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# PRELIMINARY ANALYSIS ON BACKCALCULATION OF PAVEMENT LAYER MODULI FROM SURFACE DEFLECTION DATA

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## ABSTRACT

Since the elastic moduli of pavement layers which represent the strength of a pavement structure cannot be calculated directly from surface deflection data, they are often obtained using some backcalculation procedures. The main objectives of this study are to deal with major deficiencies of traditional backcalculation procedures such as the uniqueness problem of backcalculated moduli, subjective selection of initial trial values and input data ranges, and sometimes failures to satisfy the specified convergence criteria, etc.

Burmister and Scrivner's deflection equations of a two-layer elastic pavement system were first studied. Through the use of the principles of dimensional analysis, the dominating dimensionless variables were identified and verified. These deflection equations were then inverted for modulus backcalculation using the same mechanistic variables, while assuring the unique one-to-one backcalculation relationship. Factorial BISAR runs were conducted and a new regression technique was utilized to develop prediction equations for modulus backcalculations. This study presents a new window of opportunity to backcalculate layer moduli from surface deflection data directly and instantly. Lots of research remain to be done!

## 1. INTRODUCTION

In recent years, the use of Nondestructive Deflection Testing (NDT) devices has been widely adopted to obtain surface deflection data in order to evaluate existing pavement's conditions. To estimate the elastic modulus of each pavement layer, traditional backcalculation procedures have to repetively calculate theoretical deflection data in order to match the actual surface deflection measurements within the specified error tolerance ranges. However, the selection of initial modulus values, bounds, and error ranges has great impact on the accuracy of backcalculation results. Generally speaking, backcalculation results may not be the same and the number of iterations and calculation time may be greatly influenced if the initial modulus values and ranges are specified differently. Furthermore, if the specified ranges are too small such that actual solutions are not included, the results may reach the specified bounds. Furthermore, the reasonableness of backcalculation results will be in doubt if the specified convergence criteria are not satisfied.

The main objectives of this research study are to focus on the major deficiencies of most traditional backcalculation programs and to provide means of improvements for such

conditions. Starting from multi-layered linear elastic theory for backcalculations, currently there still exist some unresolved problems, such as:

- (1) The moduli backcalculated from surface deflection data may not be unique.
- (2) There exists very few guidelines regarding to the selection of initial trial values and input data ranges which may affect the results of backcalculation dramatically.
- (3) The iterative backcalculation procedures are often very time-consuming and sometimes the specified convergence criteria are not satisfied.

To resolve these problems, this paper proposes a new backcalculation approach which integrates the concept of traditional data base approach and modern regression techniques. This approach strives to develop prediction equations which will allow direct modulus calculations from measured surface deflection data.

## 2. DEVELOPMENT OF TWO-LAYER ELASTIC THEORY

The well-known Boussinesq equations are to solve stress and displacement problems of a concentrated load acting on a single-layered uniform subgrade soil. Through Bessel function expansion of a load function, Burmister derived a surface deflection equation for any arbitrary uniformly distributed loads which is equivalent to a concentrated load acting on a two-layer elastic pavement system (Burmister, 1943). This system is composed by a vertical concentrated load  $P$ , acting on the top of pavement surface  $O$ , which is the origin of the cylindrical coordinates  $r$  and  $z$ , where downward  $z$  is treated as positive. The materials of both layers are assumed to be homogeneous, isotropic, and linear elastic. The modulus of the top layer is  $E_1$ , thickness is  $h$ ; whereas the modulus of the bottom layer is  $E_2$  and its thickness is infinite. The Poisson's ratios of both layers are assumed to be 0.5. The deflection equation is:

$$w = \frac{1.5P}{2\pi E_1} \int_0^\infty \left[ \frac{e^{2mh} + 4Nmh - N^2 e^{-2mh}}{e^{2mh} - 2N(1 + 2m^2 h^2) + N^2 e^{-2mh}} \right] J_0(mr) dm$$

$$N = \frac{E_1 - E_2}{E_1 + E_2} = \frac{1 - E_2 / E_1}{1 + E_2 / E_1} \quad (E.1)$$

Where:

- $P$  = vertical concentrated load acting on the top of pavement surface, [F].
- $h$  = thickness of the top layer, [L].
- $E_1$  = elastic modulus of the top layer, [FL<sup>-2</sup>].
- $E_2$  = elastic modulus of the bottom layer, [FL<sup>-2</sup>].
- $w$  = vertical surface deflection, [L].
- $r, z$  = cylindrical coordinates ( $r$  = horizontal direction,  $z$  = vertical direction), [L].
- $J_0(x)$  = first class of zero order Bessel function.
- $N$  = function of  $E_1$  and  $E_2$ .

where [F] and [L] represent the dimensions of force and length, respectively.

