# Loading and Curling Stress Models for Concrete Pavement Design

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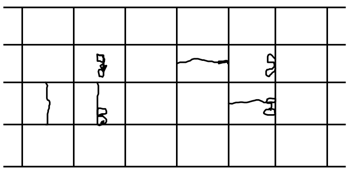
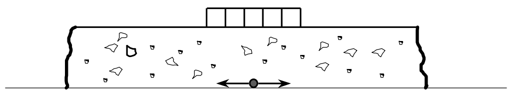
## Objective

- Develop predictive models for load, curl, and load/curl that could be utilized in a spreadsheet or PC program for rapid calculation purposes.
- May encourage use by practitioners.

## Edge Loads Cause Longitudinal And Transverse Cracks



## Load and Temperature Curling Stresses



Load Stress - Located at Edge
Curl Stress - Positive or Negative Gradient
Combined Load and Curl Stress

## Wesergaard Edge Loading Solutions

$$\sigma_e = \frac{3(1+\mu)P}{\pi(3+\mu)h^2}\left[\log_e \frac{Eh^3}{100ka^4} + 1.84 - \frac{4\mu}{3} + \frac{1-\mu}{2} + 1.18(1+2\mu)\frac{a}{\ell}\right]$$

$$\delta_e = \frac{\sqrt{2+1.2\mu}\,P}{\sqrt{Eh^3k}}\left[1 - (0.76+0.4\mu)\frac{a}{\ell}\right]$$

$$\ell = \sqrt[4]{\frac{Eh^3}{12(1-\mu^2)k}}$$

- Infinite slab length (L), Infinite slab width (W)
- Fully supported slab

## Westergaard Thermal Curling Solutions

$$\sigma_y = \sigma_0\{1 - \frac{2\cos\lambda\cosh\lambda}{\sin 2\lambda + \sinh 2\lambda}[(\tan\lambda + \tanh\lambda)\cos\frac{y}{\ell\sqrt{2}}\cosh\frac{y}{\ell\sqrt{2}} + (\tan\lambda - \tanh\lambda)\sin\frac{y}{\ell\sqrt{2}}\sinh\frac{y}{\ell\sqrt{2}}]\}$$

$$\delta_y = -\delta_0\frac{2\cos\lambda\cosh\lambda}{\sin 2\lambda + \sinh 2\lambda}[(-\tan\lambda + \tanh\lambda)\cos\frac{y}{\ell\sqrt{2}}\cosh\frac{y}{\ell\sqrt{2}} + (\tan\lambda + \tanh\lambda)\sin\frac{y}{\ell\sqrt{2}}\sinh\frac{y}{\ell\sqrt{2}}]$$

$$\sigma_0 = \frac{E\alpha\Delta T}{2(1-\mu)},\ \delta_0 = \frac{(1+\mu)\alpha\Delta T\ell^2}{h},\ \lambda = \frac{W}{\ell\sqrt{8}}$$

- Assume slab remains flat with thermal gradient
- No loss of support under the slab due to curling

## Determine Combined Stress???

**Loading (traffic)**
**Curling (thermal gradient)**

**Load + Curl Combined Stress**

## Combined Stress Finite Element Solutions

**2-Dimension - More realistic modeling of combined load and curl stresses.**

**3-Dimension - Even more realistic modeling of combined load and curl stresses, especially for stiff unbonded base.**

**FE Problems - Complexity in use of FE programs, Potential for error, Long computer run times when many solutions needed.**

## Previous Attempts To Obtain Closed-Form Models For Combination Of Load And Curl Stress

**Combined Stress = Load Stress + R * Curl Stress**

**Problem: R determined by multiple regression, Inadequate accuracy**

## Current Approach

**Utilize dimensional analysis and modern statistical modeling techniques to obtain solution that accurately reproduces FE model.**

- **Identify primary structural responses**
  - dwheel load only
  - htemperature curling only
- **Combined Load/Curl Stress =Load Stress + R * Curl Stress**
- **R = F(primary structural responses)**

## Primary Structural (Dimensionless) Variables For Load Stress

$$\pm_{WES} = \frac{P}{h^2} * f_1\left(\frac{a}{\text{a}}, «\right)$$

$$\frac{\pm_{WES}\, h^2}{P} = f_1\left(\frac{a}{\text{a}}\right)$$

$$¢_{WES} = \frac{P}{\sqrt{Eh^3 k}} * f_2\left(\frac{a}{\text{a}}, «\right)$$

$$\frac{¢_{WES}\, k\text{a}^2}{P} = f_2\left(\frac{a}{\text{a}}\right)$$

## Primary Structural (Dimensionless)Variables For Thermal Curling Stress

$$\pm_{WESC} = E \sim \Delta T * f_3\left(\frac{L}{\text{a}}, \frac{W}{\text{a}}, «\right)$$

$$\frac{\pm_{WESC}}{E} = f_3\left(\sim \Delta T, \frac{L}{\text{a}}, \frac{W}{\text{a}}\right)$$

$$¢_{WESC} = \frac{\sim \Delta T \text{a}^2}{h} * f_4\left(\frac{L}{\text{a}}, \frac{W}{\text{a}}, «\right)$$

$$\frac{¢_{WESC}\, h}{\text{a}^2} = f_4\left(\sim \Delta T, \frac{L}{\text{a}}, \frac{W}{\text{a}}\right)$$

