C.2 剛性鋪面回算之封閉型解法

CHAPTER FOUR

BACKCALCULATION OF AC/PCC PAVEMENT LAYER MODULI

Backcalculation of pavement layer moduli is done for two purposes: structural evaluation and rehabilitation design. Several uses of backcalculation results in structural evaluation were described in Chapter Three. The use of backcalculated pavement and foundation moduli in rehabilitation design (specifically, overlay design) is addressed in Chapter Five. Both of these applications of backcalculation results are demonstrated in the case studies in Chapter Six.

Much of the distress seen in AC/PCC pavements is reflected from deterioration in the underlying PCC slab. The PCC distresses which are most responsible for AC overlay deterioration are slab cracking, punchouts, joint deterioration, localized deterioration resulting from poor durability ("D" cracking and reactive aggregate distress), and deterioration of PCC and AC patches. This deterioration will also reflect through a second AC overlay unless it is identified and repaired. This requires a coordinated effort of distress surveying, nondestructive deflection testing (NDT), and coring for materials samples. The information obtained is valuable in establishing a profile of condition along the length of the project, which may be used to identify areas requiring specific repair and to determine second rehabilitation options.

Analysis of deflections measured at locations where the underlying PCC is severely deteriorated, as in the case of "D" cracking, will produce low backcalculated *in situ* PCC modulus values. These low modulus values should not be interpreted as the true stress/strain response of the PCC as a homogeneous elastic layer, but rather as an indication of the extent to which its behavior departs from that of a sound slab, i.e., the extent of the PCC's deterioration. The ability to diagnose the condition of the PCC from analysis of deflection measurements is particularly valuable in evaluation of AC/PCC pavements, since the extent of the deterioration of the PCC is often not fully evident from visible distress. In some cases, the deterioration of the PCC may be so severe and so widespread that the only feasible rehabilitation alternatives are substantial structural improvements such as a very thick AC overlay, an unbonded PCC overlay, or reconstruction. A second AC overlay must be sufficiently thick to reduce stresses and deflections in the PCC slab to low levels.

Structural evaluation using NDT data is perhaps more difficult for AC/PCC pavements than for all other pavement types. The available computer programs for backcalculation of pavement layer moduli possess a variety of theoretical and practical limitations which hinder their usefulness in AC/PCC pavement analysis. Valid and repeatable results are typically only obtained from even the best of these tools by very knowledgeable pavement engineers with considerable experience in backcalculation.

Previous research [64, 65] has demonstrated that a closed-form solution exists for backcalculation of PCC and subgrade moduli for slab-on-grade systems. One of the advantages of this direct approach to determination of pavement layer properties is its efficiency in processing deflection data. However, the direct approach has been applied only to pavement structures with PCC surfaces. This approach is not directly applicable to analysis of AC-overlaid PCC pavements, unless the influence of the AC overlay can be accounted for in interpretation of deflection basins. The adaptations to the closed-form approach which are required in order to apply it to backcalculation of AC/PCC pavement layer moduli are described in this chapter.

4.1 LIMITATIONS OF AVAILABLE BACKCALCULATION TOOLS

Most of the tools currently used for backcalculation of pavement moduli are computer programs based on multilayer elastic theory. These programs determine the *in situ* elastic moduli of pavement layers by matching deflection basin measurements to deflections predicted by multilayer elastic theory, given the layer thicknesses and Poisson's ratios and the magnitude and area of the applied load. A few backcalculation programs exist which utilize the equivalent thickness concept, i.e., reduction of a multilayer elastic system to an equivalent system of fewer layers for which a solution is more easily obtainable. Backcalculation may also be done using plate theory, i.e., two-layer elastic theory for the special case of an upper layer which exhibits pure bending (without transverse shear deformation) in response to load. The use of plate theory permits the characterization of the subgrade as either a dense liquid or as an elastic solid.

In backcalculation programs based on multilayer elastic theory, actual deflections are matched to predicted deflections in one of two ways: by iterative numerical integration of elastic layer equations, or by searching a database of deflection basins which have been generated for ranges of layer thicknesses and moduli. Backcalculation by the equivalent thickness method may also be done by iteration or by database search. Graphical procedures were used in the first backcalculation methods based on plate theory, but direct solutions may also be obtained from closed-form equations.

