Mechanistic analysis of a slab track system and its applications

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ABSTRACT: The analysis of slab tracks is similar to that of rigid highway pavements except that loads are applied to the rails connected directly to the concrete slab or through rubber booted block ties. The idealized theoretical solutions are first investigated. Together with the principles of dimensional analysis, several dominating mechanistic variables are identified and numerically verified. A systematic approach was utilized and implemented in a Visual Basic software package to study the effects of mesh fineness and element selections using the ABAQUS three dimensional (3D) finite element model (FEM) program. The track-slab system was separately analyzed using the concept of free body diagram. Based on the elastic track theory and plate theory, several dimensionless parameters were identified and various prediction models were developed. An alternative stress analysis procedure was proposed and implemented in an EXCEL spreadsheet file (TKUTRACK) using the Visual Basic for Applications software for future routine slab track analyses.

1 INTRODUCTION

Due to the superior structural capacity and economic benefits of slab tracks compared to those of traditional rail-ballast counterparts, slab track systems have become more and more popular in recent railway applications (Bilow & Randich 2000). The design of slab tracks is similar to that of rigid pavements except that "the loads are applied to the rails connected directly to the concrete slab or through rubber booted block ties" (Huang 1993). Determination of the critical structural responses of slab track system is essential to mechanistic-based design and evaluation procedures. A review of the state-of-the-art procedures (AREMA 2001) in track analysis was first conducted (Yen 2004).

Finite element models (FEM) have been successfully utilized to account for the effects of many practical conditions more realistically than theoretical solutions based on infinite slab and other idealized assumptions. With the introduction of three-dimensional (3D, ABAQUS) (Hibbitt et al. 2000) FEM and all the promising features and results reported in the literature (Hammons 1998; Kim & Hjelmstad 2000; Kuo 1996), its applications on pavement/rail engineering become inevitable. Nevertheless, due to the required running-time and complexity, 3D FEM analysis cannot be easily implemented as a part of design or structural evaluation procedure. In particular, the effects and sensitivity analysis of various design components of slab track systems were rarely investigated in the existing literature. In addition, the gaps between the available theoretical closed-form solutions and finite element solutions are often overlooked. Thus, this study strives to investigate the theoretical discrepancies, provide mesh fineness and element selection guidelines, develop adjustment factors and automated analysis procedures to account for various practical track conditions more realistically.

2 THEORETICAL SOLUTIONS

The fundamental theory due to Talbot, considering the track as a continuously and elastically supported beam subjected to a concentrated load, results in the following differential equation (Hay 1982):

$$E_s I_s \frac{d^4 y}{dx^4} = p \tag{1}$$

In which, $p = -uy$; p is the upward pressure per unit length, $[FL^{-1}]$; u is the modulus of elasticity of the track support (track stiffness modulus), $[FL^{-2}]$; y is the downward deflection, [L]; and x is the distance to any point on the deflection and bending moment curves, [L]. The track stiffness modulus u is a lumped parameter combining tie, ballast, and subgrade stiffness in one term. E_s is the modulus of elasticity of rail steel, [FL^{-2}]; I_s is the moment of inertia of the rail, [L^4]; and E_sI_s is the flexural rigidity of the rail, [FL^2]. Note that the primary dimensions are represented by [F] for force and [L] for length. This differential equation is satisfied by the following deflection equation and its successive derivatives:

$$y(x) = \frac{P}{8E_sI_s\lambda^3}\varphi_1(\lambda x)$$
$$M(x) = -E_sI_s\frac{d^2y}{dx^2} = \frac{P}{4\lambda}\varphi_3(\lambda x) \tag{2}$$

Where, P is the concentrated wheel load, [F]; M is the bending moment, [FL]; $\lambda = \sqrt[4]{u/(4E_sI_s)}$ is a damping factor, [L^{-1}]; $\varphi_1(\lambda x) = e^{-\lambda x}(\cos\lambda x + \sin\lambda x)$ and $\varphi_3(\lambda x) = e^{-\lambda x}(\cos\lambda x - \sin\lambda x)$ are Zimmerman functions.

