SIMPLIFIED STRESS ANALYSIS PROCEDURES FOR JOINTED CONCRETE PAVEMENTS

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ABSTRACT

This study focused on the development of an alternative stress estimation procedure to instantly calculate the critical stresses of jointed concrete pavements. Thus, the primary components for stress analysis including gear configurations, total wheel load, tire pressure, a widened outer lane, a tied concrete shoulder, and thermal curling due to a linear temperature differential have to be considered. The well-known ILLI-SLAB finite element program was used for the analysis. The program's applicability for stress estimation was further validated by reproducing very favorable .results to the test sections of the Taiwan's North Second Highway. With the incorporation of the principles of dimensional analysis and experimental design, a series of finite element factorial runs over a wide range of pavement designs was carefully selected and conducted. Consequently, prediction equations for stress adjustments were developed using a modern regression technique (Projection Pursuit Regression). Subsequently, a simplified stress analysis procedure was proposed and implemented in a user-friendly computer program (TKUPAV) to facilitate instant stress estimations. Together with PCA's cumulative fatigue damage equation, a modified PCA stress analysis and thickness design procedure was also proposed and incorporated into the TKUPAV program. This computer program will not only instantly perform critical stress calculations, but it may also be utilized for various structural analyses and designs of jointed concrete pavements.

INTRODUCTION

Traditionally, the Westergaard's closed-form stress solutions for a single wheel load acting on the three critical loading conditions (interior, edge, and corner) were often used in various design procedures of jointed concrete pavements. However, the actual pavement conditions are often different from Westergaard's ideal assumptions of infinite or semi-infinite slab size and full contact between the slab-subgrade interface. Besides, the effects of different gear configurations, a widened outer lane, a tied concrete shoulder, a second bonded or unbonded layer may result in very different stress responses from the Westergaard's solutions. These effects may be more accurately and realistically accounted through the use of a finite element (F.E.) computer program. Nevertheless, the difficulties of the required run time, the complexity of F.E. analysis, and the possibility of obtaining incorrect results due to the improper use of the F.E. model often prevent it from being used in practical pavement design. Thus, the main objectives of this study were to develop an alternative procedure to more conveniently calculate

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the critical stresses of jointed concrete pavements with sufficient accuracy for design purposes [*1*].

WESTERGAARD'S CLOSED-FORM SOLUTIONS

In the analysis of a slab-on-grade pavement system, Westergaard has presented closed-form solutions for three primary structural response variables, i.e., slab bending stress, slab deflection, and subgrade stress, due to a single wheel load based on medium-thick plate theory and the assumptions of an infinite or semi-infinite slab over a dense liquid (Winkler) foundation [*2*].

ILLI-SLAB FINITE ELEMENT SOLUTIONS

The basic tool for this analysis is the ILLI-SLAB F.E. computer program which was originally developed in 1977 and has been continuously revised and expanded at the University of Illinois over the years. The ILLI-SLAB model is based on classical medium-thick plate theory, and employs the 4-noded 12-degree-of-freedom plate bending elements. The Winkler foundation assumed by Westergaard is modeled as a uniform, distributed subgrade through an equivalent mass foundation. Curling analysis was not implemented until versions after June 15, 1987. The present version (March 15, 1989) [*3*] was successfully complied on available Unix-based workstations of the Civil Engineering Department at Tamkang University. With some modifications to the original codes, a micro-computer version of the program was also developed using Microsoft FORTRAN PowerStation [*4*].

RESULTS OF ACTUAL FIELD MEASUREMENTS

To further investigate the applicability of the ILLI-SLAB F. E. program for stress estimation, the actual field measurements of the test sections of Taiwan's North Second Highway [*5*] was obtained. The test sections were constructed as jointed concrete pavements with an unbonded lean concrete base and the following characteristics: (Note: 1 in. = 2.54 cm, 1 psi = 0.07 kg/cm^2, 1 pci = 0.028 kg/cm^3, 1 kip = 454 kg)