Burmister further derived a surface deflection equation for the center point of a uniformly distributed circular load acting on a two-layer elastic system. Based on the principles of dimensional analysis, the deflection equation can be simplified as follows:

$$w_c = \frac{1.5pa}{E_2} F_w \left( \frac{a}{h}, \frac{E_2}{E_1} \right) = \frac{1.5pa}{E_2} F_w \quad (E.2)$$

Where:

- $w_c$  = vertical surface deflection of the load center, [L].

- $p$  = uniformly distributed vertical pressure acting on the surface, [FL<sup>-2</sup>].  
 $a$  = radius of the circular load, [L].  
 $F_w$  = function of  $a/h$  and  $E_2/E_1$ .

Scriver further analyzed the case of a Dynaflect's load configuration acting on a two-layer pavement-subgrade system (Scriver, et al., 1973). To estimate the elastic moduli of surface layer and subgrade from the measured surface deflection data, Scriver treated everything above subgrade as a single homogeneous material to simplify the pavement as a two-layer elastic system. Since the loaded area is very small, Scriver further treated the above uniformly distributed load as a concentrated load to simplify the mathematics. For a horizontal distance  $r$  away from the origin  $O$ , the following surface deflection  $w$  is a function of  $h$ ,  $P$ ,  $E_1$ , and  $E_2$ :

$$\frac{4\pi E_1}{3P} wr = \int_{x=0}^{\infty} V * J_0(x) dx = F\left(\frac{E_2}{E_1}, \frac{r}{h}\right)$$

$$V = \frac{1 + 4Nme^{-2m} - N^2e^{-4m}}{1 - 2N(1 + 2m^2)e^{-2m} + N^2e^{-4m}} \quad (E.3)$$

Where,  $x = mr/h$  and  $V$  is a function of  $m$  and  $N$ .

For distance  $r_1$  and  $r_3$  away from the loaded center of Dynaflect, the surface deflections are assumed to be  $w_1$  and  $w_3$ , respectively. By substituting them into the above deflection equation and dividing the resulting two equations with each other, one obtains:

$$\frac{w_1 r_1}{w_3 r_3} = \frac{F_1\left(\frac{E_2}{E_1}, \frac{r_1}{h}\right)}{F_3\left(\frac{E_2}{E_1}, \frac{r_3}{h}\right)} = G\left(\frac{E_2}{E_1}, \frac{r_1}{h}, \frac{r_3}{h}\right) \quad (E.4)$$

Where,  $F_1$ ,  $F_3$ , and  $G$  are functions of  $E_2/E_1$ ,  $r_1/h$ , and  $r_3/h$ .

For a specified NDT device (such as Dynaflect), with known  $r_1$ ,  $r_3$ , and surface thickness  $h$ , one can easily find out that  $w_1 r_1 / w_3 r_3$  is a function of the modulus ratio  $E_2/E_1$  alone from the above equation.

### 3. LIMITATIONS OF BACKCALCULATION PROGRAMS

The fundamental principles of backcalculation procedures are based on pavement theories such as the multi-layer elastic theory and plate theory. The most often used multi-layer elastic theory was simplified using Odemark's equivalent thickness assumptions (Odemark, 1949). Basically, materials are assumed to be homogeneous, isotropic, and linear elastic, even though they are often far from reality. Thus, it is necessary to consider very carefully the basic assumptions and limitations of the analytical model for modulus backcalculation.

A backcalculation procedure often assumes there exists a unique combination of elastic moduli which will result in the same measured deflection data as those calculated from pavement theory, when a dynamic loading is acted on a pavement system. Thus, if the thickness of each pavement layer, load configuration, loaded area, and Poisson's ratios are known, a specific set of layer moduli may be chosen to calculate their corresponding theoretical deflections from pavement theory and compare them to the measured ones. If

their differences are not within the specified error tolerance, it is necessary to choose a new set of moduli and repeat previous process again until such condition is met. The resulting final set of layer moduli represents the strength of the pavement system. Thus, there exist unlimited sets of layer moduli which may satisfy the specified error tolerance criteria for a particular set of measured deflection data.