4.1.1 Iterative Backcalculation Programs

BISDEF [66], CHEVDEF [67], WESDEF [68], and ELSDEF are examples of iterative backcalculation programs which make repetitive calls to an elastic layer analysis subroutine (e.g., BISAR [69] for BISDEF) in order to match measured deflections to deflections predicted for program-selected layer moduli. The process stops when the measured and predicted deflections match within tolerance levels set by the user, or when the maximum number of iterations set by the user is reached. A detailed description of the solution algorithm used in these programs is given by Anderson. [70]

One limitation of iterative elastic layer backcalculation programs is that they require the user to enter starting values and ranges for the layer moduli. Unless appropriate starting values are selected, the program may never converge to a solution within the selected ranges. Some researchers have noted that there is no unique solution to the set of moduli which will produce a given deflection basin. Rather, there are as many solutions as there are layers in the pavement structure. [71, 72, 73] As a result, the solution toward which the program converges depends on the initial or "seed" modulus values selected. The boundary values must also be selected judiciously. Limits which are too narrow may prevent the program from converging to the correct solution. Limits which are too broad may allow the program to converge to an incorrect solution, particularly if inappropriate seed moduli are selected. Success with these programs thus requires not only a good knowledge of pavements and materials but also experience in backcalculation for the specific pavement type in question. It has even been suggested that iterative elastic layer backcalculation can never be truly automated until an expert system is developed to guide decisions such as selection of seed moduli. [71, 74] A second limitation of iterative elastic layer backcalculation is that it is time-consuming, increasingly so for increasing number of layers. Convergence to a solution may require several iterations for a pavement system of three or more layers.

In general, the iterative elastic layer backcalculation programs available do not perform well in analyzing AC/PCC pavements, for both of the reasons cited above. Frequently they are unable to match predicted and actual deflection basins well even when given carefully selected ranges of moduli and permitted to run several iterations. Their tendency is to underpredict the modulus of the AC surface, often going to the lower limit of the AC modulus range allowed by the user, and consequently overpredicting the modulus of the PCC slab. As a result, it is necessary to confine the AC modulus to a narrow range bracketing an appropriate value (determined by independent means, e.g., as a function of AC mix temperature), in order to obtain meaningful backcalculated modulus values for the PCC layer. The long execution time required for backcalculation of AC/PCC pavement layer moduli is also a significant limitation. Analysis of several dozen AC/PCC pavement deflection basins, such as might be measured on a highway section a few miles in length, may require several hours of program execution time even on a high-end personal computer. A considerable amount of additional time is required to code the input so that the program will execute successfully.

BOUSDEF [75] is an iterative backcalculation program similar to BISDEF, except that deflections for trial layer moduli combinations are computed not by an elastic layer subroutine but rather an equivalent thickness subroutine. This dramatically reduces execution time, which is BOUSDEF's major advantage over the BISDEF class of programs. However, the appropriateness of BOUSDEF for backcalculation of AC/PCC pavement layer moduli is questionable, owing to material behavior limitations which are inherent in the assumptions of the equivalent thickness method. These include the assumption that the pavement layers above the subgrade exhibit pure bending behavior, the assumption that all layers are fully bonded at their interfaces, the assumption that the layer moduli decrease with depth, and the assumption that the equivalent thickness of any layer (with respect to the layer below) is larger than the radius of the applied load.

4.1.2 Database Backcalculation Programs

Database backcalculation programs run much more quickly than iterative programs, but require a large amount of computer storage. Furthermore, a database backcalculation program can only be applied to situations comparable to those for which the database was generated, i.e., number of layers, material types, ranges of thicknesses and elastic moduli, interface bonding conditions, magnitude and geometry of loading, and number and spacing of sensors.

Of the backcalculation programs currently available, the database program COMDEF [76] is the only one developed specifically for AC/PCC pavements. COMDEF's database of deflection basins contains the results of more than 40,000 elastic layer program (BISAR) runs. As a result, the complete COMDEF database occupies more than 4 Megabytes of hard disk space on a personal computer. It is possible to load portions of the database corresponding to the specific cross-sections of interest to conserve hard disk space. A second and more serious limitation of COMDEF is that it requires deflections for 7 sensors at 12-inch spacings; it cannot accommodate fewer sensors or other spacings. COMDEF does not permit the user to choose whether to model the AC/PCC interface condition as bonded or unbonded, and the program's documentation does not indicate which interface condition (presumably bonded) was used in the development of the database.

MODULUS [77] is a database backcalculation program in which the deflection basin database is produced by a factorial of elastic layer program (CHEVRON) runs. MODULUS was developed for analysis of flexible pavements, but may be used to analyze AC/PCC pavements if a database of deflection basins is generated for the specific AC/PCC pavement cross-section to be analyzed. This process may take between 15 minutes and an hour depending on the complexity of the pavement structure and the capabilities of the computer used, and must be repeated for every cross-section of interest. At least 1 Megabyte of hard disk space must be available to store the generated database. Once the database is generated, analysis of deflection data proceeds fairly quickly.