1. finite slab size: 3-lane (one direction), L = 188 in., W = 148 in.
2. thickness of the top and the bottom layers: h_1 = 10 in., h_2 = 6 in.
3. concrete modulus of the top and the bottom layers: E_1 = 4.03E+06 psi, E_2 = 1.97E+06 psi
4. Poisson's ratio of the top and the bottom layers: $\mu_1 = \mu_2 = 0.20$
5. self-weight of the top and the bottom slabs: $\gamma_1 = \gamma_2 = 0.085$ pci
6. modulus of subgrade reaction: k = 481 pci
7. longitudinal joints: tied bars, spacing = 24 in., diameter = 5/8 in., Poisson's ratio = 0.2, elastic modulus = 2.9E+07 psi.
8. transverse joints: dowel bars, spacing = 12 in., diameter = 1.25 in., Poisson's ratio = 0.2, elastic modulus = 2.9E+07 psi, width of joint opening = 0.236 in., aggregate interlock factor (AGG) = 1000 psi, dowel concrete interaction (DCI) = 1.9E+06 lbs/in. (assumed).
9. with an AC outer shoulder.

A fully loaded truck with three different levels (60.7, 43.1, and 34.3 kips) of rear dual-tandem axle loads was placed near the slab corner. The gear configuration with the size of loaded area, wheel spacing and axle spacing was shown in Figure 1. At the time of testing, a positive temperature differential $\Delta T = 10.8\ ^oF$ was measured across the slab thickness; the slab thermal coefficient α was assumed 5.5E-06 $/^oF$. The resulting horizontal stresses estimated by the ILLI-SLAB program were compared to the actual measured stresses and summarized as follows:

Axle load (kips)	C 70		C 71		C 72	
	Measured (psi)	ILLI-SLAB (psi)	Measured (psi)	ILLI-SLAB (psi)	Measured (psi)	ILLI-SLAB (psi)
60.7	36.5	24.1	74.3	66.1	94.3	92.9
43.1	37.2	23.6	62.9	55.4	67.0	73.0
34.3	37.5	23.3	47.0	49.8	45.7	62.7

Note that since the sensor locations C70 = (176.2 in., 11.8 in.), C71=(176.2 in., 74 in.), and C72=(176.2 in., 136.2 in.) was actually placed 2 in. below the slab surface, the resulting ILLI-SLAB stresses (compressive x-stress) were linearly adjusted (or reduced by 40%) while making such comparisons. Apparently, fairly good agreements were achieved.

IDENTIFICATION OF MECHANISTIC VARIABLES

In reality, jointed concrete pavements consist of many single finite concrete slabs jointed by aggregate interlock, dowel bars, or tie bars. As shown in Figure 2, traffic loading may be in forms of dual wheel, tandem axle, or tridem axle. A widened outer lane may also shift the wheel loading away from Westergaard's critical loading locations. A tied concrete shoulder, a second bonded or unbonded layer may also result in different degrees of stress reductions. To account for these effects under loading only condition, the following relationship has been identified through many intensive F.E. studies for a constant Poisson's ratio (usually $\mu \approx 0.15$) [*1, 6*]:

$$\frac{\sigma h^2}{P}, \frac{\delta k \ell^2}{P}, \frac{q \ell^2}{P} = f\left(\frac{a}{\ell}, \frac{L}{\ell}, \frac{W}{\ell}, \frac{s}{\ell}, \frac{t}{\ell}, \frac{D_0}{\ell}, \frac{AGG}{k\ell}, \left(\frac{h_{efft}}{h_1}\right)^2 \right) \quad \text{(E.1)}$$

Where σ, q are slab bending stress and vertical subgrade stress, respectively, $[FL^{-2}]$; δ is the slab deflection, [L]; P = wheel load, [F]; a = the radius of the applied load, [L]; $\ell=(E*h^3/(12*(1-\mu^2)*K))^{0.25}$ is the radius of relative stiffness of the slab-subgrade system [L]; k = modulus of subgrade reaction, $[FL^{-3}]$; L, W = length and width of the finite slab, [L]; s = transverse wheel spacing, [L]; t = longitudinal axle spacing, [L]; D_0 = offset distance between the outer face of the wheel and the slab edge, [L]; AGG = aggregate interlock factor, $[FL^{-2}]$; $h_{efft} = (h_1^2 + h_2^2 * (E_2*h_2)/(E_1*h_1))^{0.5}$ is the effective thickness of two unbonded layers, [L]; h_1 , h_2 = thickness of the top slab, and the bottom slab, [L]; and E_1 , E_2 = concrete modulus of the top slab, and the bottom slab, $[FL^{-2}]$. Note that variables in both sides of the expression are all dimensionless and primary dimensions are represented by [F] for force and [L] for length.

