

# Mechanistic analysis of a slab track system and its applications

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**ABSTRACT:** The idealized theoretical solutions of slab tracks were first investigated. Together with the principles of dimensional analysis, several dominating mechanistic variables were identified and numerically verified. A systematic approach was utilized and implemented in a Visual Basic software package to study the effects of mesh fineness and element selections using the ABAQUS finite element program. The track-slab system was separately analyzed using the concept of free body diagram. Various prediction models were developed using projection pursuit regression techniques. An alternative stress analysis procedure was proposed and implemented in an EXCEL spreadsheet file (TKUTRACK) using the Visual Basic for Applications software for future routine slab track analyses.

## 1 INTRODUCTION

The design of slab tracks is similar to that of rigid pavements except that “the loads are applied to the rails connected directly to the concrete slab or through rubber booted block ties.” A review of the state-of-the-art procedures in track analysis was first conducted. With the introduction of three-dimensional (3D) ABAQUS (Hibbitt et al. 2000) finite element model (FEM) and all the promising features and results reported in the literature, its applications on pavement/rail engineering become inevitable. However, due to the required running-time and complexity, 3D FEM analysis cannot be easily implemented as a part of design or structural evaluation procedure. This study strives to investigate the theoretical discrepancies, provide mesh fineness and element selection guidelines, develop adjustment factors and automated analysis procedures to account for various practical track conditions more realistically (Yen 2004).

## 2 THEORETICAL SOLUTIONS AND PARAMETER IDENTIFICATIONS

Based on the elastic beam theory (Hay 1982) and the theory of two continuous beams on elastic foundations (Huang & Cheng 1993), the following concise relationships are subsequently re-derived and re-defined using the principles of dimensional analysis (Yen 2004):

$$\frac{yE_s I_s}{P\ell_r^3}, \frac{M}{P\ell_r} = f_1\left(\frac{x}{\ell_r}\right) \quad (1)$$

$$\frac{\delta E_s I_s}{P\ell_r^3}, \frac{M}{P\ell_r} = f_2\left(\frac{\ell_{rk}}{\ell_r}, \frac{\ell_k}{\ell_r}\right) \quad (2)$$

$$\ell_r = \sqrt[4]{\frac{E_s I_s}{u_1}}, \quad \ell_{rk} = \sqrt[4]{\frac{E_c I_c}{u_1}}, \quad \ell_k = \sqrt[4]{\frac{E_c I_c}{u_2}} \quad (3)$$

Where,  $\ell_r$  is defined as the radius of relative stiffness of the rail and track support, [L];  $\ell_{rk}$  = radius of relative stiffness of the concrete slab and track support, [L]; and  $\ell_k$  = radius of relative stiffness of the concrete slab and slab support, [L].  $y$  = downward deflection, [L];  $x$  = distance to any point on the deflection and bending moment curves, [L];  $E_s$  = modulus of elasticity of rail steel, [FL<sup>-2</sup>];  $I_s$  = moment of inertia of the rail, [L<sup>4</sup>];  $P$  = concentrated load, [F];  $\delta$  = deflection of the rail or concrete slab, [L];  $M$  = bending moment of the rail or concrete slab, [FL];  $E_c$  = modulus of elasticity of concrete slab, [FL<sup>-2</sup>];  $I_c$  = moment of inertia of the concrete slab [L<sup>4</sup>];  $u_1$  and  $u_2$  stand for the modulus of elasticity of the track support and that of the slab support, [FL<sup>-2</sup>], respectively. Note that the primary dimensions are represented by [F] for force and [L] for length.

### 3 FINITE ELEMENT MODEL IDEALIZATIONS

For 3D FEM analyses, the beam element type B31 was used to model the rails in this study. The connector element type JOINTC was adopted to model the effects of elastic constraints and load transfers of the rail fastening systems. To avoid potential stress concentration, this element is also connected to a shell element with rigid elements RB3D2 on each node was also used. The subgrade was modeled using the Foundation option. Various 3D shell and 3D solid elements are used to model concrete slabs.

A systematic approach was utilized and implemented in a Visual Basic software package to study the effects of mesh fineness and element selections using the ABAQUS program. Using vertical mesh fineness of one (or 1-layer) was found inadequate and should be avoided especially for the C3D8 and C3D8R elements. By increasing both mesh fineness, the resulting deflections of 8-node elements are very close to 20-node and 27-node elements. To achieve high accuracy and computation efficiency, it was suggested that element type C3D20 with a horizontal mesh fineness of 3 and a vertical mesh fineness of 3 be selected for further analysis (Yen 2004).

### 4 VERIFICATION OF DOMINATING MECHANISTIC VARIABLES

Various FEM runs were conducted and compared to numerically verify the aforementioned relationships. If the rail is limited in length, the normalized maximum responses will depend on the dimensionless parameter  $L/\ell_r$  alone, where L is the finite rail length, [L]. In addition, an additional dimensionless parameter  $a/\ell_r$  was also identified and verified for a continuously and elastically supported beam subjected to a uniformly distributed load. Thus, the following relationship was used to account the theoretical differences for a finite rail resting on elastic foundations subjected to a uniformly distributed load (Yen 2004):

$$\frac{\delta E_s I_s}{P \ell_r^3}, \frac{M}{P \ell_r} = f_3 \left( \frac{a}{\ell_r}, \frac{\ell_{rk}}{\ell_r}, \frac{\ell_k}{\ell_r}, \frac{L}{\ell_r} \right) \quad (4)$$

### 5 DEVELOPMENT OF AN ALTERNATIVE STRESS ANALYSIS PROCEDURE

To allow the analysis of more practical loading conditions, the substructures of a slab track system were separately analyzed. By applying the concept of free body diagram, several additional dimensionless parameters were identified based on the aforementioned elastic beam theories and the plate theory to

account for the effects of multiple steel wheel loads, the spacing of rail-fastenings, and the concrete slab.

Several series of 3D FEM factorial runs were conducted and separate databases were created. Various prediction models for stress adjustments were developed using projection pursuit regression techniques. Together with the existing two dimensional and 3D FEM prediction models for concrete pavements (Lee et al. 1996; 2004), an alternative stress analysis procedure was proposed (Yen 2004).

### 6 IMPLEMENTATION AND VERIFICATION

Finally, the proposed procedure has been implemented in an EXCEL spreadsheet file (TKUTRACK) using the Visual Basic for Applications software to facilitate future slab track analyses. Its applicability was further verified through a completely different database with very good agreements (Yen 2004).

### 7 CONCLUSIONS

The idealized theoretical solutions are first investigated. Several dominating mechanistic variables are identified and numerically verified. A systematic approach was utilized and implemented in a Visual Basic software package to study the effects of mesh fineness and element selections. The track-slab system was separately analyzed using the concept of free body diagram. Based on the elastic track theory and plate theory, several dimensionless parameters were identified and several series of factorial FEM runs were conducted. Various prediction models for stress adjustments were developed. An alternative stress analysis procedure was proposed and implemented in an EXCEL spreadsheet file (TKUTRACK) using the Visual Basic for Applications software for future routine slab track analyses.

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