4.1.3 Closed-Form Backcalculation

ILLI-BACK [65] is a backcalculation program based on closed-form solution of plate theory equations, intended for use in analysis of bare PCC pavements. ILLI-BACK executes more quickly than any other available backcalculation program, and could conceivably be used for real-time analysis of deflection data in the field. Another advantage of ILLI-BACK is that it determines a modulus of subgrade reaction (k-value, psi/inch) as well as an elastic modulus for the subgrade. Using the dense liquid (k-value) subgrade characterization, stresses at the slab edge can be calculated for use in structural evaluation and rehabilitation design. However, the current version of ILLI-BACK can only be used for bare PCC pavements. Thus, ILLI-BACK is not an appropriate tool for analysis of AC/PCC pavements, since modelling the AC as a plate fails to account for the significant compression which occurs in an AC overlay of a PCC slab. Nonetheless, for the purposes of AC/PCC pavement backcalculation, the efficiency and repeatability of the closed-form approach make it the most appealing of the available backcalculation schemes, if it can be modified to account for the behavior of the AC surface.

4.2 CLOSED-FORM BACKCALCULATION FOR BARE PCC PAVEMENTS

A simple two-parameter approach to backcalculation of surface and foundation moduli for a two-layer pavement system was proposed by Hoffman and Thompson in 1981 for flexible pavements. [78] They proposed that the deflection basin could be characterized by its AREA as defined by the following equation:

$$AREA = 6 * \left[1 + 2\left(\frac{d_{12}}{d_0}\right) + 2\left(\frac{d_{24}}{d_0}\right) + \left(\frac{d_{36}}{d_0}\right) \right] \qquad 4.1$$

where d_0 = maximum deflection at the center of the load plate, inches

d_i = deflections at 12, 24, and 36 inches from plate center, inches

AREA has units of length, rather than area, since each of the deflections is normalized with respect to d_0 in order to remove the effect of different load levels and to restrict the range of values obtained. AREA and d_0 are thus independent parameters, from which the surface and foundation moduli in a two-layer pavement system may be determined. Hoffman and Thompson developed a nomograph for backcalculation of flexible pavement surface and subgrade moduli from d_0, AREA, and the pavement thickness. AREA may be defined for more sensors or other sensor spacings if desired. The term "AREA" as used here refers to AREA for four sensors at 12-inch spacing, according to Equation 4.1.

For a bare PCC pavement, the PCC slab's *in situ* elastic modulus (E_{pcc}) and the *in situ* subgrade k-value or elastic modulus (E_s) may be backcalculated from the maximum deflection d_0, the AREA of the deflection basin, and the slab thickness. A graphical approach for doing this was developed by ERES Consultants in 1982. [79] The ILLI-SLAB finite element program was used to compute a matrix of deflection basins by varying the slab E and subgrade k for a given slab thickness. Intersecting k and E_{pcc} curves were plotted against AREA and d_0 axes, and the k and E_{pcc} for any deflection basin could then be interpolated by plotting its AREA and d_0 on the graph. In 1985, Foxworthy [49] improved the efficiency of this procedure by computerizing the interpolation process using a vectoring scheme. However, several ILLI-SLAB runs were still necessary to produce the matrix of k and E values for any given slab thickness.

Further investigation of this concept by Ioannides [64], and Ioannides, Barenberg, and Lary [65] has produced a closed-form solution procedure to replace the iterative and graphical procedures used previously, as well as the computer program ILLI-BACK for rapid analysis of deflection data for slab-on-grade pavement systems.

4.2.1 AREA versus ℓ

Research by Ioannides [64] has demonstrated that for a given load radius and sensor arrangement, a unique relationship exists between AREA and the dense liquid radius of relative stiffness of the pavement system, in which the subgrade is characterized by a k-value [80]:

$$\ell_k = \sqrt[4]{\frac{E_{pcc}\, D_{pcc}^{3}}{12\,(1 - \mu_{pcc}^{2})\, k}} \qquad 4.2$$

where ℓ_k = dense liquid radius of relative stiffness, inches

E_{pcc} = PCC elastic modulus, psi

D_{pcc} = PCC thickness, inches

μ_{pcc} = PCC Poisson's ratio

k = modulus of subgrade reaction, psi/inch

A different unique relationship exists between AREA and the elastic solid radius of relative stiffness of the pavement, in which the subgrade is characterized by an elastic modulus and a Poisson's ratio [81]:

$$\ell_e = \sqrt[3]{\frac{E_{pcc}\, D_{pcc}^{3}\,(1 - \mu_s^2)}{6\,(1 - \mu_{pcc}^2)\, E_s}} \qquad 4.3$$

where ℓ_e = elastic solid radius of relative stiffness, inches

μ_s = subgrade Poisson's ratio

E_s = subgrade elastic modulus, psi

However, no equations for AREA versus ℓ_k or ℓ_e are provided in References 64 or 65. Therefore, equations for AREA versus ℓ were derived in this study, using the equations developed by Westergaard [80], Losberg [81], and Ioannides [64] for deflection of a plate on a dense liquid or elastic solid foundation at any distance from an applied load.