Besides, the elastic moduli backcalculated from measured surface deflection data may not be unique in theory. Using Scriver's study as an example, Scriver specifically developed curves and databases for modulus backcalculation using Dynaflect's specifications where the radius of loaded area  $a$  and sensor locations  $r_1$  and  $r_3$  are fixed. From the curves of pavement thickness plotted as a function of  $w_1 r_1 / w_3 r_3$  and the modulus ratio  $E_1 / E_2$  (as shown in Figure 1), Scriver further divided this figure into four quadrants based on lines of  $w_1 r_1 / w_3 r_3 = 1$  and  $h = 11.2$  in. Thus, it can be easily found that there exists a unique solution for those two quadrants with thickness  $h$  greater than 11.2 in. However, there may be two or no solutions for the other two quadrants with thickness  $h$  less than 11.2 in. Nevertheless, this theoretical limitation is often overlooked by most traditional backcalculation programs.

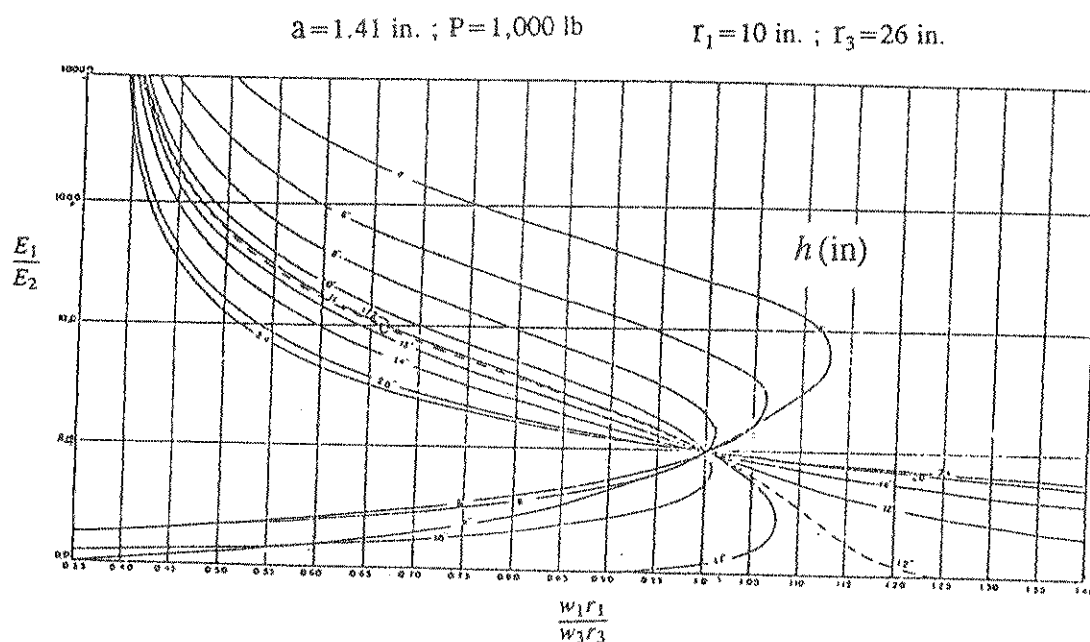


Figure 1 Scriver's Curves for Modulus Backcalculations (Scriver, et. al, 1973)

Furthermore, different specified error tolerance, initial trial modulus values and ranges may all affect the results of backcalculation in different way. Thus, efforts made in proper use of analytical models, reduction of the need to arbitrarily guess input modulus values, and calibration of nonlinear behavior of materials will all contribute to assure the reasonableness of backcalculation results.

#### 4. IDENTIFICATION OF DOMINATING MECHANISTIC VARIABLES

Since Scriver's curves are only applicable to a fixed Dynaflect's loading configuration, it is very desirable to find more general solutions for different commonly-used deflection measuring devices such as Road Raters or Falling Weight Deflectometers. Thus, additional parameters  $a$ ,  $r_1$ , and  $r_3$  have to be simultaneously considered in a backcalculation procedure.

