# 由面層撓度値回算鋪面彈性 模數的初步研究

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#### RESEARCH APPROACH

- Theoretical Investigation Burmister (1943) and Scrivner's (1973) Deflection Equations
- Identification of Functional Forms
- Validation of the Dominating Dimensionless Variables Identified
- Development of a Backcalculation Database
- Development of Backcalculation Prediction Equations
- Validation of the Proposed Prediction Equations

### **OBJECTIVES**

- Major Deficiencies of Traditional Backcalculation Procedures
  - Uniqueness Problem
  - Iterative but Time-consuming Calculation
  - Subjective Selection of Initial Trial Values and Input Data Ranges
  - Violation of the Specified Convergence Criteria
- Scope: A Two-Layer Elastic Pavement System

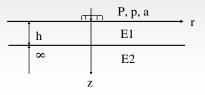
## CLASSIFICATION OF BACK-CALCULATION PROCEDURES

- Existing:
  - Iterative Approach (BISDEF, ELSDEF, etc.)
  - Data Base Approach (MODULUS)
- Proposed: (A New Approach)
  - Integrate the Concept of Traditional Database Approach and Modern Regression Techniques
  - Strive to Develop Prediction Equations to Allow "DIRECT" Modulus Calculations

# DEVELOPMENT OF TWO-LAYER ELASTIC THEORY

- Burmister (1943)
  - A Uniformly Distributed Circular Load

$$w = \frac{15pa}{E_2} F_W$$
,  $F_W = f\left(\frac{h}{a}, \frac{E_2}{E_1}\right)$ 



# DYNAFLECT'S LOAD CONFIGURATION

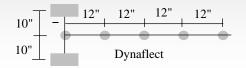
$$\frac{w_1 r_1}{w_3 r_3} = \frac{f_1\left(\frac{E_2}{E_1}, \frac{r_1}{h}\right)}{f_3\left(\frac{E_2}{E_1}, \frac{r_3}{h}\right)} = G\left(\frac{E_2}{E_1}, \frac{r_1}{h}, \frac{r_3}{h}\right)$$

- When r1, r3, and h are known ==>
- w1r1/w3r3 is a function of E2/E1

# DEVELOPMENT OF TWO-LAYER ELASTIC THEORY (continue ...)

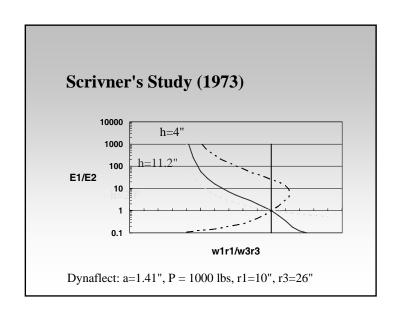
- Scrivner (1973)
  - A Concentrated Load

$$\frac{4\pi E_1}{3P}wr = f\left(\frac{E_2}{E_1}, \frac{r}{h}\right)$$



# LIMITATIONS OF BACK-CALCULATION PROGRAMS

- Assume there exists a Unique Modulus Combination
- Iterative but Time-consuming Calculations
- Results Affected by:
  - Different Initial Trial Values
  - Data Ranges
  - Specified Error Tolerance
  - Limited Number of Iterations



# UNIQUENESS PROBLEM FOR BACKCALCULATION

w1r1/w3r3	h11.2"	h<11.2"
1	Unique	None / Two
<1	Unique	None / Two

# DOMINATING MECHANISTIC VARIABLES (DIMENSIONLESS)

$$Y = \frac{4\pi E_1}{3P} w_1 r_1 = F\left(\frac{E_2}{E_1}, \frac{r_1}{h}, \frac{h}{a}\right)$$
$$\frac{w_1 r_1}{w_3 r_3} = F'\left(\frac{E_2}{E_1}, \frac{r_1}{h}, \frac{r_3}{h}, \frac{h}{a}\right)$$

- Identified through Theory for any NDT Devices
- Validated through Series of BISAR Runs

### Validation of Variable (h/a)

h/a	h	a	r1	r3	w1	w3	w1r1
	in.	in.	in.	in.	mil	mil	/w3r3
2.5	5	2	7.5	15	3.9 9	3.1 6	0.631
2.5	8	3.2	12	24	2.4 9	1.9 8	0.629
2.5	14	5.6	21	42	1.4 2	1.1 3	0.628

Note: r1/h=1.5, r3/h=3, E2/E1=100, E1=500,000 psi, E2=5,000 psi, P=1,000 lbs

### A BACKCALCULATION DATABASE

- Based on Four Dimensionless Variables Identified
- E1/E2 = 1, 2, 5, 10, 20, 50, 100, 200, 500, 1000, 2000, 5000

r1/h = 0.8, 1.2, 1.8, 2.4, 3.6, 4.8, 6.0

r3/h = 1.2, 1.8, 2.4, 3.6, 4.8, 6.0, 7.2

h/a = 0.8, 1.3, 2.5, 3.5, 5.0

(r1>r3, P = 2400 lbs, E2 = 1000 psi, h = 10 in.)

- FORTRAN Programs Written for Batch Processing
- Total of 1680 Data Sets

# 

### **USE OF THE DATABASE**

- Advantages:
  - "DIMENSIONLESS" Variables
  - Covers Almost All Data Ranges
  - Save Unnecessary Iterative Computation Time
- A Computer Program, Look-up Tables or Figures for Linear Interpolation
- "DIRECT" Calculation is Possible if "Uniqueness" is Guaranteed.