Since no thermal curling effect was considered in the above relationship, the full contact assumption between the slab-subgrade interface and the principle of superposition may be applied to the analyses. Thus, the above relationship can be broken down to a series of simple analyses for each individual effect. The adjustment factors can be separately developed to account for the effect of stress reduction due to each different loading condition.

Furthermore, the following concise relationship has been identified by Lee and Darter [*7*] for the effects of loading plus thermal curling:

$$\frac{\sigma}{E}, \frac{\delta h}{\ell^2}, \frac{qh}{k\ell^2} = f\left(\frac{a}{\ell}, \alpha\Delta T, \frac{L}{\ell}, \frac{W}{\ell}, \frac{\gamma h^2}{k\ell^2}, \frac{ph}{k\ell^4}\right) \qquad \text{(E.2)}$$

Where α is the thermal expansion coefficient, $[T^{-1}]$; ΔT is the temperature differential through the slab thickness, [T]; γ is the unit weight of the concrete slab, $[FL^{-3}]$; $D_{\gamma}=\gamma*h^2/(k*\ell^2)$; and $D_P=P*h/(k*\ell^4)$. Also note that D_{γ} was defined as the relative deflection stiffness due to self-weight of the concrete slab and the possible loss of subgrade support, whereas D_p was the relative deflection stiffness due to the external wheel load and the loss of subgrade support. The primary dimension for temperature is represented by [T].

DEVELOPMENT OF STRESS PREDICTION MODELS

A series of F. E. factorial runs were performed based on the dominating mechanistic variables (dimensionless) identified. Several BASIC programs were written to automatically generate the F. E. input files and summarize the desired outputs. The F. E. mesh was generated according to the guidelines established in earlier studies [*8*]. As proposed by Lee and Darter [*9*], a two-step modeling approach using the projection pursuit regression (PPR) technique introduced by Friedman and Stuetzle was utilized for the development of prediction models. Through the use of local smoothing techniques, the PPR attempts to model a multi-dimensional response surface as a sum of several nonparametric functions of projections of the explanatory variables. The projected terms are essentially two-dimensional curves which can be graphically represented, easily visualized, and properly formulated. Piece-wise linear or nonlinear regression techniques were then used to obtain the parameter estimates for the specified functional forms of the predictive models. This algorithm is available in the S-PLUS statistical package [*10*].

Proposed Edge Stress Prediction Models

To account for the effects of different material properties, finite slab sizes, gear configurations, and environmental effects (e.g., temperature differentials), the following equation was proposed for edge stress estimations [*1, 7*]:

$$\sigma_e = \sigma_w * R_1 * R_2 * R_3 * R_4 * R_5 + R_T * \sigma_c \qquad \text{(E.3)}$$

$$\sigma_w = \frac{3(1+\nu)P}{\pi(3+\nu)h^2}\left[\ln\frac{Eh^3}{100ka^4} + 1.84 - \frac{4}{3}\nu + \frac{1-\nu}{2} + 1.18(1+2\nu)\frac{a}{\ell}\right]$$

$$\sigma_c = \frac{CE\alpha\Delta T}{2} = \frac{E\alpha\Delta T}{2}\left[1 - \frac{2\cos\lambda\cosh\lambda}{\sin 2\lambda + \sinh 2\lambda}(\tan\lambda + \tanh\lambda)\right]$$

Where:

- σ_e = edge stress prediction, [FL^{-2}];
- σ_w = Westergaard's closed-form edge stress solution, [FL^{-2}];
- σ_c = Westergaard/Bradbury's curling stress, [FL^{-2}];
- E = elastic modulus of the slab, [FL^{-2}];
- h = slab thickness, [L];
- C = the curling stress coefficient ($\lambda = W/((8^{0.5})*\ell)$);
- R_1= adjustment factor for different gear configurations including dual-wheel, tandem axle, and tridem axle;
- R_2 = adjustment factor for finite slab length and width;
- R_3 = adjustment factor for a tied concrete shoulder;
- R_4 = adjustment factor for a widened outer lane;
- R_5 = adjustment factor for a bonded/unbonded second layer; and
- R_T = adjustment factor for the combined effect of loading plus day-time curling.

The proposed prediction models for edge stress adjustments are given in Table 1. More detailed descriptions of the development process can be found in Reference [*1*].

