

## 1<sup>st</sup> European Airport Pavement Workshop

# Study of Rigid Pavement Deflections Using 3-D Finite Element Analysis

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## INTRODUCTION

- Previous work on NDT & backcalculation analysis
  - AREA Concept
  - Closed-form Procedures: ILLI-BACK (Ioannides 1989, 1990), NUS-BACK (Li, Fwa, 1997, 1998)
  - Regression Models: Hall (1991); AASHTO (1993); Crovetto (1994)
  - Modified Deflection Ratio Procedures: Lee (1997, 1998)
  - Extensive re-backcalculation analysis of LTPP GPS test sections indicated extremely difficulties in interpreting in situ rigid pavements deflection (1997)
  - study the effects of adjacent slabs and temperature curling on rigid pavement deflections (Lee & Sheu, 2001)

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## OBJECTIVES



- Current problems in pavement analysis
    - Closed-form and 2-D analysis: inadequate
    - 3-D analysis: very complicated
    - Lack of explicit guidelines in 3-D mesh generation
    - Accuracy of the results: variable / questionable
  - Study of pavement deflection using 3-D FEM modeling
    - Study deflection convergence characteristics
    - Provide mesh generation & element selection guides
    - Develop an automatic procedure & predictive models for future evaluation
    - Bridge the gap among these solutions
- Closed-form solutions → 2-D FEM (ILLI-SLAB) → 3-D FEM (ABAQUS)

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## Comparisons of Various Solutions

- Theoretical solutions (edge loading)

$$\delta_{we} = \frac{\sqrt{2+1.2\mu}P}{\sqrt{Eh^3k}} \left[ 1 - \frac{(0.76+0.4\mu)a}{\ell} \right]$$
$$\sigma_e = \frac{3(1+\nu)P}{\pi(3+\nu)h^2} \left[ \ln\left(\frac{Eh^3}{100ka^4}\right) + 1.84 - \frac{4\nu}{3} + \frac{1-\nu}{2} + \frac{1.18(1+2\nu)a}{\ell} \right]$$

- 2-D Model: simple, short analysis time, more assumptions, can model actual pavement well
- 3-D Model: more complicated, longer analysis time, not easy to model, can best model actual pavement structural response
- Scope: a single loaded slab (Winkler foundation)

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## PARAMETER ANALYSIS AND MODEL BUILDING

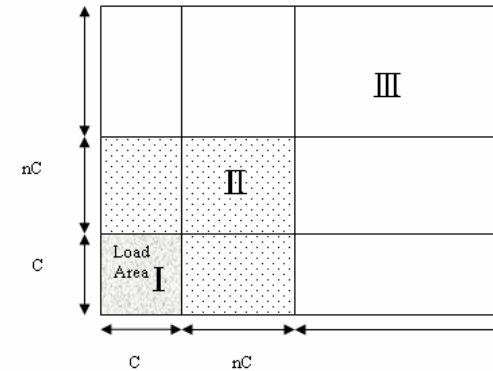
- Definition of mesh generation
- Deflection / Stress convergence characteristics
  - 3-D shell elements (Horizontal mesh fineness)
  - 3-D solid elements (Horizontal and vertical mesh)
- Convergence characteristics due to different slab thicknesses and load sizes
  - Deflection convergence & Stress convergence
- Determine of the length of finer mesh
- Recommendations on mesh fineness & element selection

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## Definition of Mesh Generation



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## Convergence Characteristics of Mesh Fineness Study

- Input Parameters / Dominating Parameters

$L=197$  in.,  $W=197$  in.       $a/\ell=0.1$   
 $E=1,200,000$  psi,  $h=8.5$  in.       $L/\ell=7$   
 $k=100$  pci,  $P=2,250$  lbs       $W/\ell=7$   
 $C=5$  in.,  $p=90$  psi,  $\nu=0.15$        $h/a=3$

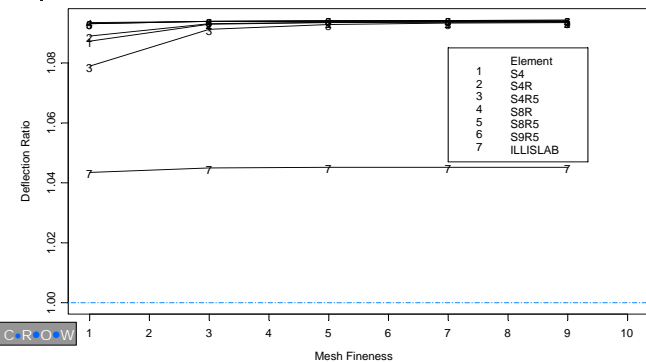
- Mesh fineness: horizontal 1~10, vertical 1~4, Zone I = Zone II, Extended ( $8^*C$ )
- Deflection ratios