Thus, the following functions relating all previously discussed dimensionless mechanistic variables may be derived based on Burmister and Scrivner's equations using the principles of dimensional analysis:

$$Y = \frac{4\pi E_1}{3P} w_1 r_1 = F\left(\frac{E_2}{E_1}, \frac{r_1}{h}, \frac{h}{a}\right) \quad (\text{E.5})$$

$$\frac{w_1 r_1}{w_3 r_3} = F\left(\frac{E_2}{E_1}, \frac{r_1}{h}, \frac{r_3}{h}, \frac{h}{a}\right) \quad (\text{E.6})$$

The above relationships were numerically validated through a series of BISAR runs (*Shell Oil Co., 1978*). Briefly speaking, the values of  $w_1 r_1 / w_3 r_3$  were confirmed to be unaffected by any changes in  $E_1$ ,  $E_2$ ,  $r_1$ ,  $r_3$ ,  $h$ , and  $a$  values as long as the aforementioned four dimensionless ratios remain constant. More detailed information about this validation process can be found in literature (*Chen, 1994*).

## 5. DEVELOPMENT OF A BACKCALCULATION DATABASE

A backcalculation database was created by selecting proper ranges of dimensionless variables  $E_1 / E_2$ ,  $r_1 / h$ ,  $r_3 / h$ , and  $h / a$  in equation (E.6). Having those four parameters fixed as constants, the values of  $w_1 r_1 / w_3 r_3$  will not be affected by any changes in other input parameters. The ranges of  $E_1 / E_2$  were selected based on the possible modulus values of surface and subgrade materials, whereas the load radii and sensor locations of most popular nondestructive testing devices, and surface thickness are considered to determine the ranges of  $r_1 / h$ ,  $r_3 / h$ , and  $h / a$  for a broader representation of practical pavement systems.

Assuming  $P = 2,400$  lbs,  $E_2 = 1,000$  psi, and  $h = 10$  in., the ranges of four dimensionless variables  $E_1 / E_2$ ,  $r_1 / h$ ,  $r_3 / h$ , and  $h / a$  were selected as follows: (where  $r_1 > r_3$  in order to avoid unnecessary calculations such that a total of 1680 sets of data were obtained.)

$$E_1 / E_2 = 1, 2, 5, 10, 20, 50, 100, 200, 500, 1000, 2000, 5000$$

$$r_1 / h = 0.8, 1.2, 1.8, 2.4, 3.6, 4.8, 6.0$$

$$r_3 / h = 1.2, 1.8, 2.4, 3.6, 4.8, 6.0, 7.2$$

$$h / a = 0.8, 1.3, 2.5, 3.5, 5.0$$

FORTRAN programs were written to automatically generate the input files for a batch of BISAR runs (*Shell Oil Co., 1978*) based on the above parameters. The resulting deflection data were also directly retrieved from the BISAR outputs. Thus, a backcalculation database containing all pertinent input variables along with the deflection data was created for analysis.

## 6. DEVELOPMENT OF PREDICTION MODELS

### 6.1 Application of Projection Pursuit Regression Technique

Projection Pursuit Regression (PPR) techniques introduced by Friedman and Stuetzle (*1981*) strives to model the response surface ( $y$ 's) as a sum of nonparametric functions of projections of the predictor variables ( $x$ 's) through the use of local smoothing techniques. Assuming there exists a true model:

$$y = \bar{y} + \sum_{m=1}^{M_0} \beta_m \phi_m(a_m^T x) + \varepsilon \quad (\text{E.7})$$

Where  $x = (x_1, x_2, \dots, x_p)^T$  denotes the vector of predictor variables,  $\bar{y}$  is the expected (or mean) value of response variable,  $\beta_m$  is the regression coefficient, and  $\epsilon$  is the residual or random error. The PPR algorithm strives to minimize the mean squared residuals over all possible combinations of  $\beta_m$ ,  $\phi_m$ , and  $\alpha_m$  values. Conceptually, the explanatory variables  $x$ 's are projected onto the direction vectors  $a_1, a_2, \dots, a_m$ , to get the lengths of the projections  $\alpha_m^T x$ , where  $m = 1, \dots, M_0$ . An optimization technique is also used to find the best combinations of nonlinear transformations  $\phi_1, \phi_2, \dots, \phi_m$  for the multidimensional response surface.  $\phi_m(\alpha_m^T x)$  stands for the unknown nonparametric transformation functions of the projected lengths  $\alpha_m^T x$  to be estimated.