Furthermore, based on the idealization of two continuous beams on elastic foundations, the following differential equations are re-derived (Hetenyi 1974; Wang 1987; Huang & Cheng 1993):

$$E_sI_s\frac{d^4y_1}{dx^4} = p_1$$
$$E_cI_c\frac{d^4y_2}{dx^4} + p_1 = p_2 \tag{3}$$

In which, $p_1 = -u_1(y_1 - y_2)$; $p_2 = -u_2y_2$; p_1 and p_2 stand for the upward pressure per unit length on the rail and concrete slab, [FL^{-1}]; y_1 and y_2 represent the deflections of the rail and concrete slab, [L]; u_1 and u_2 stand for the modulus of elasticity of the track support and that of the slab support, [FL^{-2}], respectively. In addition, E_s is the modulus of elasticity of rail steel, [FL^{-2}]; I_s is the moment of inertia of the rail, [L^4]; E_c is the modulus of elasticity of concrete slab, [FL^{-2}]; and I_c is the moment of inertia of the concrete slab [L^4].

By assuming $l=u_1/(E_sI_s)$, $m=u_1/(E_cI_c)$, $n=u_2/(E_cI_c)$, and substituting $y_1 = Ae^{\lambda x}$, $y_2 = Be^{\lambda x}$ into the above equation, one can obtain: $A\lambda^4 + l(A-B) = 0$ and $B\lambda^4 - mA + (m+n)B = 0$. By converting them into $B/A = (\lambda^4 + l)/l = m/(\lambda^4 + m + n)$ and setting $U=\lambda^4$, the solutions of a second degree polynomial of U are obtained as U_1, U_2= $1/2*\left(-(l+m+n) \pm \sqrt{(l+m+n)^2 - 4*l*n}\right)$. After assuming $\kappa=\sqrt[4]{-U_1/4}$ and $\omega=\sqrt[4]{-U_2/4}$, the general solutions for the deflections of rail steel and concrete slab can be obtained. By applying the proper boundary conditions and assuming $\xi = (U_1 + l)/l$ and $\eta = (U_2 + l)/l$, one can obtain the following deflection equations and its successive derivatives which satisfy the aforementioned differential equations:

$$y_1 = \left[\frac{\varepsilon}{\kappa^3}\varphi_1(\kappa x) - \frac{1}{\omega^3}\varphi_1(\omega x)\right]\cdot\rho^*$$
$$y_2 = \eta\left[\frac{1}{\kappa^3}\varphi_1(\kappa x) - \frac{1}{\omega^3}\varphi_1(\omega x)\right]\cdot\rho^* \tag{4}$$

$$M_1 = -2E_sI_s\cdot\left[-\frac{\varepsilon}{\kappa}\varphi_3(\kappa x) + \frac{1}{\omega}\varphi_3(\omega x)\right]\cdot\rho^*$$
$$M_2 = -2E_cI_c\eta\cdot\left[-\frac{1}{\kappa}\varphi_3(\kappa x) + \frac{1}{\omega}\varphi_3(\omega x)\right]\cdot\rho^* \tag{5}$$

Where, y_1 and y_2 are the deflections of the rail and concrete slab; M_1 and M_2 are the bending moments of the rail and concrete slab, [FL], respectively. $\varepsilon = \frac{\eta}{\xi} = \frac{U_2 + l}{U_1 + l}$ and $\rho^* = \frac{1}{4(\varepsilon - 1)}\cdot\frac{P}{2E_sI_s}$. $\varphi_1(\kappa x)$, $\varphi_3(\kappa x)$, $\varphi_1(\omega x)$, $\varphi_3(\omega x)$ are Zimmerman functions as previously defined. The maximum deflections (δ_s and δ_c) and bending moments (M_s and M_c) occurring under the load where x=0 are as follows:

$$\delta_s = \left[\frac{\varepsilon}{\kappa^3} - \frac{1}{\omega^3}\right]\cdot\frac{1}{4(\varepsilon-1)}\cdot\frac{P}{2E_sI_s}$$
$$\delta_c = \eta\left[\frac{1}{\kappa^3} - \frac{1}{\omega^3}\right]\cdot\frac{1}{4(\varepsilon-1)}\cdot\frac{P}{2E_sI_s} \tag{6}$$

$$M_s = \left[\frac{\varepsilon}{\kappa} - \frac{1}{\omega}\right]\cdot\frac{1}{4(\varepsilon-1)}\cdot P$$
$$M_c = \eta\cdot\left[\frac{1}{\kappa} - \frac{1}{\omega}\right]\cdot\frac{1}{4(\varepsilon-1)}\cdot\frac{E_cI_c}{E_sI_s}P \tag{7}$$

The critical rail or concrete slab stress can then be determined by $\sigma = M*c/I$, in which σ is the critical stress of the rail or concrete slab, [FL^{-2}]; M is the maximum bending moment of the rail or the concrete slab, [FL], c is the distance from the base of the rail or concrete slab to its neutral axis, [L]; I is the moment of inertia of the rail or concrete slab, [L^4].