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## Edge Deflection Ratio (2-D Shell)

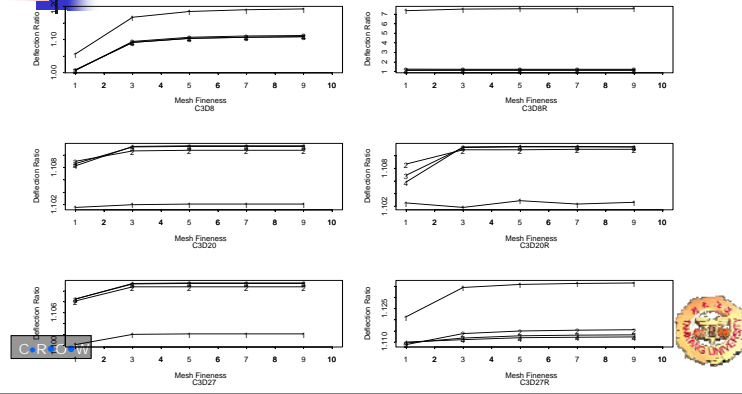


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## Edge Deflection Ratio (3-D Solid)



## Deflection convergence

- 3-D Solid elements > 2-D Shell elements > ILLI-SLAB > Westergaard solutions
- Similar deflections: S8R, S8R5, and S9R5 elements (S9R5=S8R5)
- Coarser mesh: S4R5<S4<S4R
- Vertical mesh fineness = 1 layer → NG
- Finer vertical mesh (w/ Horizontal=3)
  - C3D20, C3D27, and C3D20R deflections increase
  - C3D8 and C3D8R deflections decrease
- Mesh fineness recommendation: horizontal=3, vertical = 3 (layers), C3D20 or C3D27 elements



## Convergence Characteristics due to Thickness and Load Size Effects

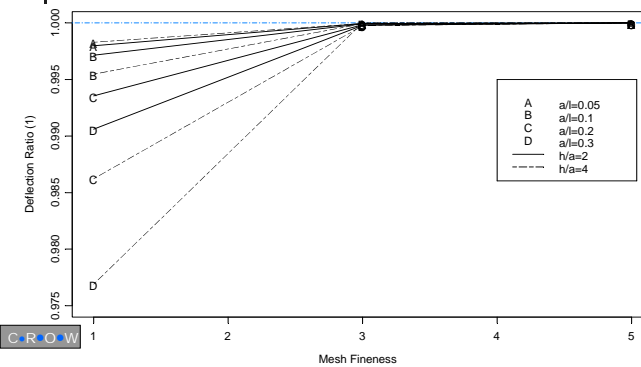
- Use C3D27 elements
- Horizontal mesh fineness (w/ vertical=3)
- Vertical mesh fineness (w/ horizontal=3)
- Cases Analyzed

$a/l$	$h/a$	C	h	E	k
		in.	in.	psi	pci
0.05	2	10	11.284	1.99E+08	150
0.1	2	15	16.926	2.48E+07	200
0.2	2	5	5.642	1.03E+06	400
0.3	2	7.5	8.463	3.83E+05	500
0.05	4	5	11.284	3.31E+07	400
0.1	4	10	22.568	6.20E+06	600
0.2	4	6	13.541	5.82E+04	150
0.3	4	7	15.797	1.16E+04	130

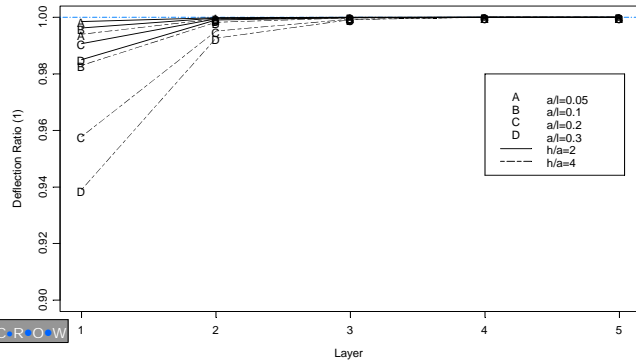
註:  $L/l=7, W/l=7, p=90\text{psi}$



## Edge Deflection Ratio (vertical=3)



## Edge Deflection Ratio (Horizontal=3)



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## Thickness and Load Size Effects