# PROJECTION PURSUIT REGRESSION

- "Projection" (PPR) Algorithm by Friedman and Stuetzle, 1981
- Capable of Modeling Variable Interactions
- Model the Response Surface as a Sum of Nonparametric Prediction Functions of Explanatory Variables Using Local Smoothing Techniques

### "PROJECTION" (PPR) ALGORITHM

$$y = \overline{y} + \sum_{m=1}^{M_0} \beta_m \phi_m (a_m^T x) + \varepsilon$$

Minimizing the Mean Squared Residuals:

$$E[\varepsilon^2] = Minimum$$

# PROPOSED PREDICTION EQUATIONS FOR "DIRECT" CALCULATION - E1/E2

$$\log_{10}\left(\frac{E_1}{E_2}\right) = f\left(\frac{w_1\eta}{w_3r_3}, \frac{h}{a}, \frac{\eta}{h}, \frac{r_3}{h}, \frac{\eta}{r_3}, \frac{\eta}{a}, \frac{r_3}{a}\right)$$

Statistics and Limits:

$$N = 1247$$
,  $R^2 = 0.995$ ,  $SEE = 0.064$ ,  $CV = 2.8\%$ 

$$1 \le \frac{E_1}{E_2} \le 5000$$
,  $0.8 \le \frac{\eta}{h} \le 6.0$ ,  $1.2 \le \frac{r_3}{h} \le 7.2$ 

$$0.8 \le \frac{h}{a} \le 5.0, \ \frac{w_1 r_1}{w_3 r_3} \le 1.0, \ r_1 > r_3$$

### TWO-STEP MODELING APPROACH

- Use PPR Algorithm to Break down the Multi-Dimensional Response Surface into a Sum of Several Smooth Projected Curves, Which Are Graphically Representable in Two Dimensions
- Use Traditional Linear and Nonlinear Regressions to Model Each Projected Curves Individually

# PROPOSED PREDICTION EQUATIONS FOR "DIRECT" CALCULATION - E1

$$\log\left(\frac{4\pi E_1}{3P}w_1\eta\right) = f\left(\log\left(\frac{E_1}{E_2}\right), \frac{h}{a}, \frac{\eta}{h}, \log\left(\frac{E_1}{E_2}\right) * \frac{\eta}{h}, \frac{\eta}{a}\right)$$

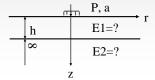
Statistics and Limits:

$$N = 420, R^2 = 0.999, SEE = 0.033, CV = 1.9\%$$

$$1 \le \frac{E_1}{E_2} \le 5000$$
,  $0.8 \le \frac{r_1}{h} \le 6.0$ ,  $0.8 \le \frac{h}{a} \le 5.0$ 

# VALIDATION - A NUMERICAL EXAMPLE

- A Two-Layer System h=10 in, P=3,000 lbs, a=7.69 in NDT Devices, r1=36 in, r3=60 in
- BISAR
   If E1=1.0 Mpsi, E2=5.0 Kpsi Known
   ==> w1=0.00386 in, w3=0.0028 in
- Backcalculation
  - (1) BISDEF Trials
  - (2) Proposed Approach



#### THE PROPOSED APPROACH

(1) Use for "Direct" Calculation
Dimensionless Variables:
 h/a = 1.3, r1/h = 3.6
 r3/h = 6.0, w1r1/w3r3 = 0.827
Use of the Prediction Models:
 ==> log(E1/E2) = 2.28826, E1/E2 = 194.2
 ==> log(Y) = 2.2475, Y = 176.8068

==> E1 = 911,259 psi, E2 = 4,692 psi

### **BISDEF TRIALS**

E1	E2	E1	E2	E1	E2	Withi
Start	Start	Range	Range			n Toler-
Mpsi	Kpsi	Mp	Kpsi	Mpsi	Kps	ance*
0.5	3		1~5	1.61	4.7	Y, N
		0.1~	0		1	
1.61	4.71		1~1	ERR	ER	-
etc.	etc.	0.1~	0		R	
1.1	4.0		1~8	0.98	5.2	Y, N
		0.8~			8	

- \* Absolute Sum of \$\frac{1}{9}\$ Difference
- Change in Modulus Values

### THE PROPOSED APPROACH

(2) Use as a Pre-Processor

Assist in Selection of Initial Trial Values, Data Ranges to Speed Up the Convergence

E1	E2	E1	E2	E1	E2	Withi
Star	t Start	Range	Range			n Toler-
Mps	i Kpsi	Mp	Kpsi	Mpsi	Kps	ance*
		si			i	
0.91	4.69		1~1	0.98	5.1	Y, Y
		0.1~	2		3	

1.2

### **CONCLUSIONS**

- Discussed the "Uniqueness" Problem and Short Comings of Traditional Approach
- Proposed an Alternative Approach Using Database and Modern Regressions
- Identified Dominating Dimensionless Variables for More Complete Coverage
- Strive to Develop Prediction Equations to Allow "DIRECT" Modulus Calculations

### LOOKING AHEAD ...

- Further Improve the Prediction Accuracy
- Investigate a Three- or Four-Layer System
- Must also Assure "Uniqueness" of Solutions
- Possible Use of Local Regression Techniques or Any Database Search Algorithms
- Lots of Research Remain to be Done!

### **CONCLUSIONS** (continue ...)

- Tentative Applications:
  - A Calculator, a Spreadsheet, or a Computer Program for "Direct" Modulus Calculations (Instantly)
  - A Pre-Processor of Traditional Backcalculation Programs
  - In-field Modulus Determinations and NDT Data Checking
- Obtain More Accurate and Consistent Results