Proposed Corner / Interior Stress Prediction Models

Similar approach was adopted to develop separate prediction models for corner / interior stress adjustments. More detailed information was given in References [*1, 11, 12, 13, 14*].

TKUPAV PROGRAM DEVELOPMENT

The research findings were incorporated into a window-based computer program (TKUPAV), which was developed using the Microsoft Visual Basic software package [*15*], to instantly perform critical stress estimations. The TKUPAV program was designed to be highly user-friendly and thus came with many well-organized graphical interfaces, selection menus, and command buttons for easy use. Together with PCA's cumulative fatigue damage equation, a modified PCA stress analysis and thickness design procedure was also implemented in the TKUPAV program for practical trial applications. Both English and Chinese versions of the program are available. An example input screen of the program was shown in Figure 3.

CONCLUSIONS AND RECOMMENDATIONS

An alternative procedure for the determination of the critical stresses of jointed concrete pavements was developed under this study. The effects of a finite slab size, different gear configurations, a widened outer lane, a tied concrete shoulder, a second bonded or unbonded layer, and thermal curling due to a linear temperature differential were considered. The ILLI-SLAB program's applicability for stress estimation was further validated by reproducing very favorable results to the test sections of the Taiwan's North Second Highway. Based on the dimensionless mechanistic variables identified, prediction equations for stress adjustments were developed using a modern regression technique (Projection Pursuit Regression). Subsequently, a simplified stress analysis procedure was proposed and implemented in a user-friendly computer program (TKUPAV) to facilitate instant stress estimations. Together with PCA's cumulative fatigue damage equation, a modified PCA stress analysis and thickness design procedure was also incorporated in the TKUPAV program. This computer program will not only instantly perform critical stress calculations, but it may also be utilized for various analyses and designs of concrete pavements.

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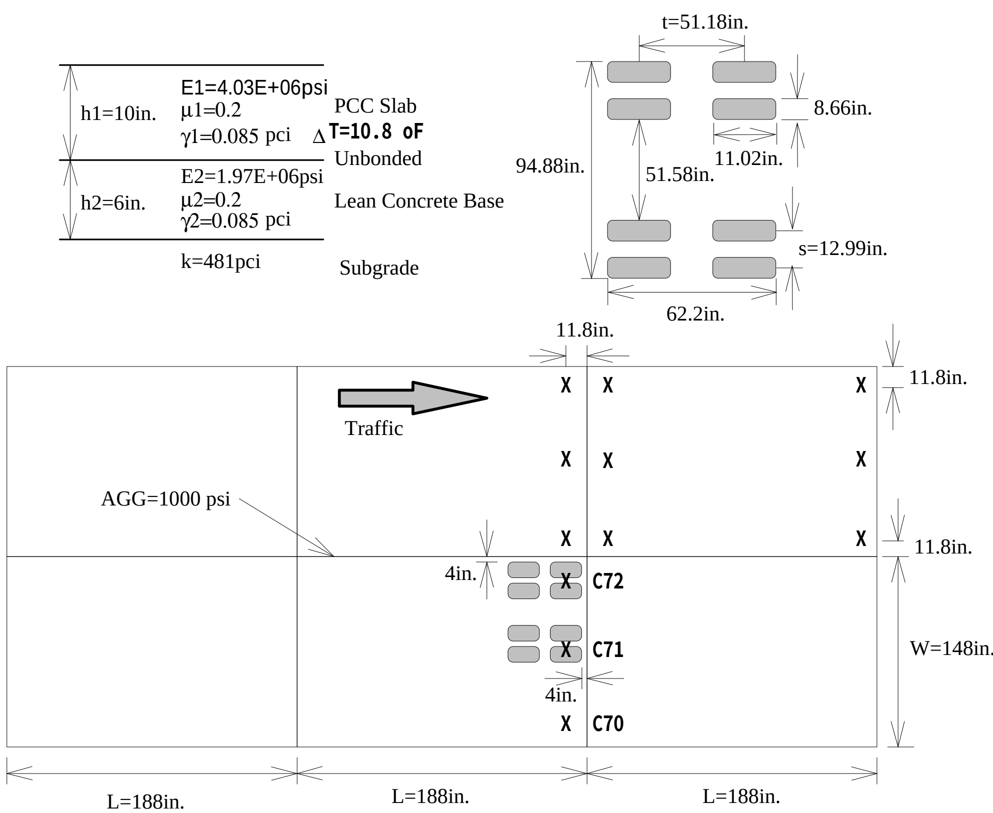