This new regression technique is available in the S-PLUS statistical package (*Statistical Sciences, Inc., 1993*). The two-step modeling approach proposed by Lee and Darter (1993) was adopted to find the best fit of the response surface using proper functional forms.

## 6.2 Development of Backcalculation Prediction Equations

Based on the aforementioned two-layer elastic theory and the results of dimensional analysis, the solutions of  $E_1/E_2$  may not be unique if one tries to derive its relation with  $w_1 r_1 / w_3 r_3$ ,  $r_1/h$ ,  $r_3/h$ , and  $h/a$  using equation (E.6). In other words, the following equation may have only one, two, or even no solutions.

$$\frac{E_2}{E_1} = G\left(\frac{w_1 r_1}{w_3 r_3}, \frac{r_1}{h}, \frac{r_3}{h}, \frac{h}{a}\right) \quad (\text{E.8})$$

Thus, Figure 1 may be sub-divided into four regions using lines of  $E_1/E_2 = 1$  and  $w_1 r_1 / w_3 r_3 = 1$  for this analysis. To illustrate the possibility of developing prediction equations for modulus backcalculation, this paper only limited to the subregion of  $E_1/E_2 > 1$  and  $w_1 r_1 / w_3 r_3 \leq 1$ , where unique one-to-one relationship was guaranteed. Note that these four subregions are different from Scrivner's original four quadrants. Subsequently, nonlinear prediction equations were developed to approximate this five dimensional response surface using the proposed two-step modeling technique.

Since the selected ranges of  $E_1/E_2$  were very large, it was decided to take a logarithm transformation of this variable to minimize difficulties which may occur in the subsequent regression analysis. After considerable amounts of PPR trials, the following prediction equations together with summary regression statistics were proposed for this sub-region:

$$\log_{10}\left(\frac{E_1}{E_2}\right) = 2.283 + 0.948 \Phi_1 + 0.514 \Phi_2 + 0.253 \Phi_3$$

$$\Phi_1 = \begin{cases} 0.864 + 16.373 (A1) + 56.154 (A1)^2 + 101.2 (A1)^3 + 65.23 (A1)^4 & \text{if } A1 \leq -0.05 \\ 1.637 + 72.73 (A1) + 2056.2 (A1)^2 + 35709.4 (A1)^3 + 242630.6 (A1)^4 & \text{if } -0.05 < A1 \end{cases}$$

$$\Phi_2 = \begin{cases} -2.006 + 1.388 (A2) - 0.146 (A2)^2 + 0.083 (A2)^3 - 0.024 (A2)^4 & \text{if } A2 \leq 3.0 \\ 10.388 - 9.964 (A2) + 3.759 (A2)^2 - 0.590 (A2)^3 + 0.034 (A2)^4 & \text{if } 3.0 < A2 \end{cases}$$

$$\Phi_3 = \begin{cases} 20.760 + 46.171 (A3) + 40.127 (A3)^2 + 15.617 (A3)^3 + 2.224 (A3)^4 & \text{if } A3 \leq -1.5 \\ 0.488 - 1.031 (A3) - 0.563 (A3)^2 + 0.206 (A3)^3 + 0.053 (A3)^4 & \text{if } -1.5 < A3 \leq 0 \\ 0.462 - 0.841 (A3) - 5.209 (A3)^2 + 4.505 (A3)^3 - 1.552 (A3)^4 & \text{if } 0 < A3 \end{cases}$$

$$\begin{aligned}
A1 &= -0.699x_1 + 0.00046x_2 - 0.00059x_3 + 0.00151x_4 + 0.715x_5 - 0.00013x_6 - 0.00003x_7 \\
A2 &= -0.419x_1 - 0.0864x_2 + 0.813x_3 + 0.167x_4 - 0.355x_5 - 0.0420x_6 + 0.0298x_7 \\
A3 &= 0.681x_1 - 0.0998x_2 + 0.383x_3 - 0.307x_4 + 0.534x_5 + 0.0153x_6 - 0.00084x_7 \\
X &= [x_1, x_2, \dots, x_7] = \left[ \frac{w_1 r_1}{w_3 r_3}, \frac{h}{a}, \frac{r_1}{h}, \frac{r_3}{h}, \frac{r_1 * h}{h r_3}, \frac{r_1 * h}{h a}, \frac{r_3 * h}{h a} \right] \quad (E.9)
\end{aligned}$$