4.2.1.1 AREA versus ℓ_k

The equations for deflection of a PCC slab on a dense liquid foundation load are given by Losberg [81]:

If $0 < s < a/\ell_k$:

$$w(s) = \frac{P}{\pi \left(\frac{a}{\ell_k}\right)^2} \frac{\ell_k^2}{D} \left(1 - c_1\, ber\ s - c_2\, bei\ s \right) \qquad 4.4$$

If $s > a/\ell_k$:

$$w(s) = \frac{P}{\pi \left(\frac{a}{\ell_k}\right)^2} \frac{\ell_k^2}{D} \left(c_3\, ker\ s + c_4\, kei\ s \right) \qquad 4.5$$

where $w(s)$ = deflection at s

r = distance from center of applied load

s = r / ℓ_k

P = load

a = load radius

ber = Kelvin function of order 0, real (derivative is ber$^{/}$)

bei = Kelvin function of order 0, imaginary (derivative is bei$^{/}$)

ker = modified Kelvin function of order 0, real (derivative is ker$^{/}$)

kei = modified Kelvin function of order 0, imaginary (derivative is kei$^{/}$)

$$D = \frac{E_{pcc} D_{pcc}^{3}}{12 (1 - \mu_{pcc}^{2})} \qquad 4.6$$

Equations for the four constants c_1 through c_4 were developed by Ioannides [64]:

$$c_1 = 1 - \pi \left(\frac{a}{\ell_k}\right)^2 \frac{w_0 D}{P \ell_k^2} \qquad 4.7$$

$$w_0 = \left(\frac{P}{8 k \ell_k^2}\right) \left\{ 1 + \left(\frac{1}{2 \pi}\right) \left[ln \left(\frac{a}{2 \ell_k}\right) + \gamma - 1.25 \right] \left(\frac{a}{\ell_k}\right)^2 \right\} \qquad 4.8$$

$$c_2 = \frac{\left(\frac{a}{\ell_k}\right)^2}{2} \left[\ln \left(\frac{2 \ell_k}{a}\right) + 0.5 - \gamma + \left(\frac{a}{\ell_k}\right)^2 \left(\frac{2\pi}{64}\right) \right] \qquad 4.9$$

γ = Euler's constant, 0.57721566490

$$c_3 = A - B\,c_4 \tag{4.10}$$

$$A = \frac{1 - c_1\,ber\left(\frac{a}{\ell_k}\right) - c_2\,bei\left(\frac{a}{\ell_k}\right)}{ker\left(\frac{a}{\ell_k}\right)} \tag{4.11}$$

$$B = \frac{kei\left(\frac{a}{\ell_k}\right)}{ker\left(\frac{a}{\ell_k}\right)} \tag{4.12}$$

$$c_4 = \frac{F - G\,A}{1 - G\,B} \tag{4.13}$$

$$F = \frac{-c_1\,ber'\left(\frac{a}{\ell_k}\right) - c_2\,bei'\left(\frac{a}{\ell_k}\right)}{kei'\left(\frac{a}{\ell_k}\right)} \tag{4.14}$$

$$G = \frac{ker'\left(\frac{a}{\ell_k}\right)}{kei'\left(\frac{a}{\ell_k}\right)} \tag{4.15}$$

These equations were solved for radial distances of 0, 12, 24, and 36 inches and for ℓ_k values from 15 to 80 using the IMSL [82, 83] library of functions available on the Apollo network of UNIX workstations at the University of Illinois. The deflections computed were used to obtain AREAs corresponding to each value of ℓ_k. The results are illustrated by the curve labelled "dense liquid" in Figure 4.1.