Figure 1 - Test Sections of Taiwan's Second Highway

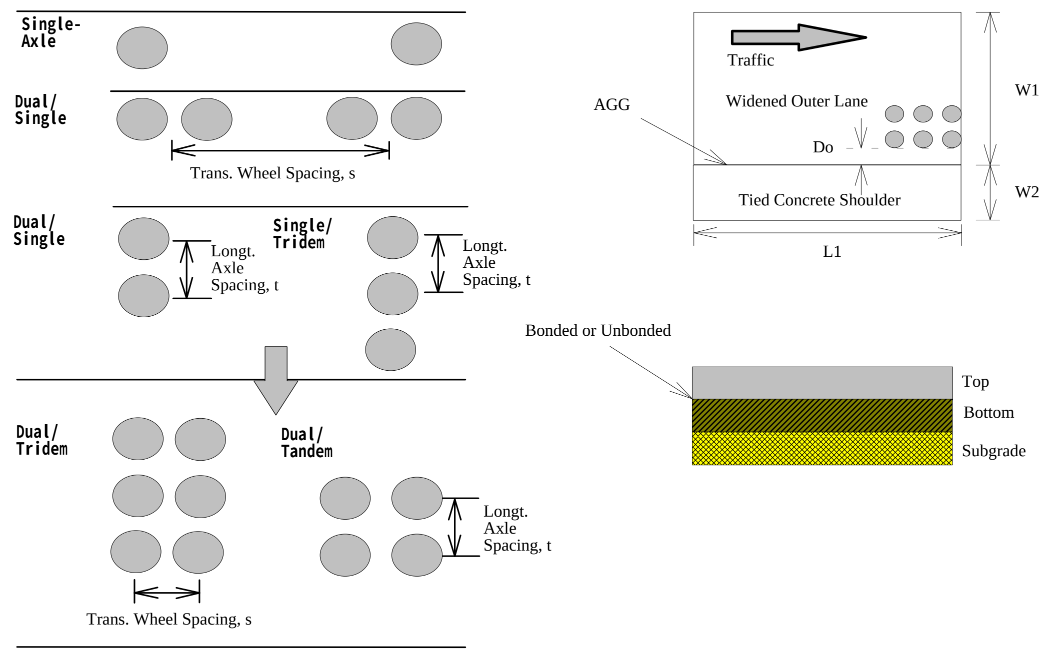

Figure 2 Various Conditions of Jointed Concrete Pavements

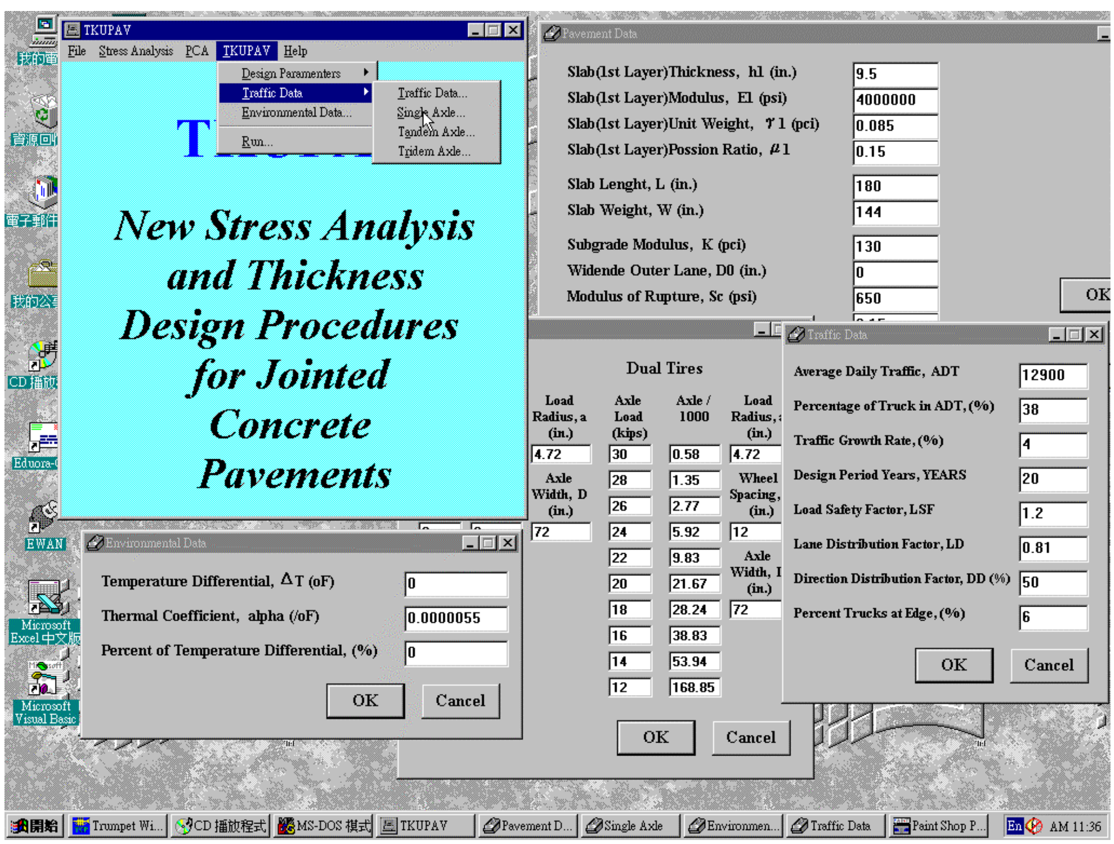

Figure 3 Sample Input Screens of the TKUPAV Program

Table 1 Proposed Prediction Models for Edge Stress Adjustments

Dual Wheel (Single Axle)	$R_1 = 0.56197 + 0.09313\Phi_1 + 0.0065\Phi_2$ $\Phi_1 = \begin{cases} -0.043 + 0.452(A1) + 0.075(A1)^2 & \text{if } A1 \le -2 \\ 2.997 + 6.278(A1) + 4.122(A1)^2 + 0.964(A1)^3 & \text{if } A1 > -2 \end{cases}$ $\Phi_2 = \begin{cases} -1.461 - 4.460(A2) + 392.524(A2)^2 + 2955.995(A2)^3 + 4914.455(A2)^4 & \text{if } A2 \le 0 \\ -1.425 + 45.240(A2) - 309.329(A2)^2 + 832.054(A2)^3 - 765.888(A2)^4 & \text{if } A2 > 0 \end{cases}$ $A1 = -0.7919x1 + 0.60762x2 + 0.06072x3$ $A2 = 0.01799x1 - 0.88168x2 + 0.4715x3$ $X = [x1, x2, x3] = \left[\frac{s}{a}, \frac{a}{a}, \frac{s \times a}{a^2}\right]$	$0.05 \le \frac{a}{a} \le 0.4$ $0 \le \frac{s}{a} \le 4.0$