Statistics:  $N=1247$ ,  $R^2=0.995$ ,  $SEE=0.0645$ ,  $CV=2.8\%$   
Limits:  $1 \leq E_1/E_2 \leq 5000$ ,  $0.8 \leq r_1/h \leq 6$ ,  $1.2 \leq r_3/h \leq 7.2$ ,  
 $0.8 \leq h/a \leq 5.0$ ,  $w_1 r_1 / w_3 r_3 \leq 1$ ,  $r_1 > r_3$

Where,  $N$  is the number of observations,  $R^2$  is the coefficient of determination,  $SEE$  is the standard error of estimates, and  $CV$  is the coefficient of variation. Note that it is also possible to increase the degree of accuracy (or the coefficient of determination  $R^2$  approaching to 1.0) by increasing the number of variables and projected terms, but the number of parameter estimates and complexity of the model will also be increased undesirably. A balance point between the accuracy and complexity of the model has to be selected when developing these equations.

Similarly, an additional prediction model was developed to best estimate the modulus of the first pavement layer using equation (E.5) for ease of calculation. The proposed PPR model is summarized as follows:

$$\begin{aligned}
\log_{10}(Y) &= \log_{10} \left( \frac{4\pi E_1}{3P} w_1 r_1 \right) = 1.677 + 1.035 \Phi_1 + 0.0788 \Phi_2 \\
\Phi_1 &= \begin{cases} -1.625 + 0.801(A1) + 0.021(A1)^2 - 0.017(A1)^3 + 0.001(A1)^4 & \text{if } A1 \leq 2.5 \\ -2.654 + 2.017(A1) - 0.508(A1)^2 + 0.083(A1)^3 - 0.005(A1)^4 & \text{if } 2.5 < A1 \end{cases} \\
\Phi_2 &= \begin{cases} 0.301 + 1.701(A2) - 0.023(A2)^2 - 0.014(A2)^3 - 0.003(A2)^4 & \text{if } A2 \leq 0 \\ 0.295 + 1.149(A2) - 0.225(A2)^2 - 0.262(A2)^3 + 0.078(A2)^4 & \text{if } 0 < A2 \leq 2.5 \\ 1.845 - 0.316(A2) - 0.079(A2)^2 + 0.012(A2)^3 - 0.001(A2)^4 & \text{if } 2.5 < A2 \end{cases} \\
A1 &= 0.998x_1 + 0.0130x_2 - 0.0147x_3 + 0.0633x_4 - 0.00202x_5 \\
A2 &= -0.827x_1 - 0.107x_2 + 0.472x_3 + 0.284x_4 + 0.0303x_5 \\
X &= [x_1, x_2, \dots, x_5] = \left[ \log_{10} \left( \frac{E_1}{E_2} \right), \frac{h}{a}, \frac{r_1}{h}, \log_{10} \left( \frac{E_1}{E_2} \right) * \frac{r_1}{h}, \frac{h * r_1}{a * h} \right] \quad (E.10)
\end{aligned}$$

Statistics:  $N=420$ ,  $R^2=0.9989$ ,  $SEE=0.03266$ ,  $CV=1.95\%$   
Limits:  $1 \leq E_1/E_2 \leq 5000$ ,  $0.8 \leq r_1/h \leq 6$ ,  $0.8 \leq h/a \leq 5.0$

Thus, the approximate modulus ratio of  $E_1/E_2$  can be estimated using equation (E.9). By substituting the obtained  $E_1/E_2$  ratio into equation (E.10), the modulus of the top layer  $E_1$  can be easily estimated. Thus, the elastic modulus of the bottom layer  $E_2$  can be directly calculated from the equation  $E_2 = E_1 * (E_2/E_1)$ .

## 7. VALIDATION OF THE PROPOSED PREDICTION EQUATIONS

Suppose there exists a two-layer elastic system with a surface thickness  $h = 10$  in., a uniform circular wheel load  $P = 3,000$  lbs, and the radius of the circular load  $a = 7.69$  in. The NDT deflection measurements are located at  $r_1 = 36$  in. and  $r_3 = 60$  in. If the elastic moduli of the top and bottom layers ( $E_1 = 1,000,000$  psi and  $E_2 = 5,000$  psi) are known, the



corresponding theoretical BISAR deflections can then be obtained as  $w_1 = 0.00386$  in. and  $w_3 = 0.0028$  in.