3 PARAMETER IDENTIFICATIONS

Based on the elastic beam theory and the theory of two continuous beams on elastic foundations, the following parameters are subsequently defined:

$$\ell_r = \sqrt[4]{\frac{E_s I_s}{u_1}} \quad , \quad \ell_{rk} = \sqrt[4]{\frac{E_c I_c}{u_1}} \quad , \quad \ell_k = \sqrt[4]{\frac{E_c I_c}{u_2}} \tag{8}$$

In which, ℓ_r is defined as the radius of relative stiffness of the rail and track support, [L]; ℓ_{rk} is defined as the radius of relative stiffness of the concrete slab and track support, [L]; and ℓ_k is defined as the radius of relative stiffness of the concrete slab and slab support, [L].

Applying the principles of dimensional analysis to the solutions of elastic beam theory and the theory of two continuous beams on elastic foundations, i.e., equations (2), (6) and (7), the following concise relationships are obtained:

$$\frac{yE_s I_s}{P\ell_r^{\ 3}}, \frac{M}{P\ell_r} = f_1\left(\frac{x}{\ell_r}\right) \tag{9}$$

$$\frac{\delta E_s I_s}{P\ell_r^{\ 3}}, \frac{M}{P\ell_r} = f_2\left(\frac{\ell_{rk}}{\ell_r}, \frac{\ell_k}{\ell_r}\right) \tag{10}$$

Where, y is the downward deflection, [L]; x is the distance to any point on the deflection and bending moment curves, [L]; δ is maximum deflection of the rail or concrete slab, [L]; M is the maximum bending moment of the rail or concrete slab, [FL]. Also note that all variables in both sides are dimensionless. $(\delta E_s I_s)/(P\ell_r^3)$ and $M/(P\ell_r)$ are the normalized deflection and normalized moment parameters, respectively.

4 FINITE ELEMENT MODEL IDEALIZATIONS

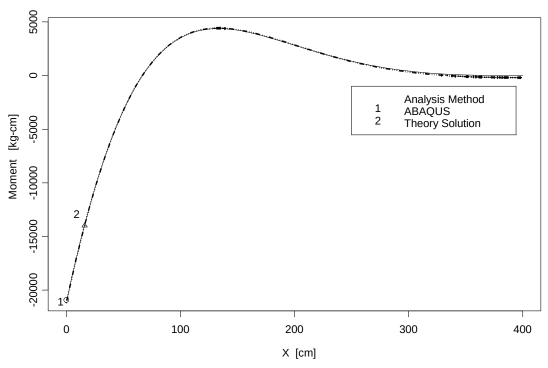
According to earlier literature (Kuo 1996; Bao 1998), the ABAQUS 3D FEM (Hibbitt et al. 2000) was selected for this study. Various elements will be carefully chosen to simulate different components of the slab track system. In particular, 3D shell and 3D solid elements, beam elements, and spring elements will be used to model concrete slabs, rails, and various rail fastenings as well as the subgrade support, respectively.

4.1 *Two dimensional model building and verifications*

Various beam elements such as B21, B31, B31H, B32, B32H, and B31OS defined in the ABAQUS library can be used to model the rails. According to the literature, using the 2-node linear beam element types B21 and B31 can adequately achieve the desired accuracy while considering the efficiency of the required computation time and resources. Assuming a longitudinal rail resting on a Winkler foundation with the following input parameters: P=9.81kN, E_s=206GPa, I_s=3090cm^4, u=49.05MPa, the infinite rail length L=400cm, the spacing of rail support (or fastenings) s = 0.58m, the beam element type B21 was used to model the rails and the foundation was modeled as SPRING elements. Using a very fine mesh with 400 beam elements and a symmetry option, the resulting FEM solutions were compared to the closed-form solutions with excellent agreements as shown in Figure 1.

Furthermore, considering two continuous beams on elastic foundations using the same element types and similar input parameters, the resulting ABAQUS maximum deflections and bending moments were found to be in relatively good agreements with the closed-form solutions as well (Yen 2004).