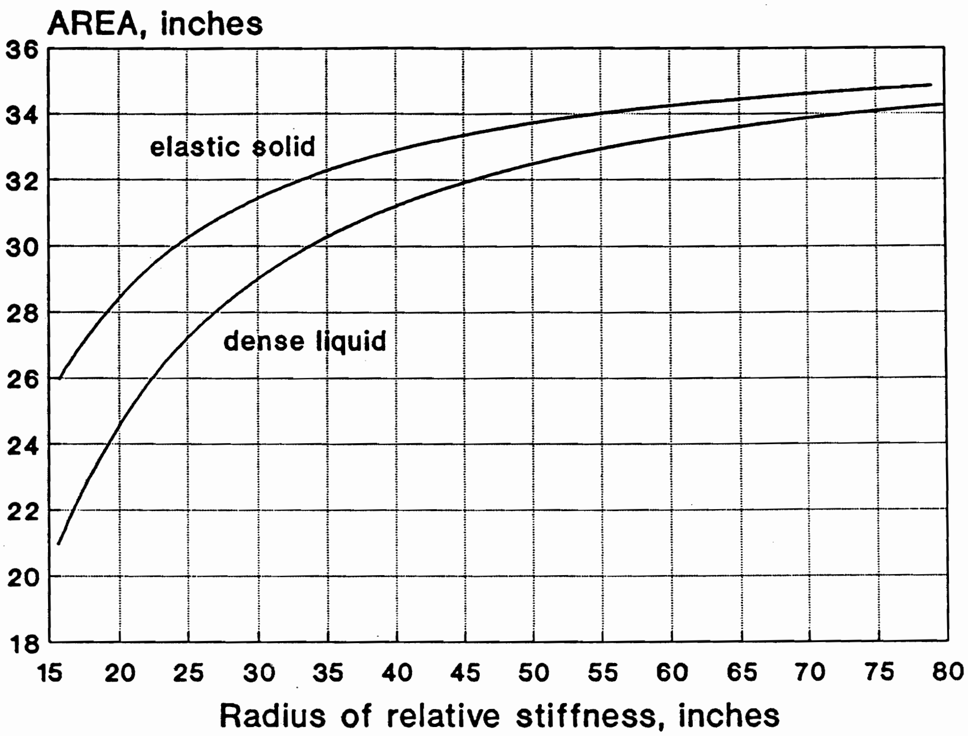

Figure 4.1 Deflection AREA versus radius of relative stiffness.

Since the curve asymptotically approaches an AREA value of 36 inches, an appropriate and meaningful equation form for modelling the relationship of AREA to ℓ is that of an asymptotic regression model, also called a monomolecular growth model. [84] Such a model has the following general form:

$$AREA = k_1 - k_2 \, e^{-k_3 \, \ell^{k_4}} \qquad 4.16$$

where k_1 is the asymptotic AREA value, k_2 is a parameter for the range of ℓ values, and k_3 and k_4 are scale parameters which govern the rate of growth. To predict ℓ as a function of AREA, the model may be rearranged to yield the following general form:

$$\ell = \left[\frac{ln\left(\frac{k_1 - AREA}{k_2}\right)}{-k_3} \right]^{\frac{1}{k_4}} \qquad 4.17$$

The SAS statistical analysis software [33] was used to determine the parameters for each model by nonlinear regression. The model obtained for ℓ_k versus AREA is given by Equation 4.18. The fit of the model to the data is excellent, as illustrated by Figure 4.2. Values obtained from ILLI-BACK runs are also shown for comparison.

$$\ell_k = \left[\frac{ln\left(\frac{36 - AREA}{1812.279133}\right)}{-2.559340} \right]^{4.387009} \qquad 4.18$$

R^2 = 99.99 percent (predicted versus actual values)

σ_Y = 0.097 inches (standard error of the estimate)

n = 63

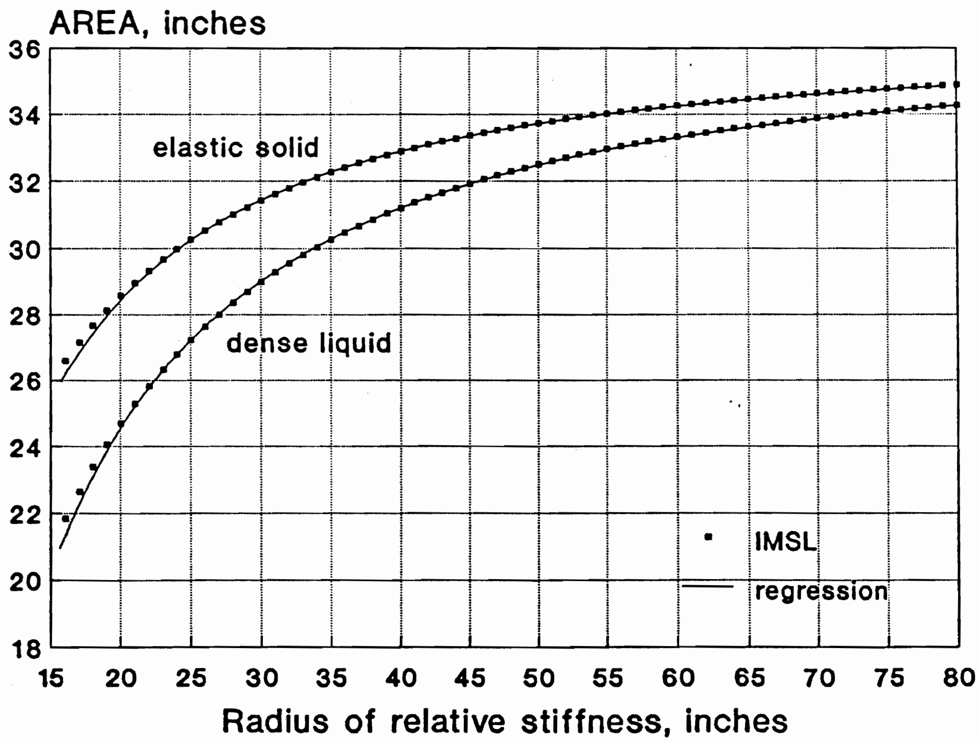

Figure 4.2 Comparison of AREA versus ℓ values and regression models.