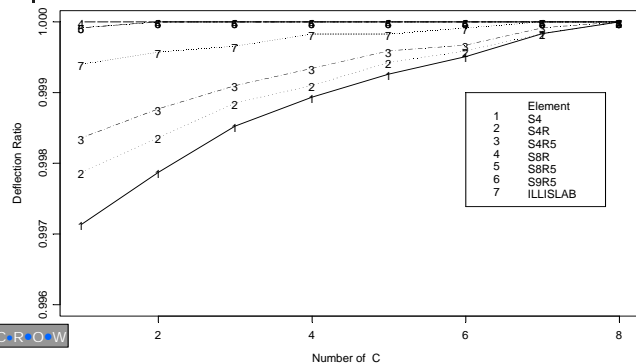
- Deflection convergence
  - Finer horizontal or vertical mesh → Deflections increase to convergence
- Sensitivity: Vertical mesh > Horizontal mesh
- Smaller  $h/a$  & smaller  $a/l$  conditions have better convergence characteristics
- Larger  $h/a$  & larger  $a/l$  → More difficult to converge
- Recommendation: horizontal=3, vertical = 3 → good convergence & efficiency

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## Determining the Length of Finer Mesh (nC) - Deflection Ratio



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## Mesh Generation and Element Selection Recommendations

- Mesh fineness: Horizontal=3, Vertical=3, Length of Finer Mesh (Zone II)=3\*C
  - Vertical mesh fineness = 1 layer → NG
- Element selection: 20- or 27-point solid elements
  - 8-point solid elements → NG (different from 20- and 27-point elements solutions)
  - reduced integration or 20-point element → stress reduction 1~2%
  - The execution time of 20-point elements ≈ 60% of 27-point elements

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## IDENTIFICATION OF ADDITIONAL DIMENSIONLESS PARAMETER

$$\frac{\sigma h^2}{P} = f\left(\frac{a}{\ell}, \frac{L}{\ell}, \frac{W}{\ell}, \frac{h}{a}\right)$$

$$\frac{\delta k \ell^2}{P} = f\left(\frac{a}{\ell}, \frac{L}{\ell}, \frac{W}{\ell}, \frac{h}{a}\right)$$

Inspired by the solutions & charts of Burmister's layered theory (1943, 1945)

h/a	a/ℓ	L/ℓ	h	L=W	E	k	P	σh <sup>2</sup> /P	δ*kℓ <sup>2</sup> /P
			cm	m	GPa	MN/m <sup>3</sup>	kN		
3.0	0.1	2	21.5	1.43	13.78	44.1	7.8	2.3983	1.0200
3.0	0.1	2	32.2	2.15	10.34	22.0	22.5	2.3984	1.0199
3.0	0.1	2	32.2	2.15	13.78	29.4	20.0	2.3984	1.0199
3.0	0.1	2	43.0	2.87	24.12	38.6	44.5	2.3985	1.0200
3.0	0.1	2	43.0	2.87	31.01	49.6	66.8	2.3989	1.0200
3.0	0.1	2	53.7	3.58	20.67	26.4	90.4	2.3976	1.0200
3.0	0.1	2	53.7	3.58	27.56	35.2	41.7	2.3983	1.0199
3.0	0.1	2	64.5	4.30	13.78	14.7	120.2	2.3993	1.0199
3.0	0.1	2	64.5	4.30	41.34	44.1	90.1	2.3982	1.0199
3.0	0.1	2	64.5	4.30	27.56	29.4	100.1	2.3985	1.0200
6.0	0.2	3	43.0	1.07	13.78	5639.9	7.8	1.3991	1.1148
6.0	0.2	3	64.5	1.61	10.34	2820.0	22.5	1.3993	1.1147
6.0	0.2	3	64.5	1.61	13.78	3760.0	20.0	1.3994	1.1147
6.0	0.2	3	86.0	2.15	24.12	4934.9	44.5	1.3991	1.1147
6.0	0.2	3	86.0	2.15	31.01	6344.9	66.8	1.3987	1.1147
6.0	0.2	3	107.5	2.69	20.67	3384.0	90.4	1.3989	1.1148
6.0	0.2	3	107.5	2.69	27.56	4512.0	41.7	1.3991	1.1147
6.0	0.2	3	129.0	3.22	13.78	1880.0	120.2	1.3990	1.1147
6.0	0.2	3	129.0	3.22	41.34	5639.9	90.1	1.3993	1.1147
6.0	0.2	3	129.0	3.22	27.56	3760.0	100.1	1.3992	1.1147

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## Discussion of the Additional Parameter Identification