## Two Additional Dimensionless Parameters Identified

$$D_{\gamma} = \frac{\gamma h^2}{k\ell^2}$$

$$D_P = \frac{Ph}{k\ell^4} = 12(1-\mu^2) * \frac{P}{Eh^2}$$

**$D_{\gamma}$ and $D_P$ are dimensionless parameters that represent the relative deflection stiffness due to the self-weight of the concrete slab, external wheel load, and the possible loss of subgrade support due to curling.**

## LOADING ONLY

**Finite Slab Length Effect**

$$R_L = \frac{\sigma_{FE}}{\sigma_{WES}} = F\left(\frac{a}{\ell}, \frac{L}{\ell}\right)$$

**Finite Slab Width Effect**

$$R_W = \frac{\sigma_{FE}}{\sigma_{WES}} = F\left(\frac{a}{\ell}, \frac{W}{\ell}\right)$$

## LOADING ONLY (Continued)

**Where:**

**$R_L$= adjustment (multiplication) factor for finite slab length effect**

**$R_W$ = adjustment factor for finite slab width effect**

**$\sigma_{wes}$ =Westergaard's edge stress solution**

**$\sigma_e$ = edge stress determined by the finite element model**

**F =function to be derived from finite element outputs and statistical modeling techniques**

## THERMAL CURLING ONLY

$$R_C = \frac{\sigma_{FE}}{\sigma_{WESC}} = F\left(\alpha\Delta T, \frac{L}{\ell}, \frac{W}{\ell}, \frac{\gamma h^2}{k\ell^2}\right)$$

**Where:**

**$R_c$ = adjustment factor for thermal curling**

**$\sigma_{wesc}$ = Westergaard/ Bradbury's edge stress solution**

**$\sigma_e$ = edge stress determined by the finite element model**

**F = function to be derived from finite element outputs and statistical modeling techniques**

## LOADING AND THERMAL CURLING

$$\sigma_T = R_L * R_W * \sigma_{WES} + R_T * \sigma_{WESC}$$

$$R_T = F\left(\frac{a}{\ell}, \alpha\Delta T, \frac{L}{\ell}, \frac{W}{\ell}, \frac{\gamma h^2}{k\ell^2}, \frac{Ph}{k\ell^4}\right)$$

**Where:**

**T = total combined load and curl edge stress**

**$R_L$*$R_W$*$\sigma_{wes}$ = Westergaard edge loading stress adjusted for slab length and width effects**

**$\sigma_{wesc}$ = Westergaard/ Bradbury edge curling stress**

**RT = adjustment factor for the effect of loading plus thermal curling**

**F = function to be derived from finite element outputs and statistical modeling techniques**

## DEVELOPMENT OF MODELS

- **S-PLUS Statistical Package includes modern modeling techniques**
- **Used the "projection" algorithm to breakdown the multi-dimensional response surface into a sum of several smooth projected curves**
- **Used traditional linear and nonlinear regression techniques to obtain the parameter estimates of each projected curve and the overall regression statistics**

## Using this modeling approach, the following "projection" model was developed for RL

$R_L$ Z 0.940 H 0.0799 ¤$_1$ ($A_1$)

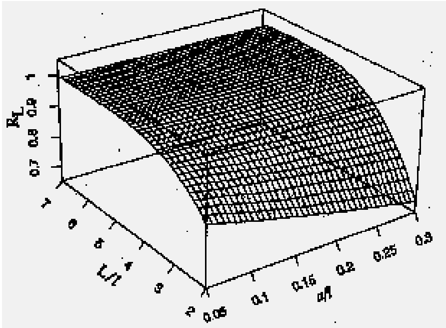
¤$_1$ ($A_1$) Z J4.031 H $\frac{1}{0.203 \text{ H } 0.0345 A_1^{\text{J}3.304}}$

$A_1$ Z J0.944*($a$/ ª) H 0.331($L$/ ª)

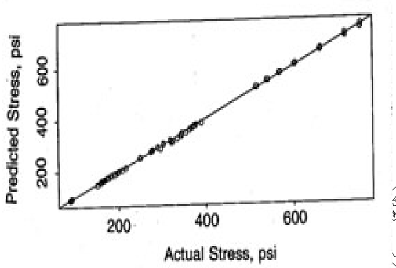
Statistics : N = 36 , R2 = 0.994 , SEE = 0.063 ,