4.2.1.2 AREA versus ℓ_e

For a PCC slab on an elastic solid foundation, the deflection at any distance from the applied load is given by the following equation from Losberg [81]:

$$w(r) = \frac{2\,P}{\pi\, a\, C} \int_0^{\infty} \frac{J_0\,(\alpha r)\, J_1(\alpha a)}{\alpha\,(1 + \alpha^3\, \ell_e^{\,3})}\, d\alpha \qquad 4.19$$

$$C = \frac{E_s}{(1 - \mu_s^2)} \qquad 4.20$$

This integral was solved for radial distances of 0, 12, 24, and 36 inches and for ℓ_e values from 15 to 80 using the IMSL [82, 83] library on the Apollo network. The deflections computed were used to obtain an AREA corresponding to each value of ℓ_e. The results are illustrated by the curve labelled "elastic solid" in Figure 4.1.

The model obtained for ℓ_e versus AREA is given by Equation 4.21. The fit of the model to the data is excellent, as illustrated by Figure 4.2. Values obtained from ILLI-BACK runs are also shown for comparison.

$$\ell_e = \left[\frac{\ln\left(\dfrac{36 - AREA}{4521.676303}\right)}{-3.645555} \right]^{5.334281} \qquad 4.21$$

Residual R^2 = 99.99 percent

σ_Y = 0.118 inches

n = 83

4.2.2 Backcalculation of Subgrade k or E_s

With AREA calculated from measured deflections, ℓ_k or ℓ_e may be obtained from Equation 4.18 or 4.21. The k-value may be obtained by rearrangement of Westergaard's interior deflection equation [65, 80].

$$k = \left(\frac{P}{8\, d_0\, \ell_k^2}\right)\left\{1 + \left(\frac{1}{2\pi}\right)\left[ln\left(\frac{a}{2\,\ell_k}\right) + \gamma - 1.25\right]\left(\frac{a}{\ell_k}\right)^2\right\} \qquad 4.22$$

Figure 4.3 was developed from Equations 4.18 and 4.22 for load P = 9000 pounds and load radius a = 5.9055 inches. For loads within about 2000 pounds of this value, the deflections d_0, d_{12}, d_{24}, and d_{36} may be scaled linearly to 9000-pound deflections.

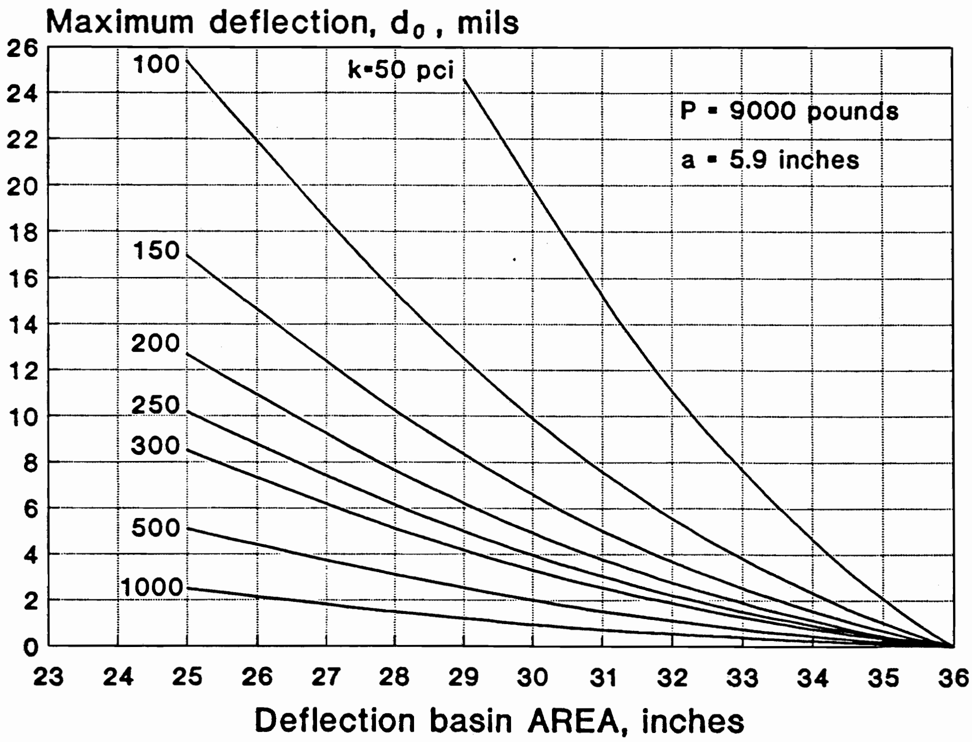

Figure 4.3 Determination of k-value from d_0 and AREA.

