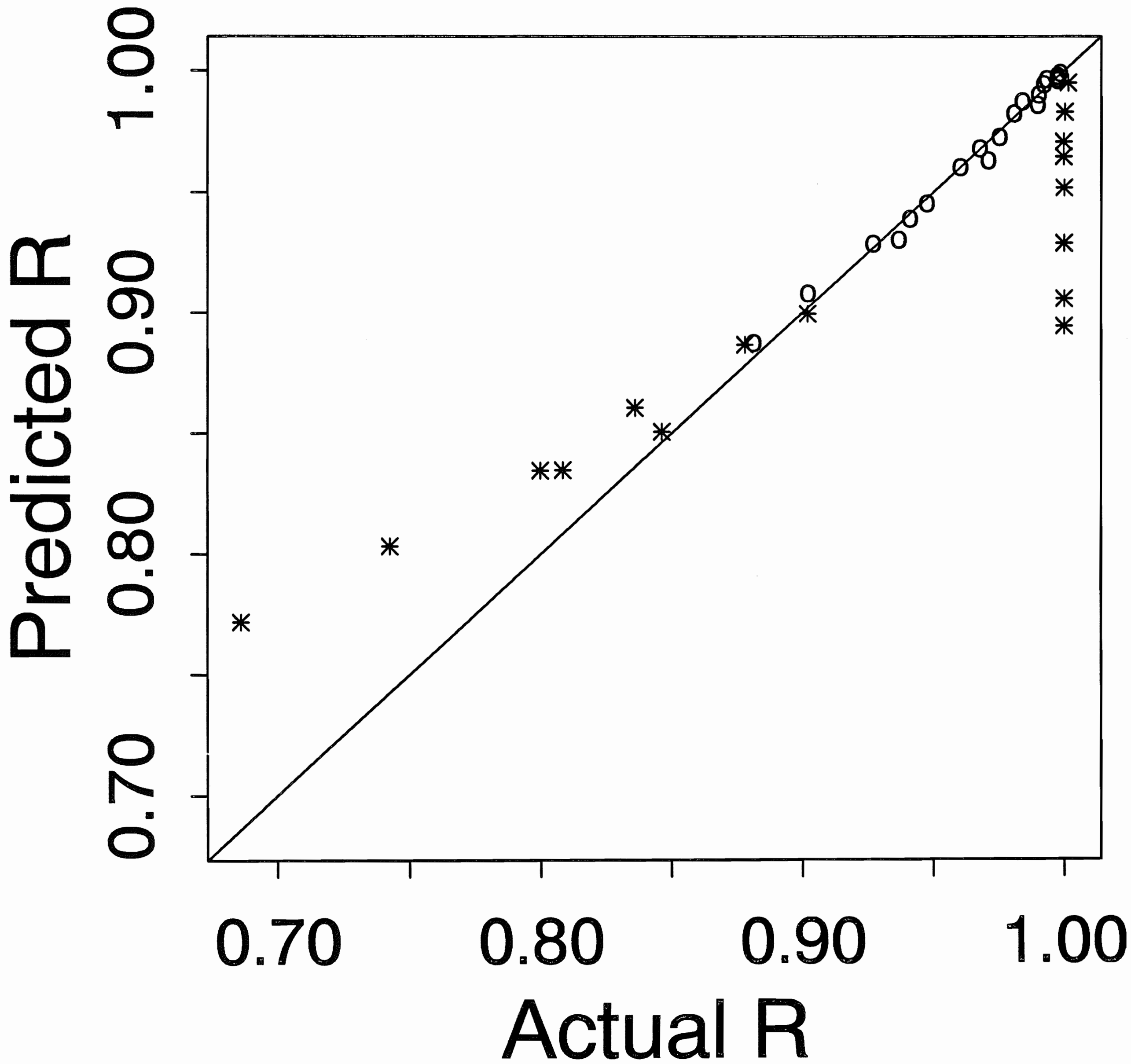
- Use Models #2 and #3 for prediction within and extrapolation beyond the specified ranges

- Model #2: good prediction within the range
  unacceptable results when extrapolated

