NEW PREDICTIVE MODELINGTECHNIQUES FOR PAVEMENTS

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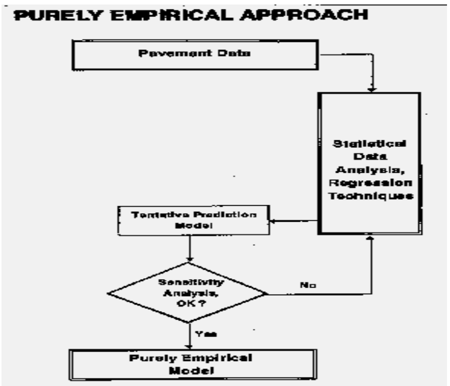
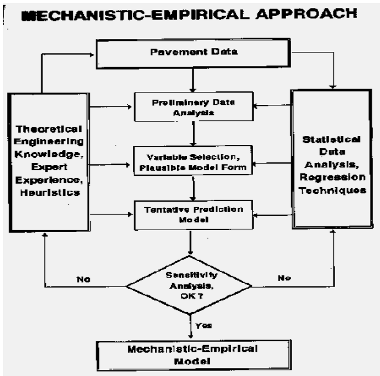
PROBLEM STATEMENT

- Various applications of pavement prediction models (new pavement design, pavement evaluation and rehabilitation plan, pavement management programming, etc.)
- Accuracy of prediction is very inconsistent and often very poor
- Existing models often fail to satisfy some tatistical assumptions and engineering boundary conditions
- Lack of guidelines for model development





OBJECTIVES

- Investigate the advantages and disadvantages ofthe current modeling procedures and techniques
- Introduce modern regression techniques
- Propose a systematic statistical and engineering approach for model development
- Demonstrate the proposed modeling procedures

TRADITIONAL REGRESSION TECHNIQUES

- Multiple Linear Regression:

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \dots + \beta_{p-1} x_{p-1} + \varepsilon$$

- Nonlinear Regression:

$$y = F(\beta_0, \beta_1, \dots, \beta_{p-1}, x_1, x_2, \dots, x_k) + \varepsilon$$

Both minimizing the sum of squared residuals:

$$RSS(\hat{\beta}) = \sum_{i=1}^{n} [r_i(\hat{\beta})]^2$$

MULTIPLE LINEAR REGRESSION

- Preliminary or explanatory analysis of linear relationships of a group of important variables
- Stepwise and all-subset regressions used for automatic variable selection
- Very sensitive to the presence of outliers and influential data points
- Regression diagnostics based on delete-one statistics are often masked by some groups of influential observations

NONLINEAR REGRESSION

- Can handle a complicated nonlinear model
- Model specifications: assuming a descriptive model form and guessing initial parameter estimates (specifying bounds if necessary)
- Very sensitive to the presence of outliers and influential data points
- Often fail to satisfy convergence criterion and some statistical assumptions
- Parameter estimates often insignificant or toward wrong direction in physical interpretations

MODERN REGRESSION TECHNIQUE

- Projection Pursuit Regression (PPREG, "Projection") Algorithm:
  - —capable of modeling variable interactions (Friedman and Stuetzle, 1981)
  - —attempting to model the response surface as a sum of nonparametric functions of projections of the explanatory variables through the use of local smoothing techniques

"PROJECTION" (PPREG) ALGORITHM

$$y = \bar{y} + \sum_{m=1}^{M_0} \beta_m \phi_m(\mathbf{a}_m^T x) + \varepsilon$$

$$E\left[\phi_m(\mathbf{a}_m^T x)\right] = 0 \ , \ E\left[\phi_m^2(\mathbf{a}_m^T x)\right] = 1$$

Minimizing the mean squared residuals:

$$E\left[\mathbf{r}^2\right] = E\left[\mathbf{y} - \bar{\mathbf{y}} - \sum_{m=1}^{M_0} \beta_m \phi_m(\mathbf{a}_m^T x)\right]^2$$

A CASE STUDY: EDGE STRESS DUE TOLOADING AND FINITE SLAB LENGTH EFFECT

- Determine the maximum bending stress at the longitudinal edge of the slab
- Finite element model can not be easily implemented as a part of a design procedure
- To illustrate the advantages of introducing mechanistic variables and selecting proper functional forms in model development