The elastic modulus of the subgrade E_s may be obtained by rearrangement of Losberg's deflection equation [81]:

$$E_s = \left[\frac{2\,P\,(1 - \mu_s^2)}{d_0\,\ell_e} \right] \left[0.19245 - 0.0272 \left(\frac{a}{\ell_e} \right)^2 + 0.0199 \left(\frac{a}{\ell_e} \right)^2 ln \left(\frac{a}{\ell_e} \right) \right] \qquad 4.23$$

Figure 4.4 was developed from Equations 4.21 and 4.23 for load P = 9000 pounds, load radius a = 5.9055 inches, and subgrade Poisson's ratio μ_s = 0.50. Again, for loads within about 2000 pounds of this value, the deflections d_0, d_{12}, d_{24}, and d_{36} may be scaled linearly to 9000-pound deflections.

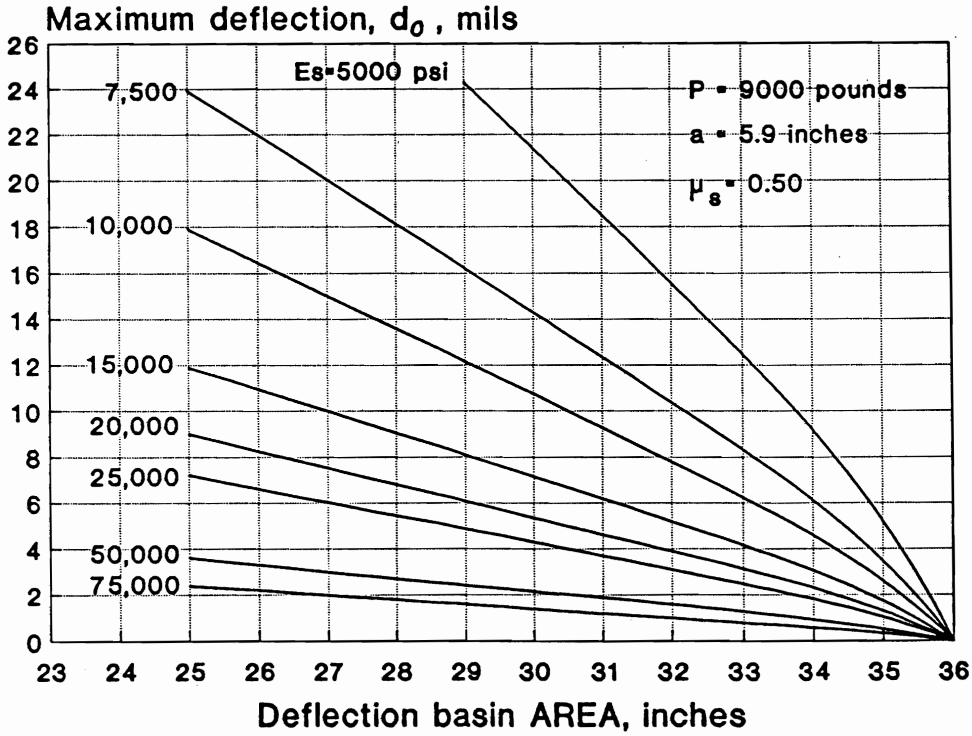

Figure 4.4 Determination of E_s from d_0 and AREA.

4.2.3 PCC Elastic Modulus

The elastic modulus of the PCC slab may be determined using the appropriate (dense liquid or elastic solid) definition of the radius of relative stiffness. Figure 4.5 was developed from Equations 4.2 and 4.18, assuming a PCC Poisson's ratio μ_{pcc} = 0.15 and load radius a = 5.9055 inches. Figure 4.6 was developed from Equations 4.3 and 4.21, assuming μ_{pcc} = 0.15, μ_s = 0.50, and a = 5.9055 inches. For either support characterization, the PCC elastic modulus E_{pcc} may be determined for a known value of slab thickness, D_{pcc}.

4.3 BACKCALCULATION FOR AC/PCC PAVEMENTS

In order to apply this closed-form backcalculation approach to AC/PCC pavements, deflections measured on the existing AC surface must be adjusted to account for the influence of the AC layer. The steps for doing this are described in this section.

4.3.1 AC Elastic Modulus

An existing AC/PCC pavement cannot be properly modelled as a slab on grade, since the AC overlay exhibits not only bending but also significant compression. To determine the amount of compression that occurs in the AC overlay, the elastic modulus of the AC layer must be determined. The recommended method for determining E_{ac} is to monitor the temperature of the AC mix during deflection testing, conduct diametral resilient modulus tests on cores obtained from the AC surface, and to establish a relationship between E_{ac} and temperature for use in assigning a modulus value to each deflection basin.

The AC mix temperature may be measured directly during deflection testing, as described in Chapter Three. If measured AC mix temperatures are not available, they may be approximately estimated from pavement surface and air temperatures using procedures developed by Southgate [85], Shell [86], the Asphalt Institute [87], or Hoffman and Thompson [78]. Pavement surface temperature may be monitored during deflection testing using a hand-held infrared sensing device which is aimed at the pavement. The mean air temperature for the five days prior to deflection testing, which is an input to some of the referenced methods for estimating mix temperature, may be obtained from a local weather station or other local sources.