(a)



(b)

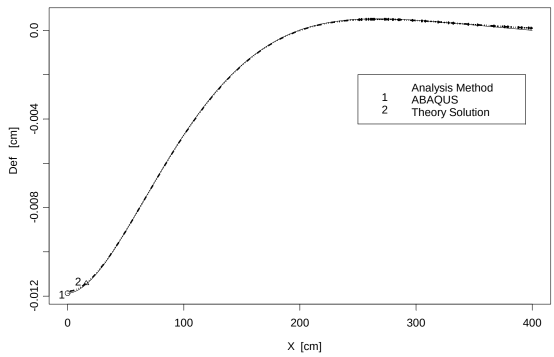


Figure 1. Comparison of ABAQUS solutions with closed-form solutions: (a) rail deflection; (b) rail bending moment.

4.2 *3D model building and convergence study*

As for 3D FEM analyses, the beam element type B31 was used to model the rails in this study. Different from the earlier literature (Kuo 1996; Bao 1998), the connector element type JOINTC was adopted to model the effects of elastic constraints and load transfers of the rail fastening systems. This element can provide the connection between two nodes while allowing the inputs of a spring constant and the dashpot damping. In addition, to avoid potential stress concentration this element is also connected to a shell element with rigid elements RB3D2 on each node was also used to uniformly distribute the load to the concrete slab. The subgrade was modeled as a Winkler foundation or dense liquid foundation. Since identical results are obtained using either SPRING elements or the Foundation option, the latter was chosen to model the subgrade.

Various 3D shell and 3D solid elements are used to model concrete slabs. These include both linear and quadratic elements employing both full and reduced integration. Two types of thin shell elements (4-node, 8-node, and 9-node) are considered: those satisfy the thin shell theory (the Kirchhoff constraint) analytically and those converge to thin shell theory numerically as the thickness decreases. The selected 3D solid elements (8-node, 20-node, and 21~27-node) include first-order (linear) and second-order (quadratic) interpolation elements. A brief summary of the characteristics of 3D shell and 3D solid elements from the ABAQUS library is also available (Wu 2003).

A single finite slab track system resting on a Winkler foundation under a concentrated load with the following input parameters: P=9.81kN, E_s=235GPa, I_s=2000cm^4, u_1=105MPa, finite slab/rail length=4.8m, finite slab width W=3.84m, slab thickness h_c=16cm, E_c=19.6GPa, u2=56.8MPa, the size of rail fastenings 24*24 cm^2, the spacing of the rail support (or rail fastenings) s=0.60m was chosen for the convergence study. A systematic approach was utilized and implemented in a Visual Basic software package to study the effects of mesh fineness and element selections using the ABAQUS program. This program was developed to automatically construct FEM models, generate the input files, conduct the runs, as well as summarize the results.

Mesh generation in the horizontal direction generally includes the following steps: generation of finer mesh at the loaded area (Zone I) and at its neighborhood area (Zone II), and progressively increasing to coarser mesh further away (Zone III). The horizontal mesh fineness is defined as the ratio of the length of the loaded area to the selected element length. In addition, Zone I and Zone II were chosen to have the same mesh fineness. The length of the Zone II in the longitudinal direction was set to 3 times the spacing of the rail fastenings, whereas the length of the Zone II in the transverse direction was chosen as 2 times the length of the loaded area in this study for consistency and efficiency considerations. The mesh of Zone III was decided as 4 times coarser than Zone 1 (Wu 2003; Lee et al. 2004). The slab thickness was evenly divided into up to 4 sub-layers for vertical mesh fineness study.

Using vertical mesh fineness of one (or 1-layer) was found inadequate and should be avoided especially for the C3D8 and C3D8R elements. By increasing both mesh fineness, the resulting deflections of 8-node elements are very close to 20-node and 27-node elements. Generally speaking, the rail deflections of all 3D solid elements are about 16% lower than the closed-form solutions, whereas the slab deflections are approximately 10% higher than the closed-form solutions. These results may be explained by their theoretical discrepancies in allowing or disallowing compressions within elements. To achieve high accuracy and computation efficiency, it was suggested that element type C3D20 with a horizontal mesh fineness of 3 and a vertical mesh fineness of 3 be selected for further analysis (Yen 2004).

5 VERIFICATION OF DOMINATING MECHANISTIC VARIABLES

Various FEM runs were conducted and compared to the solutions given in equation (10) to numerically verify the relationship for a continuously and elastically supported beam subjected to a concentrated load. Keeping the dimensionless parameter x/ℓ_r constant while changing other input variables, the normalized deflection $(yE_sI_s)/(P\ell_r^3)$ and normalized moment $M/(P\ell_r)$ remain unchanged. If the rail is limited in length, the normalized maximum responses will depend on the dimensionless parameter L/ℓ_r alone, where L is the finite rail length, [L].