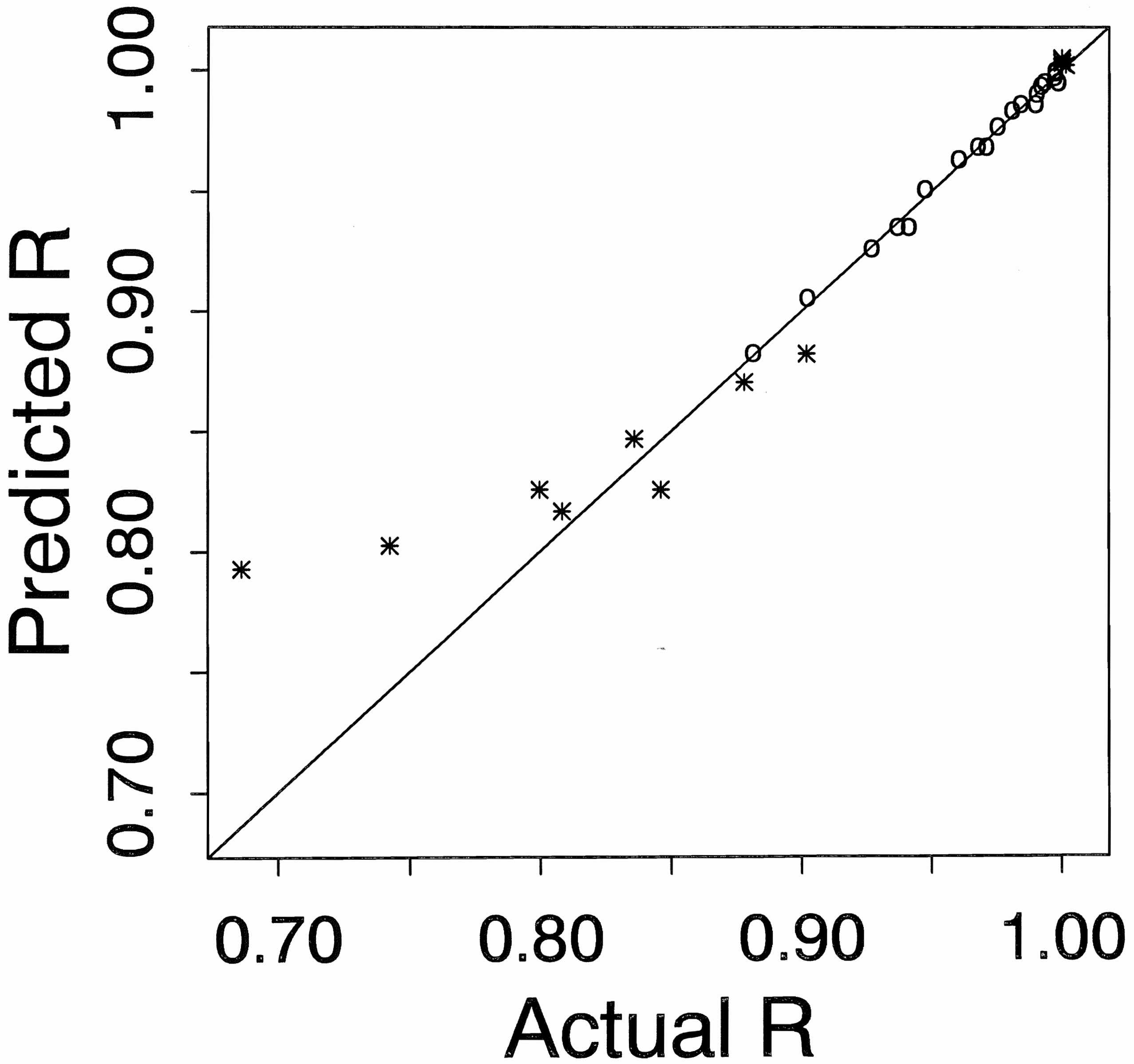
- Model #3: good prediction within the range
  acceptable results when extrapolated

- Conclusions: correct functional forms provide more comprehensive insights of the model

Model #2



Model #3



CONCLUSIONS

- Investigated the advantages and disadvantages of the current modeling procedures and techniques

- Introduced one modern regression technique - PPREG or "Projection" algorithm

- Proposed a systematic statistical and engineering approach for model development (emphasizing on subject-related engineering knowledge and selecting proper functional forms)

- Demonstrated the proposed modeling procedures in a case study