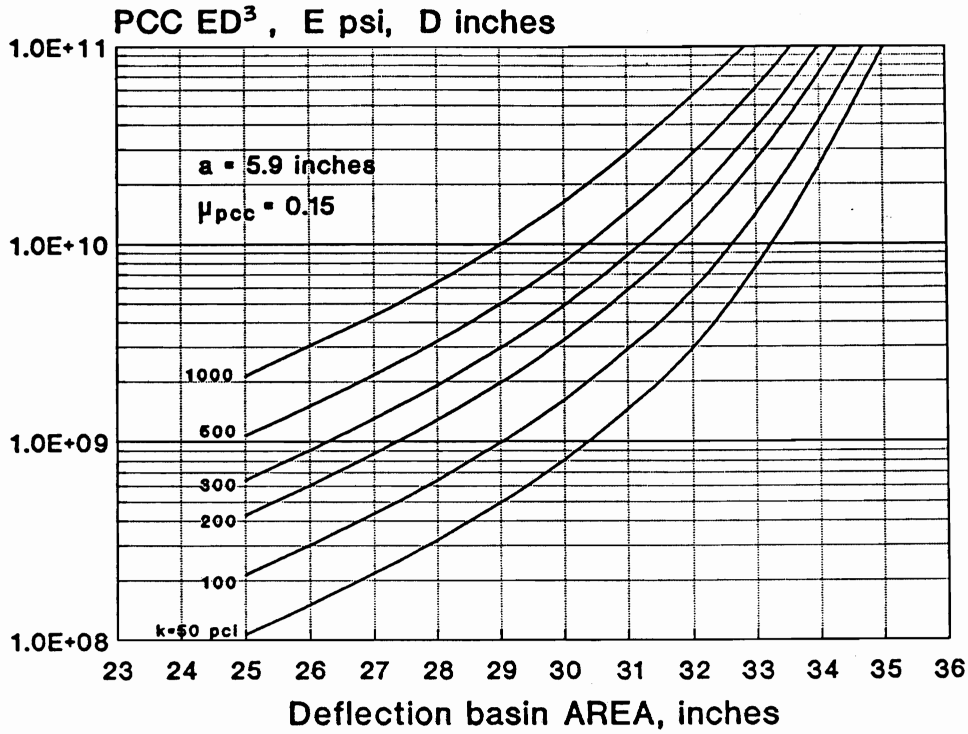

Figure 4.5 Determination of E_{pcc} from k-value, AREA, and slab thickness.

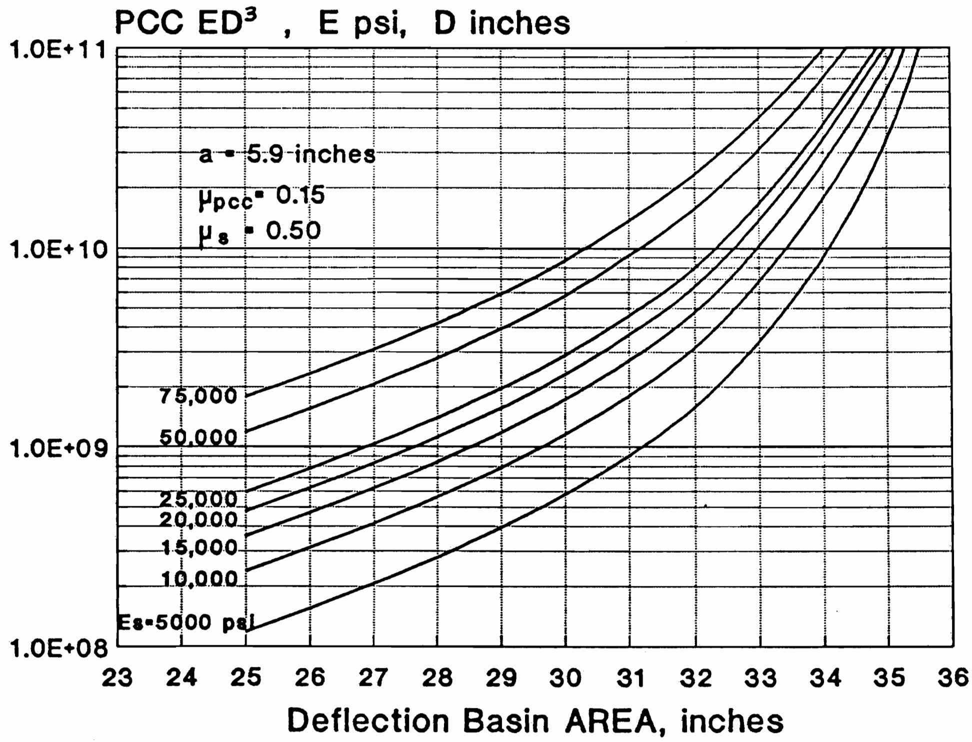

Figure 4.6 Determination of E_{pcc} from E_s, AREA, and slab thickness.