Similarly, a series of FEM runs were conducted to numerically verify the relationship for a continuously and elastically supported beam subjected to a uniformly distributed load (q). Keeping the dimensionless parameter a/ℓ_r constant while changing other input variables, the normalized maximum responses remain unchanged. In addition, many cases were analyzed to numerically verify the relationship given in equation (10) for two continuous beams on elastic foundations. By keeping these two dimensionless parameters (ℓ_{rk}/ℓ_r and ℓ_k/ℓ_r) constant, the normalized maximum responses remain unchanged as well. Subsequently, the following concise relationship is identified to account the theoretical differences for a finite rail resting on elastic foundations subjected to a uniformly distributed load (Yen 2004):

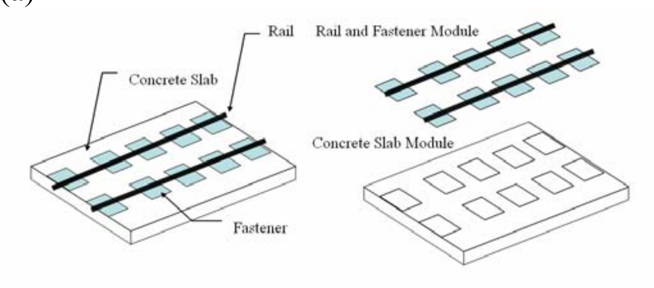
$$\frac{\delta E_s I_s}{P\ell_r^{\ 3}},\frac{M}{P\ell_r} = f_3\left(\frac{a}{\ell_r},\frac{\ell_{rk}}{\ell_r},\frac{\ell_k}{\ell_r},\frac{L}{\ell_r}\right) \tag{11}$$

6 DEVELOPMENT OF AN ALTERNATIVE STRESS ANALYSIS PROCEDURE

A slab track system consists of the rails, rail-pad-fastener systems, concrete slab, and subgrade. Traffic loadings are applied to the rails connected directly to the concrete slab or through rubber booted block ties. A different gear load configuration, a finite slab width or length, and a second bonded or unbonded layer may also result in different degrees of stress reduction. The effect of a temperature differential or moisture gradient may alter the magnitude of critical stresses. Thorough treatments of various combinations of all such conditions are very challenging due to the complexity and vast amount of required computation time in 3D FEM analysis.

To allow the analysis of more practical loading conditions, the substructures of a slab track system as shown in Figure 2 were separately analyzed (Yen, 2004). By applying the concept of free body diagram, several additional dimensionless parameters were identified based on the aforementioned elastic beam theories and the plate theory to account for the effects of multiple steel wheel loads, the spacing of rail-fastenings, and the concrete slab. Several series of 3D FEM factorial runs were conducted using beam element type B31 and C3D20 solid elements with a horizontal mesh fineness of 3 and a vertical mesh fineness of 3. Separate databases were created based on the relationship given by equation (11).

(a)



(b)

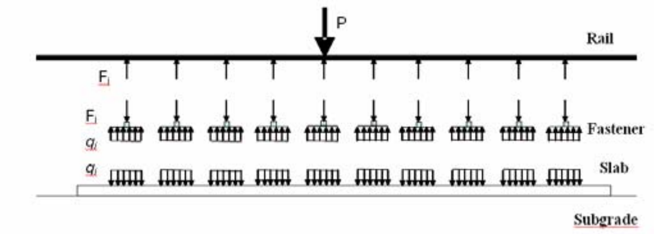


Figure 2. Illustration of the (a) substructures and (b) the free body diagrams of a slab track system.

Various prediction models for stress adjustments were developed using a two-step modeling procedure utilizing projection pursuit regression techniques proposed by Lee and Darter (1994). The projection pursuit regression (PPR), strives to model the response surface (y's) as a sum of nonparametric functions of projections of the predictor variables (x's) through the use of super smoothers (Friedman & Stuetzle 1981). The S-PLUS statistical package (Insightful Corp. 2003), which has been widely used by statisticians, was selected for the analysis due to the availability of this regression technique. With the help of the PPR, a multi-dimensional response surface is broken down into the sum of several smooth projected curves which are graphically representable in two dimensions. Plausible model forms and applicable boundary conditions may then be easily identified and specified through visual inspection and/or engineering knowledge of physical relationships to model these individual projected curves separately. Traditional parametric regression techniques such as linear, piecewise-linear, and nonlinear regressions are then utilized for these purposes with higher confidence in the parameter estimates.