Now, consider a backcalculation problem if one assumes  $w_1$  and  $w_3$  are known, and  $E_1$  and  $E_2$  are the unknown moduli to be calculated in the above case. The dimensionless mechanistic variables are first calculated as:  $h/a = 1.3$ ,  $r_1/h = 3.6$ ,  $r_3/h = 6.0$ , and  $w_1 r_1 / w_3 r_3 = 0.827$ . Based on the proposed equation (E.9), one can get  $A1 = -0.14267$ ,  $A2 = 3.28976$ ,  $A3 = 0.35660$ ,  $\phi_1 = -0.59584$ ,  $\phi_2 = 1.26694$ , and  $\phi_3 = -0.32111$ . Thus,  $\log_{10}(E_1 / E_2) = 2.28826$  or  $E_1 / E_2 = 194.20$  can be calculated instantly. By substituting  $E_1 / E_2 = 194.20$  into the proposed equation (E.10), one can get  $A1 = 2.75956$ ,  $A2 = 2.15223$ ,  $\phi_1 = 0.49000$ , and  $\phi_2 = 0.80058$ . Therefore,  $\log_{10}(Y) = 2.2475$ , or  $Y = 176.8068$  can be easily obtained. Finally, the elastic moduli of the top and bottom layers are calculated as  $E_1 = 911,259$  psi and  $E_2 = 4,692$  psi, respectively.

In summary, these proposed prediction equations not only turned out to be very accurate representations of the modulus backcalculation relationships, but they can also be obtained instantly. This obviously provides considerable improvements in terms of efficiency and accuracy to traditional backcalculation programs, such as BISDEF program (Bush, 1985).

## 8. CONCLUSIONS

This paper first studies Burmister and Scrivner's deflection equations of a two-layer elastic pavement system. Through the use of the principles of dimensional analysis, the dominating dimensionless parameters (or mechanistic variables) were identified and numerically verified by a series of BISAR program runs. Secondly, this deflection function was converted to a new functional form for backcalculation of elastic moduli using the same dimensionless parameters. Main efforts were also placed to assure the unique one-to-one relationship of the above function. Factorial BISAR runs were performed to develop a backcalculation database based on some selected ranges of these dimensionless parameters. This study also presents a new backcalculation procedure by developing prediction equations using a new regression technique to calculate layer moduli from surface deflection data directly and instantly.

Apparently, this new procedure can offer numerous practical applications for modulus backcalculations. The procedure by itself alone can be the main body of a new backcalculation program with known prediction accuracy. In addition, the procedure can be used as a tool to assist the selection of initial trial values, input data ranges to speed up the convergence process when existing iterative backcalculation programs are used. Furthermore, the layer moduli of an existing pavement system can be calculated promptly at the same time when NDT testings are conducted using this procedure. Thus, possible measurement errors when collecting surface deflection data at different locations can be minimized or promptly adjusted in fields. Consequently, the backcalculated modulus values will be more consistent and accurate.

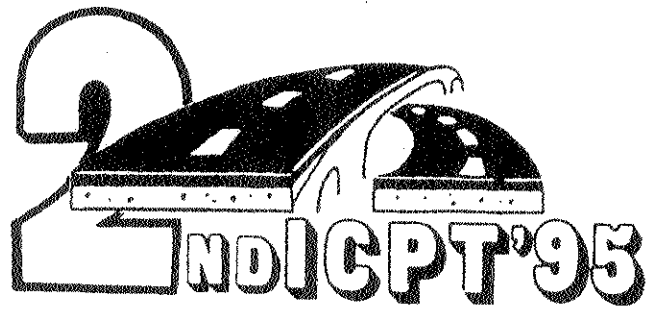
Continuing research efforts to provide more complete coverage of a pavement system are currently underway. For a three-layer or a four-layer pavement system, the dominating dimensionless parameters may be obtained using the principles of dimensional analysis. The accuracy of prediction equations may also be improved while minimizing the complexity. There are lots of research remain to be done!

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