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Mechanistic Analysis of a Slab Track System and Its Applications

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Introduction

- Slab Track Systems More Structural Capacity and Economic Benefits
- Slab Track Similar to Rigid Pavements
- 3D Finite Element not Easily Implemented
- The Gaps Between Theoretical Solutions & Finite Element

Research Objectives/Approach

- Investigate the Theoretical Discrepancies
- Provide Mesh Fineness & Element Selection Guidelines
- Develop Adjustment Factors
- Develop Automated Analysis Procedures
- To Account for Various Practical Track Conditions More Realistically

Continuously and Elastically Supported Beam System

$$p = uy$$

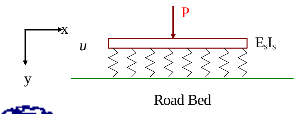
$$E_s I_s \frac{d^4 y}{dx^4} = p$$

$$M = -E_s I_s \frac{d^2 y}{dx^2}$$

$$y(x) = \frac{P}{8 E_s I_s \lambda^3} \varphi_1(\lambda x)$$

$$M(x) = -E_s I_s \frac{d^2 y}{dx^2} = \frac{P}{4\lambda} \varphi_3(\lambda x)$$

$$\lambda = \sqrt[4]{u/(4 E_s I_s)}$$



Zimmerman Functions

$$\varphi_1(\lambda x) = e^{-\lambda x}(\cos \lambda x + \sin \lambda x)$$

$$\varphi_3(\lambda x) = e^{-\lambda x}(\cos \lambda x - \sin \lambda x)$$

Two Continuous Beams on Elastic Foundations Theory

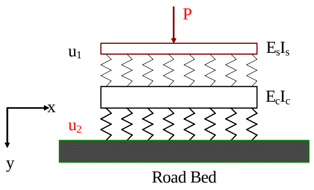
- Formulation of a Slab Track System

$$p_2 = -u_2 y_2$$

$$p_1 = -u_1(y_1 - y_2)$$

$$E_s I_s \frac{d^4 y_1}{dx^4} = p_1$$

$$E_c I_c \frac{d^4 y_2}{dx^4} + p_1 = p_2$$



Theoretical Solutions

$$y_1 = \left[\frac{\varepsilon}{\kappa^3}\varphi_1(\kappa x) - \frac{1}{\omega^3}\varphi_1(\omega x)\right]\cdot \rho^*$$

$$y_2 = \eta\left[\frac{1}{\kappa^3}\varphi_1(\kappa x) - \frac{1}{\omega^3}\varphi_1(\omega x)\right]\cdot \rho^*$$

$$M_1 = -2E_s I_s \cdot \left[-\frac{\varepsilon}{\kappa}\varphi_3(\kappa x) + \frac{1}{\omega}\varphi_3(\omega x)\right]\cdot \rho^*$$

$$M_2 = -2E_c I_c \eta \cdot \left[-\frac{1}{\kappa}\varphi_3(\kappa x) + \frac{1}{\omega}\varphi_3(\omega x)\right]\cdot \rho^*$$

Parameter Identifications (1/2)

- Radius of Relative Stiffness of Rigid Pavement $\ell = \sqrt[4]{\frac{E_c h_c}{12(1-v^2)}}$
- Radius of Relative Stiffness of :
  - The Rail and Track Support, ℓ_r
  - The Concrete Slab and Track, ℓ_{rk}
  - Concrete Slab and Slab Support, ℓ_k

$$\ell_r = \sqrt[4]{\frac{E_s I_s}{u_1}} \quad , \quad \ell_{rk} = \sqrt[4]{\frac{E_c I_c}{u_1}} \quad , \quad \ell_k = \sqrt[4]{\frac{E_c I_c}{u_2}}$$

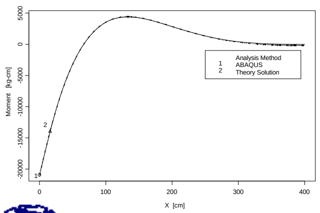
Parameter Identifications (2/2)

- The Principles of Dimensional Analysis
- Parameters Identifications & Verifications
  - Dimensionless Variables:
    - Normalized Deflection Parameters
    - Normalized Moment Parameters

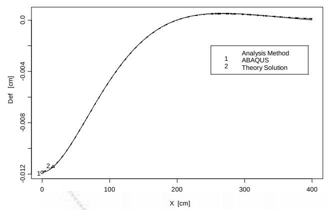
$$\frac{yE_s I_s}{P\ell_r^{\,3}}, \frac{M}{P\ell_r} = f_1\left(\frac{x}{\ell_r}\right) \qquad \frac{\delta E_s I_s}{P\ell_r^{\,3}}, \frac{M}{P\ell_r} = f_2\left(\frac{\ell_{rk}}{\ell_r}, \frac{\ell_k}{\ell_r}\right)$$

Two Dimensional Model Building and Verifications

- Comparison of ABAQUS solutions with closed-form solutions



Rail bending moment



Rail deflection

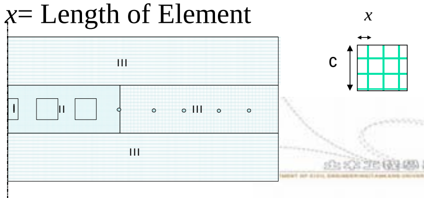
Rail: Beam Elements, Foundation: Spring Elements

Three Dimensional Model Building & Convergence Study

- Systematic Approach
  - Visual Basic & FORTRAN Programs
  - Construct FEM Models, Input Data, Batch Runs, & Summarize the Results Automatically.
- Mesh Fineness Study
- Element Selections Study
- Model Building
- Accuracy & Efficiency