THREE DIFFERENT APPROACHES

- Use arbitrary but "best" linear combinations of individual variables (Darter, 1977)
- Introduce as many mechanistic variables as possible and also find "best" linear combinations of them (Salsilli, 1991)
- Introduce as many mechanistic variables as possible and also find the best functional forms using the **"Projection" (PPREG)** algorithm

ARBITRARY LINEAR COMBINATIONS OFVARIABLES

¬ **Perform a large factorial of finite element runs:**

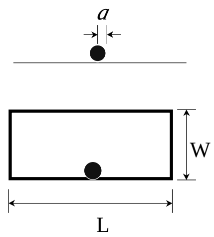
1. Slab length, L = 15, 20, 25, 30 ft
2. Slab thickness, h = 8, 10, 14 in.
3. Foundation support, k = 50, 200, 500 pci

(E = 5 Mpsi, W = 12 ft, loaded area = 12 x 15 in2)

¬ **Resulting model for edge stress prediction:**

$$\pm = \frac{P}{18h^2}\left(17.4 - 0.05\frac{h^3}{k} + 7.4\log\frac{h^3}{k}\right)$$

Concrete Pavement	*Mechanistic Variables*
Stress, Deflection	± h^2/P, ¢ kª $^2/P$
	$a/$ª
	$a/$ª, $L/$ª, $W/$ª



EDGE STRESS DUE TO LOADINGR

$$R = \frac{\pm_i}{\pm_w} = f\left(\frac{a}{ª}, \frac{L}{ª}\right)$$

$$ª = \sqrt[4]{\frac{E h^3}{12(1 - « ^2)k}}$$

R= adjustment (multiplication) factor for the finite slab length effect

$\pm_w$ =Westergaard's edge stress solution

$\pm_i$ =Edge stress determined by thefiniteelement model

INTRODUCING MECHANISTIC VARIABLES

¬ A small factorial of finite element runs:

a / ª : 0.05, 0.1, 0.2, 0.3

L / ª : 3.0, 4.0, 5.0, 7.0

¬ Resulting model (Model #2):

$$R \; Z \; 0.58 - 0.53\left(\frac{a}{ª}\right) H \; 0.18\left(\frac{L}{ª}\right) J \; 0.02\left(\frac{L}{ª}\right)^2 H \; 0.11\left(\frac{a}{ª}\right)\left(\frac{L}{ª}\right)$$

Limits: **3 á L / ª á 5, 0.05 á a / ª á 0.3**

N = 12, R^2 = 0.996, SEE = 0.0028, CV = 0.29%

PROPER FUNCTIONAL FORMS

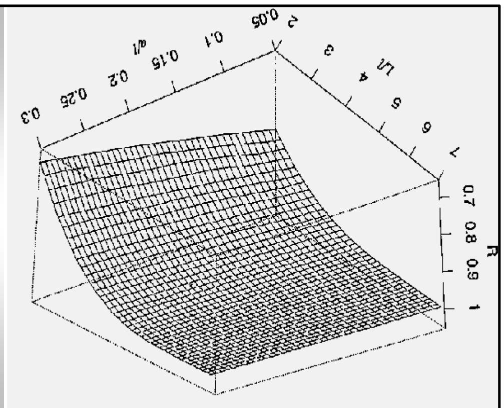
¬ A small factorial of finite element runs:

***a*/ª : 0.05, 0.1, 0.2, 0.3**

L/ª : 2.0, 2.5, 3.0, 3.5, 4.0, 4.5, 5.0, 6.0, 7.0

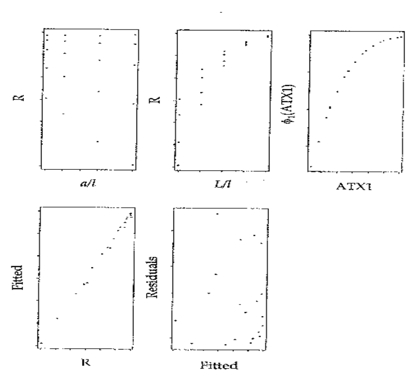
¬ Use of the "Projection" (PPREG) algorithm to select proper functional forms

¬ Discussion of "prediction" within and "extrapolation" beyond the specified ranges



USE OF THE "PROJECTION" ALGORITHM

- The 3-dimensional response surface is broken down into a sum of several smooth projected curves, which are graphically representable in two dimensions.
- Plausible functional forms and applicable boundary conditions may then be easily identified and specified.
- Traditional linear and nonlinear regressions are then utilized to model each projected curve individually.



