MECHANISTIC DESIGN MODELS OF LOADING AND THERMAL CURLING IN CONCRETE PAVEMENTS

- Objectives:
  To Develop Separate Edge Stress Prediction Models Due to the Individual and Combination Effects of Loading and Thermal Curling Using the ILLI-SLAB Finite Element Program

- Main Regression Algorithm Used:
  "Projection" (PPREG) Algorithm

- Major Findings:
  - Two Additional Dimensionless Parameters Identified
  - Previous Problems Using Dimensional Analysis Are Now Resolved
  - Three Very Accurate and Dimensionally Correct Predictive Models Developed

TWO ADDITIONAL DIMENSIONLESS PARAMETERS IDENTIFIED

1. Westergaard's Edge Loading Solutions:

$$\sigma_w = \frac{3(1+\mu)P}{\pi(3+\mu)h^2}\left[\log_e \frac{Eh^3}{100ka^4} + 1.84 - \frac{4\mu}{3} + \frac{1-\mu}{2} + 1.18(1+2\mu)\frac{a}{l}\right]$$

$$\delta_w = \frac{\sqrt{2+1.2\mu}\,P}{\sqrt{Eh^3k}}\left[1 - (0.76 + 0.4\mu)\frac{a}{l}\right]$$

$$l = \sqrt[4]{\frac{Eh^3}{12(1-\mu^2)k}}$$

2. Westergaard's Thermal Curling Solutions (an infinitely long strip):

$$\sigma_y = \sigma_0 \{1 - \frac{2\cos\lambda\cosh\lambda}{\sin 2\lambda + \sinh 2\lambda} [(\tan\lambda + \tanh\lambda)\cos\frac{y}{l\sqrt{2}}\cosh\frac{y}{l\sqrt{2}} + (\tan\lambda - \tanh\lambda)\sin\frac{y}{l\sqrt{2}}\sinh\frac{y}{l\sqrt{2}}] \}$$

$$\delta_y = -\delta_0 \frac{2\cos\lambda\cosh\lambda}{\sin 2\lambda + \sinh 2\lambda} [(-\tan\lambda + \tanh\lambda)\cos\frac{y}{l\sqrt{2}}\cosh\frac{y}{l\sqrt{2}} + (\tan\lambda + \tanh\lambda)\sin\frac{y}{l\sqrt{2}}\sinh\frac{y}{l\sqrt{2}}]$$

$$\sigma_0 = \frac{E\alpha\Delta T}{2(1-\mu)}, \quad \delta_0 = \frac{(1+\mu)\,\alpha\,\Delta T\, l^2}{h}, \quad \lambda = \frac{W}{l\sqrt{8}}$$

TWO ADDITIONAL DIMENSIONLESS PARAMETERS IDENTIFIED (CONTINUED)

1. Westergaard's Edge Loading Solutions:

$$\sigma_w = \frac{P}{h^2} * f_1\left(\frac{a}{l}, \mu\right)$$

$$\frac{\sigma h^2}{P} = f\left(\frac{a}{l}\right)$$

$$\delta_w = \frac{P}{\sqrt{Eh^3k}} * f_2\left(\frac{a}{l}, \mu\right)$$

$$\frac{\delta k l^2}{P} = f\left(\frac{a}{l}\right)$$

$$l = \sqrt[4]{\frac{Eh^3}{12(1 - \mu^2)k}}$$

2. Westergaard's Thermal Curling Solutions:

$$\sigma_y = E\alpha\Delta T * f_3\left(\frac{L}{l} \text{ or } \frac{W}{l}, \mu\right)$$

$$\frac{\sigma}{E} = f\left(\alpha\Delta T, \frac{L}{l}, \frac{W}{l}\right)$$

$$\delta_y = \frac{\alpha \Delta T\, l^2}{h} * f_4\left(\frac{L}{l} \text{ or } \frac{W}{l}, \mu\right)$$

$$\frac{\delta h}{\ell^2} = f\left(\alpha\Delta T, \frac{L}{\ell}, \frac{W}{\ell}\right)$$

3. Deflection at the Center of the ILLI-SLAB Program:

$$\delta = \frac{\gamma h}{k}$$

4. Two Additional Dimensionless Parameters Identified:

$$D_\gamma = \frac{\gamma h^2}{k l^2}$$

$$\frac{\gamma \ell}{k} \Big/ \left(\frac{\ell^2}{h}\right)$$

$$D_P = \frac{Ph}{kl^4} = 12(1 - \mu^2)\frac{P}{Eh^2}$$

$$\frac{P}{k\ell^2} \Big/ \left(\frac{\ell^2}{h}\right)$$

where D_γ and D_P are dimensionless parameters to represent the relative deflection stiffness due to the self-weight of the concrete slab, external wheel load, and the possible loss of subgrade support.