Together with the existing two dimensional and 3D FEM prediction models for concrete pavements (Lee et al. 1996; 2004), the following systematic stress analysis procedure was proposed (Yen 2004):

1. Input the axle load (P) and the pertinent parameters of a slab track system;
2. Convert the input parameters into dominating mechanistic variables (ℓ_{rk}/ℓ_r, ℓ_k/ℓ_r, s/ℓ_r, and x/ℓ_r);
3. Determine the maximum reaction force of fasteners (F_0), $F_0 = P * R_{F_0}$, where $R_{F_0} = f_1(\ell_{rk}/\ell_r, \ell_k/\ell_r, s/\ell_r)$;
4. Calculate each reaction force of fasteners (F_i), $F_i = F_0 * R_{F_i} * R_{D_a}$, in which $R_{F_i} = f_2(x/\ell_r)$, $R_{D_a} = f_3(D_a/\ell_r)$, and D_a is the tandem axle spacing;
5. Convert each F_i into uniform load (q_i) applied on the concrete slab and calculate the corresponding interior slab stress (σ_{wi}) using Westergaard's closed-form solution;
6. Determine the critical slab stress, $\sigma_i = \sum_{i=0}^{n}(\sigma_{wi} * R_{D_0})$, where $R_{D_0} = f_3(a/\ell, D_0/\ell)$ is the stress reduction at a distance (D_0) away from the critical location; and
7. Make critical stress adjustment due to the effects of finite slab length (L_c) and slab width (W_c) and the rail spacing (t), $\sigma_{FEM} = \sigma_i * R_{LW} * R_t$, where $R_{LW} = f_4(a/\ell, L_c/\ell, W_c/\ell)$ and $R_t = f_5(a/\ell, t/\ell)$.

Note that the adjustment factors (f_1 through f_5) stand for the predictive models developed based on the corresponding dominating mechanistic variables.

7 IMPLEMENTATION AND VERIFICATION

Finally, the proposed procedure has been implemented in an EXCEL spreadsheet file (TKUTRACK) using the Visual Basic for Applications software to facilitate future routine slab track analyses. Its applicability was further verified through a completely different database generated using different input parameters. As shown in Figure 3, very good agreement has been observed when comparing the TKUTRACK predicted stresses versus the resulting ABAQUS slab stresses. More detailed information regarding the tentative applications of the proposed analytical procedure can be found in the literature (Yen 2004).

8 CONCLUSIONS

This study strives to bridge the gap between the closed-form theoretical solutions and the numerical results obtained from 3D FEM analyses. The idealized theoretical solutions are first investigated. Together with the principles of dimensional analysis, several dominating mechanistic variables are identified and numerically verified. A systematic approach was utilized and implemented in a Visual Basic software package to study the effects of mesh fineness and element selections using the ABAQUS 3D FEM program. To achieve high accuracy and computation efficiency, it was recommended that element type C3D20 with a horizontal mesh fineness of 3 and a vertical mesh fineness of 3 be selected.

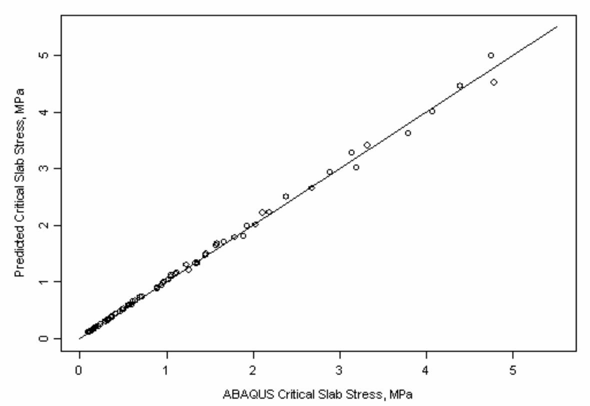


Figure 3. Verification of the critical slab stress for TKUTRACK and ABAQUS.

The track-slab system was separately analyzed using the concept of free body diagram. Based on the elastic track theory and plate theory, several dimensionless parameters were identified and several series of factorial FEM runs were conducted. Various prediction models for stress adjustments were developed using a two-step modeling procedure utilizing projection pursuit regression techniques. An alternative stress analysis procedure was proposed and implemented in an EXCEL spreadsheet file (TKUTRACK) using the Visual Basic for Applications software for future routine slab track analyses.

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