Tandem Axle (Single Wheel)	$R_1 = 0.58306 + 0.19316\Phi_1 + 0.06236\,\Phi_2$ $\Phi_1 = \begin{cases} 0.159 + 1.604(A1) + 0.820(A1)^2 + 0.135(A1)^3 & \text{if } (A1) \le -1 \\ 1.319 + 4.509(A1) + 1.760(A1)^2 - 0.914(A1)^3 & \text{if } (A1) > -1 \end{cases}$ $\Phi_2 = \begin{cases} 2.151 + 11.020(A2) - 2.894(A2)^2 & \text{if } (A2) \le -0.2 \\ 2.210 + 11.770(A2) - 16.209(A2)^2 - 70.589(A2) & \text{if } (A2) > -0.2 \end{cases}$ $A1 = -0.51308x1 + 0.85264x2 + 0.08604x3 - 0.04849x4$ $A2 = -0.07313x1 - 0.93937x2 + 0.33502x3 + 0.00055x4$ $X = [x1, x2, x3, x4] = \left[\frac{t}{a}, \frac{a}{a}, \frac{t \times a}{a^2}, \frac{t}{a}\right]$	$0.1 \le \frac{a}{a} \le 0.4$ $0 \le \frac{t}{a} \le 1.6$
Tridem Axle (Single Wheel)	$R_1 = 0.44485 + 0.17726\Phi_1 + 0.02072\,\Phi_2$ $\Phi_1 = \begin{cases} 0.230 + 1.078(A1) + 0.177(A1)^2 & \text{if } A1 \le -1 \\ 2.480 + 6.329(A1) + 3.363(A1)^2 & \text{if } A1 > -1 \end{cases}$ $\Phi_2 = \begin{cases} -1.754 + 11.049(A2) + 8.611(A2)^2 & \text{if } A2 \le 0.12 \\ -2.398 + 20.152(A2) - 15.813(A2)^2 & \text{if } A2 > 0.12 \end{cases}$ $A1 = -0.54456x1 + 0.83346x2 - 0.09349x3 - 0.00724x4$ $A2 = 0.05007x1 + 0.87037x2 - 0.48983x3 + 0.00362x4$ $X = [x1, x2, x3, x4] = \left[\frac{t}{a}, \frac{a}{a}, \frac{t \times a}{a^2}, \frac{t}{a}\right]$	$0.05 \le \frac{a}{a} \le 0.4$ $0 \le \frac{t}{a} \le 3$
Finite Slab Length	$R_2 = 0.9399 + 0.07986\Phi_1$ $\Phi_1 = -4.0308 + \frac{1}{0.2029 + 0.0345A1^{-3.3043}}$ $A1 = -0.9436\frac{a}{a} + 0.3310\frac{L}{a}$	$2 \le \frac{L}{a} \le 7$ $0.05 \le \frac{a}{a} \le 0.3$
Finite Slab Width	$R_2 = 1.00477 + 0.01214\Phi_1$ $\Phi_1 = -0.5344 + 1.654(1 - A1)^{-10.7412}$ $A1 = 0.9951\frac{a}{a} - 0.09856\frac{W}{a}$	$2 \le \frac{W}{a} \le 7$ $0.05 \le \frac{a}{a} \le 0.3$
Tied Concrete Shoulder [12]	$R_3 = \begin{cases} 0.99864 - 0.51237(x1) - 0.0672\ln(x2) + 0.00315\ln^2(x2) \\ +0.015936(x1)^2 * \ln^2(x2) \\ 1.04284 - 0.84692(x1) - 0.0009299\ln(x2) + 0.06837(x1)\ln(x2) \\ +0.63417(x1)^2 + 0.0042\ln^2(x2) - 0.000629(x1) * \ln(x2)^3 \quad \text{if } x1 > 5 \end{cases}$ $X = [x1, x2] = \left[\frac{a}{a}, \frac{AGG}{ka}\right]$	$0.05 \le \frac{a}{a} \le 0.3$ $5 \le \frac{AGG}{ka}$