LOADING ONLY

1. Finite Slab Length Effect:

$$R_L = \frac{\sigma_i}{\sigma_w} = f\left(\frac{a}{l}, \frac{L}{l}\right)$$

2. Finite Slab Width Effect:

$$R_W = \frac{\sigma_i}{\sigma_w} = f\left(\frac{a}{l}, \frac{W}{l}\right)$$

Where:

R_L = an adjustment (multiplication) factor for the finite slab length effect;

R_W = an adjustment factor for the finite slab width effect;

σ_w = Westergaard's edge stress solution, $[FL^{-2}]$; and

σ_i = edge stress determined by the finite element model, $[FL^{-2}]$.

THERMAL CURLING ONLY

$$R_c = \frac{\sigma_i}{\sigma_c} = f\left(\alpha \Delta T\ ,\ \frac{L}{l}\ ,\ \frac{W}{l}\ ,\ \frac{\gamma\, h^2}{k l^2}\right)$$

Where:

R_c = an adjustment factor for thermal curling;

σ_c = Westergaard/Bradbury's edge stress solution, $[FL^{-2}]$; and

σ_i = edge stress determined by the finite element model, $[FL^{-2}]$.

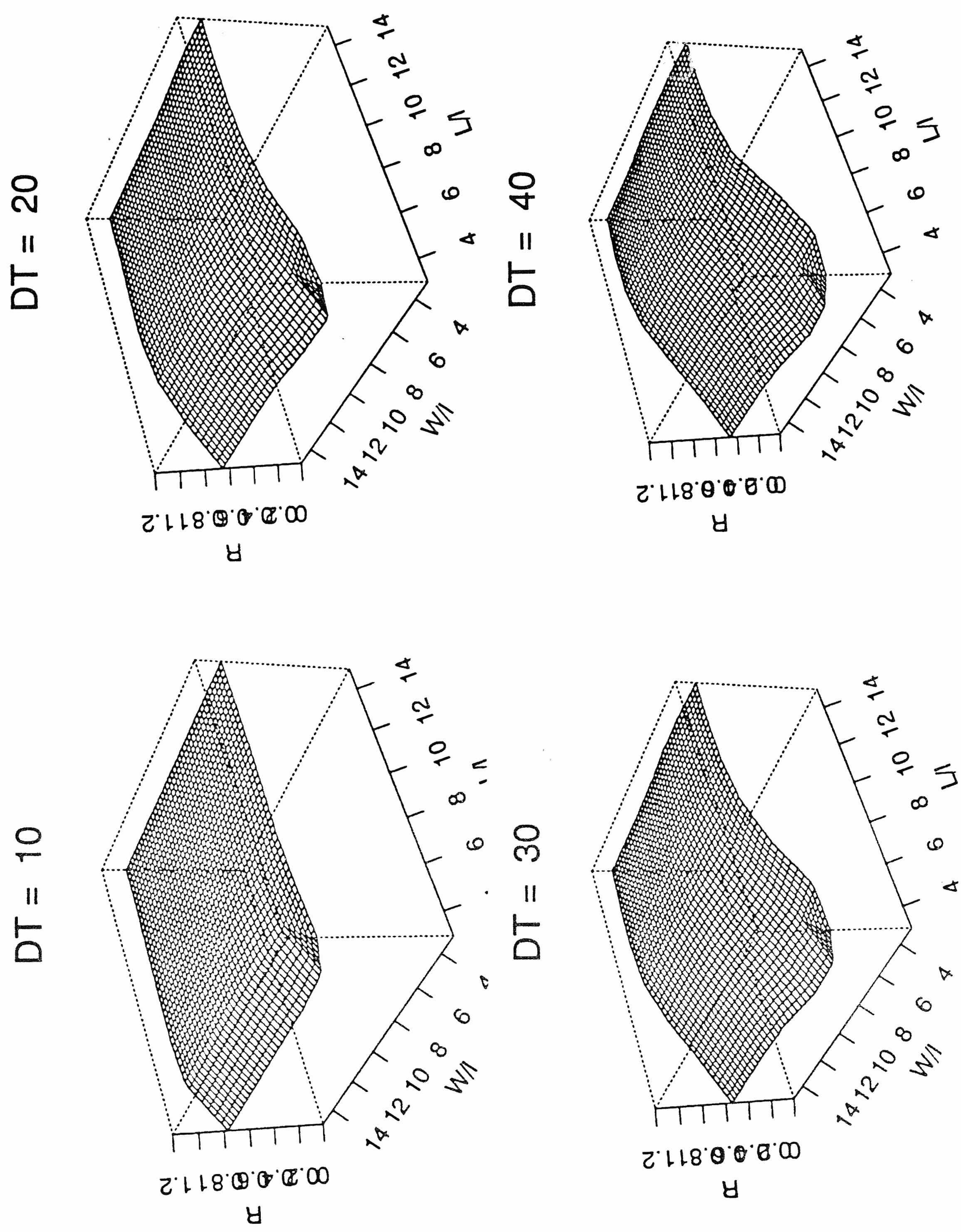