Mesh Fineness Study

- Vertical Mesh Fineness: 3 Layer
- Horizontal Mesh Fineness
  - Mesh Fineness=c/*x*
  - c=Length of The Loaded Area
  - *x*= Length of Element



Results of Element Selections Study

- Rail: B31
- Fastening Systems: JOINTC & RB3D2
- Concrete Slab:
  - Using Shell Element: S8R5
  - Using Solid Element: C3D20

Verification of Dominating Mechanistic Variables

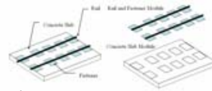
- Continuously and Elastically Supported Beam

$$\frac{yE_sI_s}{P\ell_r^{3}}, \frac{M}{P\ell_r} = f_1\left(\frac{x}{\ell_r}, \frac{L}{\ell_r}\right)$$

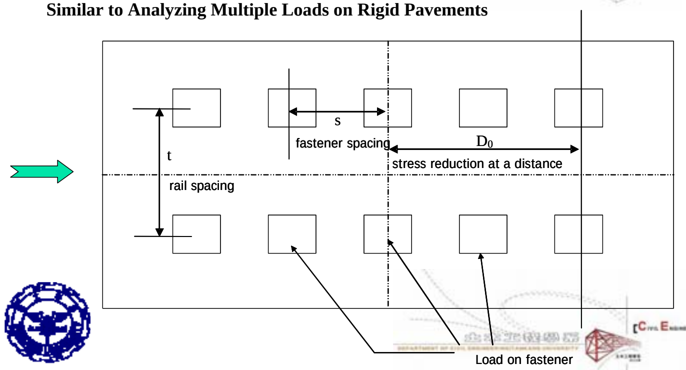
- Two Continuous Beams on Elastic Foundations

$$\frac{\delta E_sI_s}{P\ell_r^{3}}, \frac{M}{P\ell_r} = f_3\left(\frac{a}{\ell_r}, \frac{\ell_{rk}}{\ell_r}, \frac{\ell_k}{\ell_r}, \frac{L}{\ell_r}\right)$$

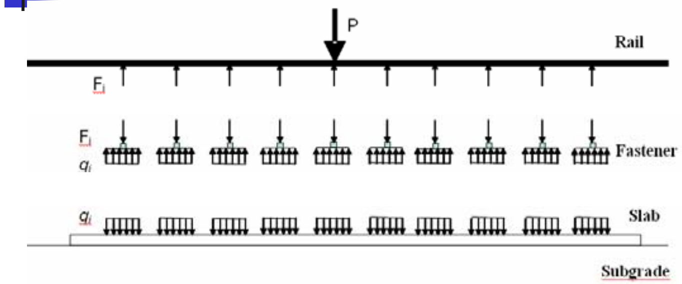
Substructures of a Slab Track System



Similar to Analyzing Multiple Loads on Rigid Pavements



Concept of Free Body Diagram



Alternative Stress Analysis Procedure

- Stress Prediction Models
  - Dimensionless Parameters
  - Databases Generated from Factorial Runs of ABAQUS 3-D FEM Models
  - Projection Pursuit Regression (S-Plus)
- Elastic Track Theory (Rail & Fastening Systems)
- Plate Theory (Concrete Slab)

Stress Analysis Procedure (1/2)

1. Input Load P
2. Convert ℓ_{rk}/ℓ_r, ℓ_{rk}/ℓ_r, s/ℓ_r, x/ℓ_r
3. Determine Max. Reaction Force $F_0 = P*R_{F_0}$

   $$R_{F_0} = f_1(\ell_{rk}/\ell_r, \ell_k/\ell_r, s/\ell_r)$$

4. Calculate Each Reaction Force $F_i = F_0 * R_{F_i} * R_{D_a}$

   $$R_{F_i} = f_2(x/\ell_r) \qquad R_{D_a} = f_3(D_a/\ell_r)$$

Stress Analysis Procedure (2/2)

5. Convert $F_i \rightarrow q_i \rightarrow \sigma_{wi}$
6. Determine Critical Slab Stress

   $$\sigma_i = \sum_{i=0}^{n}(\sigma_{wi} * R_{D_0}) \qquad R_{D_0} = f_3(a/\ell, D_0/\ell)$$

7. Make Critical Stress Adjustments

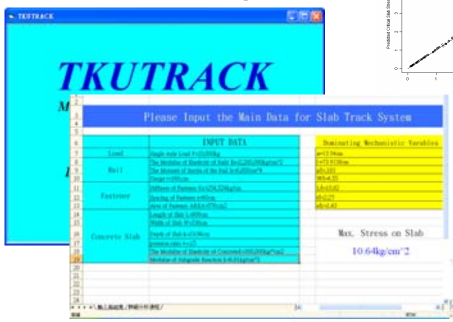
   $$\sigma_{FEM} = \sigma_i * R_{LW} * R_t$$

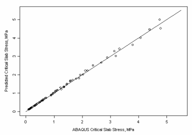
   $$R_{LW} = f_4(a/\ell, L_c/\ell, W_c/\ell)$$

   $$R_t = f_5(a/\ell, t/\ell)$$

Implementation and Verification

- TKUTRACK Program





Conclusions (1/2)

- Bridge the Gap
  - Theoretical Solutions & FEM Analyses
- Principles of Dimensional Analysis
  - Identified & Verified Mechanistic Variables
- Systematic Approach
  - Optimum Mesh Fineness
  - Appropriate Elements Selections
  - Building Effective Model

Conclusions (2/2)

- Using the Concept of Free Body Diagram
  - Substructures of a Slab Track System
  - Elastic Track Theory & Plate theory
- Alternative Stress Analysis Procedure
  - Dimensionless Parameters
  - Stress Prediction Models
  - Rigid Pavement Study
- TKUTRACK

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