RESULTING ONE-TERM "PROJECTION" MODEL

Model #3:

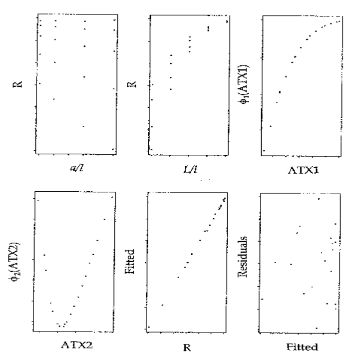
$$R = 0.967 + 0.033\phi_1(ATX_1)$$

$$\phi_1(ATX_1) = -5.587 + \frac{1}{0.147 + 0.263\,ATX_1^{-5.17}}$$

$$ATX_1 = -0.895\frac{a}{l} + 0.447\frac{L}{l}$$

$$Limits:\ 3 \le \frac{L}{l} \le 5\ ,\ 0.05 \le \frac{a}{l} \le 0.3$$

N = 20, R2 = 0.994, SEE = 0.0027, CV = 0.28%



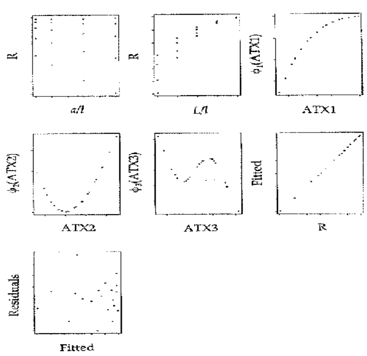
RESULTING TWO-TERM "PROJECTION" MODEL

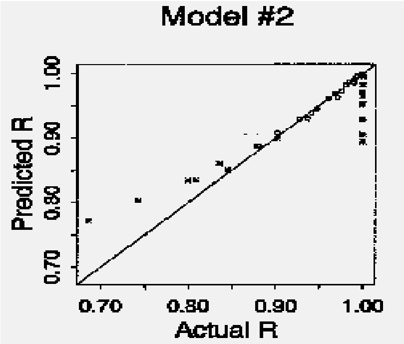
$$R = 0.967 + 0.033\phi_1(ATX_1) + 0.0021\phi_2(ATX_2)$$

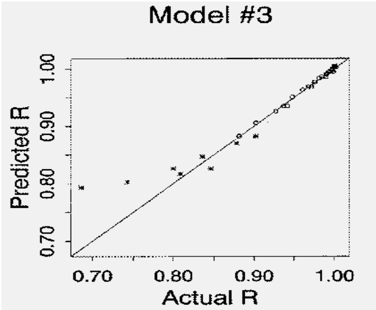
$$ATX_1 = -0.895\frac{a}{l} + 0.447\frac{L}{l}$$

$$ATX_2 = 0.997\frac{a}{l} + 0.0779\frac{L}{l}$$

Note: The second projected term contributes little to the prediction of R (i.e. $_2 = 0.0021$ vs. $_1 = 0.033$).







PREDICTION AND EXTRAPOLATION

- Use Models #2 and #3 for prediction within and extrapolation beyond the specified ranges
- Model #2: good prediction within the range unacceptable results when extrapolated
- Model #3: good prediction within the range acceptable results when extrapolated
- Conclusions: correct functional forms provide more comprehensive insights of the model

CONCLUSIONS

- Investigated the advantages and disadvantages of the current modeling procedures and techniques
- Introduced one moden regression technique - PPREG or "Projection" algorithms
- Proposed a systematic statistical and engineering approach for model development (emphasizing on subject-related engineering knowledge and selecting proper functional forms)
- Demonstrated the proposed modeling procedures in a case study