Figure 6.5 - Effect of a Positive Temperature Differential, a Finite Slab Length, and a Finite Slab Width

LOADING AND THERMAL CURLING

$$R_T = \frac{\sigma_i - \sigma_L}{\sigma_c} = f\left(\frac{a}{l}, \alpha \Delta T, \frac{L}{l}, \frac{W}{l}, \frac{\gamma h^2}{kl^2}, \frac{Ph}{kl^4}\right)$$

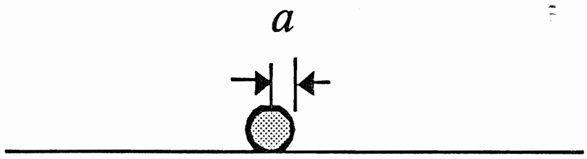
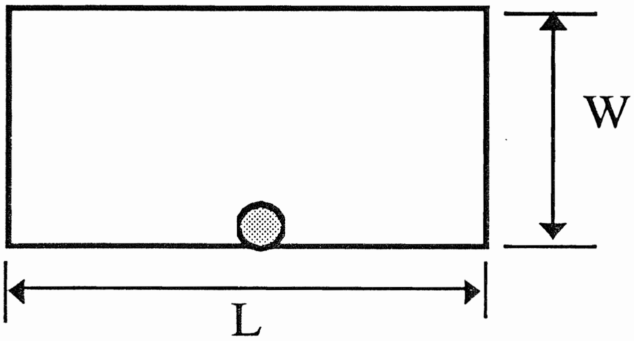
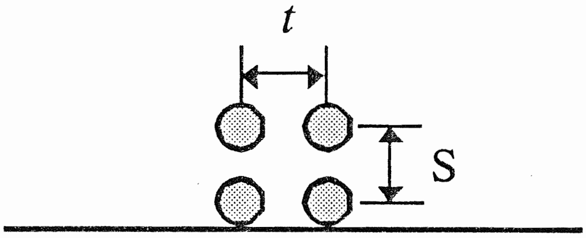
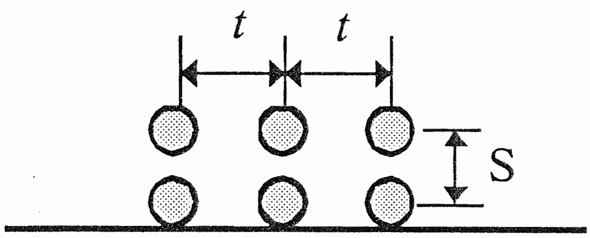
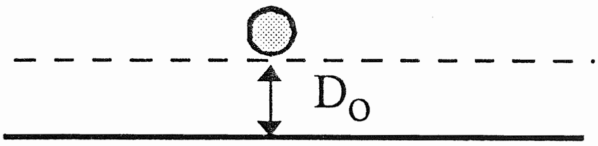
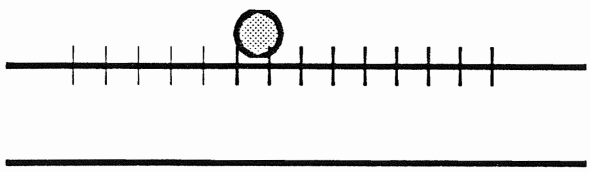
Where:

σ_i = total edge stress determined by the finite element model, $[FL^{-2}]$;

σ_L = edge stress determined by the finite element model due to wheel loading alone, $[FL^{-2}]$;

σ_c = Westergaard/Bradbury's edge stress solution, $[FL^{-2}]$; and

R_T = an adjustment factor for the effect of loading plus thermal curling.

Concrete Pavement	Mechanistic Variables
	a/l
	a/l, L/l, W/l
	a/l, t/l, S/l
	a/l, t/l, S/l
	a/l, D_O/l
	a/l, AGG/kl