CV = 0.67 %

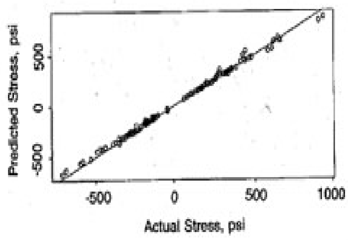
Limits : 2á $L$/ ª á 7 , 0.05á $a$/ ª á 0.3



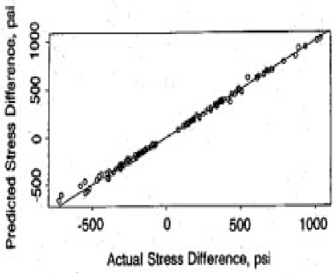

## Validation of the Proposed Models for Loading Only




## Validation of the Proposed Models for Curling Only




## Validation of the Proposed Loading and Curling Models




## Numerical Example

**Pavement slab with the following characteristics:**

**E = 5.5 Mpsi, K = 250 pci**

**L = 10 ft, W = 12 ft, h = 12 in.**

**= 0.087 pci, = 0.15, = 5.5 x $10^{-6}$ / $^{o}$F**

**Loading: Single wheel load of 9,000 lbs with a loaded rectangle of the size of 10x10 in2 is applied.**

**Thermal Gradient: Linear temperature differential of +20 oF (day-time condition) through the slab thickness.**

**Determine: Critical edge stresses due to loading and loading plus curling.**

## Numerical Example (Continued)

**Solution:**

**The equivalent radius of the load area is a = 5.64 in. and the radius of relative stiffness of the slab-subgrade system is = 31.20 in.**

**The dominating mechanistic variables are:**

a / ª Z 0.18 , L / ª Z 3.83 , W / ª Z 4.60

~a T Z H11.0 E J 05 ,

D ¥ Z 2.27 E J 05 , D$_P$ Z 29.99 E J 0.5

**Westergaard's solutions:**

$_{wes}$ **= 346 psi for loading only**

$_{wesc}$ **= 118 psi for curling only.**

## Numerical Example (Continued)

**Adjustment factors for finite slab length and width fo loading only:**

**$R_L$ = 0.968, $R_W$ = 1.000**

**Loading only edge stress calculated by the proposed models:**

**$_L$= $_{wes}$ * $R_L$ * $R_W$ = 346 * 0.968 * 1.000 = 335 psi**

**Actual FE load only edge stress = 344 psi**

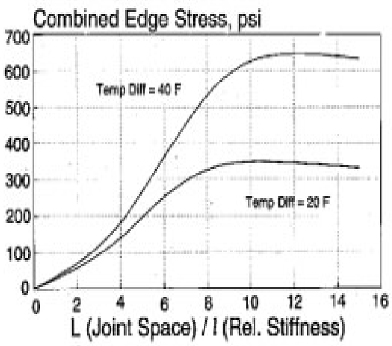
**Adjustment factor for loading plus curling:**

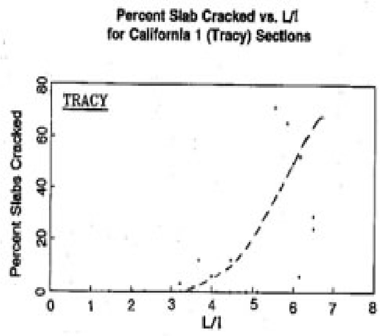
**$R_T$ = 0.732**

**Predicted combined load and curl edge stress determined by the proposed model is:**

**$_T$= $_L$+ $R_T$ * $_{wesc}$ = 335 + 0.732 * 118 = 421 psi**

**FE total combined edge stress = 436 psi.**




## Conclusions

- **Closed-form predictive models were developed for edge stresses (load and curl) using dimensional analysis and modern modeling techniques.**
- **Previous research using dimensional analysis included the major independent variables:**
  - **normalized load radius (a/ ),**
  - **normalized slab length (L/ ),**
  - **normalized slab width (W/ ), and**
  - **dimensionless product ( T) of a temperature differential and thermal expansion coefficient.**

**However, the actual structural response to a temperature influence could not be adequately described using only these four parameters.**

## Conclusions (Continued)

- **Two additional dimensionless parameters (D and D$_P$) representing the relative deflection stiffness due to the self- weight of the concrete slab, the applied load, and loss of support from curling were identified.**
- **A new modeling approach was used which makes use of projection pursuit regression algorithm and linear and nonlinear regression techniques to develop the proposed predictive models.**
- **The new models use only the dominating structural variables as opposed to earlier attempts using arbitrary linear combinations of input parameters with very few insights to the actual relationships among the variables.**

## Conclusions (Continued)

- **Consequently, closed-form mechanistic design models that have been carefully validated are ready for implementation on a spreadsheet or computer program. These stress models turned out to be very accurate representations of the 2-D finite element model.**
- **The new models were also properly formulated to satisfy applicable engineering boundary conditions. They are also simple, easy to comprehend, dimensionally correct, and may be extrapolated to wider ranges of other input parameters.**