Table 1 Proposed Prediction Models for Edge Stress Adjustments (Continue ...)

Widened Outer Lane	$R_4 = 0.61711 + 0.15373\ \Phi_1 + 0.02504\ \Phi_2$ $\Phi_1 = \begin{cases} 0.693 + 1.279(A1) + 0.369(A1)^2 + 0.037(A1)^3 & \text{if} \quad (A1) \le -2.5 \\ 2.839 + 8.234(A1) + 8.158(A1)^2 + 3.608(A1)^3 + 0.576(A1)^4 & \text{if} \quad (A1) > -2.5 \end{cases}$ $\Phi_2 = \begin{cases} -2.285 + 5.921(A2) - 6.001(A2)^2 + 7.743(A2)^3 & \text{if} \quad (A2) \le 0.5 \\ -3.008 + 4.693(A2) + 4.334(A2)^2 - 2.167(A2)^3 & \text{if} \quad (A2) > 0.5 \end{cases}$ $A1 = -0.98868x1 - 0.12214x2 - 0.08717x3$ $A2 = 0.19802x1 + 0.98019x2 + 0.00305x3$ $X = [x1, x2, x3] = \left[\frac{D_0}{a}, \frac{a}{a}, \frac{D_0}{a} \right]$	$0.1 \le \frac{a}{a} \le 0.4$ $0 \le \frac{D_0}{a} \le 2$
Unbonded Second Layer	$R_5 = 0.72692 + 0.14272\Phi_1 + 0.00933\Phi_2$ $\Phi_1 = \begin{cases} 3.31765 + 2.4036(A1) & \text{if} \quad A1 \le -1.4 \\ 5.72684 + 4.10244(A1) & \text{if} \quad A1 > -1.4 \end{cases}$ $\Phi_2 = \begin{cases} 14.535 - 20.351(A2) + 5.986(A2)^2 & \text{if} \quad A2 \le 1.2 \\ 1.619 - 8.367(A2) + 4.877(A2)^2 & \text{if} \quad A2 > 1.2 \end{cases}$ $A1 = 0.11914x1 - 0.99288x2$ $A2 = 0.65518x1 + 0.75547x2$ $h_{efft} = \sqrt{h_1^2 + \frac{E_2 h_2}{E_1 h_1} h_2^2}$, $X = [x1, x2] = \left[\frac{a}{a}, \left(\frac{h_{efft}}{h1} \right)^2 \right]$	$0.05 \le \frac{a}{a} \le 0.4$ $1 \le \left(\frac{h_{efft}}{h_1} \right)^2 \le 2$
Bonded Second Layer	$\sim = \frac{(1/2)h_1(h_1 + h_2)}{h_1 + h_2(E_1 / E_2)}$, $= (1/2)(h_1 + h_2) - \sim$ $h_{1f} = \sqrt[3]{h_1^3 + 12h_1\ ^2}$, $h_{2f} = \sqrt[3]{h_2^3 + 12h_2\ ^2}$ $hefft = \sqrt{h_{1f}^2 + \left(\frac{h_{2f}}{h_{1f}} \right) h_{2f}^2}$, $X = [x1, x2] = \left[\frac{a}{a}, \left(\frac{h_{efft}}{h_{1f}} \right)^2 \right]$ Use the above unbonded prediction model to calculate R_5	(Same as above)
Load plus Day-time Curling	$R_T = 0.94825 + 0.15054\Phi_1(A1) + 0.03724\Phi_2(A2) + 0.03395\Phi_3(A3)$ $\Phi_1(A1) = \begin{cases} -2.5575 + 0.8003\ (A1) - 0.8003(A1)^2 & \text{if} \quad A1 \le 3 \\ -2.6338 + 1.1038(A1) - 0.0914(A1)^2 & \text{if } 3 < A1 \le 7 \\ 0.7564 - 0.0155(A1) & \text{if} \quad A1 > 7 \end{cases}$ $\Phi_2(ATX2) = \begin{cases} -0.6788 + 0.0.8003(A2) - 0.8003(A2)^2 & \text{if} \quad A2 \le 3 \\ 3.7674 - 2.297(A2) + 0.2963(A2)^2 & \text{if } 3 < A2 \le 7 \\ -7.0337 + 1.2945(A2) & \text{if} \quad A2 > 7 \end{cases}$ $\Phi_3(ATX3) = \begin{cases} 4.0843 + 4.8241\ (A3) & \text{if} \quad A3 \le 3 \\ 0.1815 + 0.0541(A3) - 1.0899(A3)^2 & \text{if } -1 < A3 \le 0.5 \\ 0.0453 + 0.0383(A3) & \text{if} \quad A3 > 0.5 \end{cases}$ $ATX1 = -0.04724X1 + 0.56954 * X2 - 0.08408X3 + 0.20033X4 - 0.26647X5 + 0.00375X6 + 0.73881X7 - 0.01142X8 + 0.0953X9 + 0.01121X10$ $ATX2 = 0.03869X1 + 0.35781X2 + 0.09078X3 - 0.04054X4 + 0.86388X5 + 0.01635X6 - 0.31246X7 + 0.00552X8 - 0.12677X9 - 0.01765X10$ $ATX3 = 0.58567X1 + 0.25804X2 + 0.14784X3 + 0.14984X4 + 0.12743X5 - 0.05012X6 + 0.72295X7 - 0.0131X8 - 0.01304X9 - 0.06591X10$ $X = [x1, x2, x3, ..., x10]$ $= \left[\frac{W}{a}, \frac{L}{a}, ADT, \frac{a}{a}, DG, DP, \frac{L}{a} * \frac{a}{a}, \frac{L}{a} * ADT, DG * \frac{L}{a}, DG * \frac{W}{a} \right]$	$0.05 \le \frac{a}{a} \le 0.3$ $3 \le \frac{W}{a} \le 11$ $\frac{W}{a} = \frac{L}{a}$ $1.06 \le DG \le 9.93$ $2.61 \le DP \le 140.74$ $5.5 \le ADT \le 22$ $DG = d¥ \times 10^5$ $DP = dp \times 10^5$ $ADT = \sim \times \Delta T \times 10^6$