- Identification of the additional parameter ( $h/a$ ) was originally inspired by Burmister's layered theory (Burmister, 1943, 1945)
- Other literature also indicated that analytical solutions derived for thick elastic plates are governed by the ratio of a circular load radius ( $a$ ) to the thickness of the slab ( $h$ ). Different  $a/h$  ratios were used to compute the maximum bending stress ( $\sigma$ ) in terms of the percent of the applied pressure ( $p$ ) (Shi & Yao, 1989; Van Cauwelaert, 1990; Ioannides & Khazanovich, 1994; Khazanovich & Ioannides, 1995).
- The conventional Westergaard's ordinary theory solution results in an overestimate in the bending stress. The correction introduced by Westergaard's special theory results in bending stress reduction, bringing it in line with Burmister's layered solutions (Ioannides & Khazanovich, 1994).

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## DEVELOPMENT OF DATABASES AND PREDICTION MODELS

- Development Of An Automated Analysis Program
- ABAQUS batch processing & Databases
  - $L/\ell = 2\sim 7$  (step by 1)
  - $W/\ell = 2\sim 7$  (step by 1)
  - $a/\ell = 0.05, 0.1\sim 0.5$  (step by 0.1)
  - $h/a = 0.5\sim 6.0$  (step by 0.5)
  - C3D27, Horizontal=3, Vertical=3, Zone II=3\*C
- Deflection adjustment prediction model

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## Estimation of 3-D FE Solutions

- Deflection Ratio ( $1/R1=0\sim 1$ )

$$R1 = \frac{\delta_{3-D \text{ FEM}}}{\delta_W} = f\left(\frac{a}{\ell}, \frac{L}{\ell}, \frac{W}{\ell}, \frac{h}{a}\right)$$

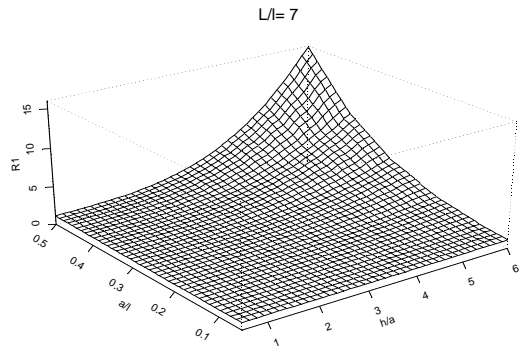
$$\delta_{3-D \text{ FEM}} = \delta_W \times R1$$

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## Edge Deflection Ratio ( $L/\ell=7$ )

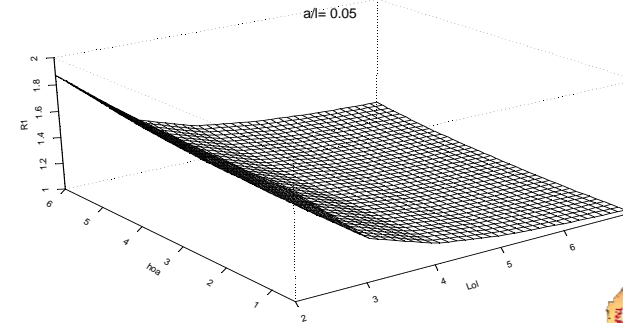


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## Edge Deflection Ratio ( $a/\ell=0.05$ )

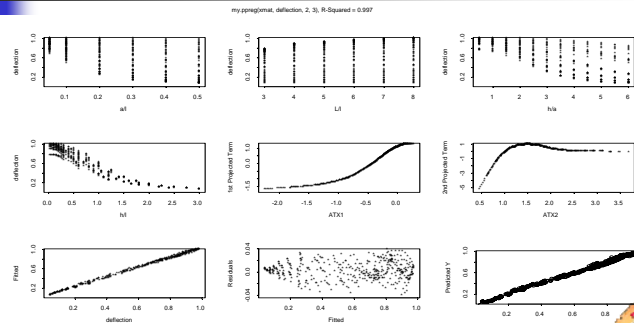


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## Deflection Ratio ( $1/R1$ ) Predictive Model



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## DISCUSSIONS & CONCLUSIONS<sup>(1)</sup>

- Compare 2-D and 3-D FEM results
- Expand the closed-form solutions and previous 2-D FEM findings to facilitate 3-D analysis
- Deflection: 3-D Solid elements > 2-D Shell elements > ILLI-SLAB > Westergaard solutions
- Vertical mesh fineness = 1 layer → NG

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## DISCUSSIONS & CONCLUSIONS<sup>(2)</sup>

- Sensitivity: Vertical mesh > Horizontal mesh > Length of Finer Mesh (Zone II)
- Smaller  $h/a$  & smaller  $a/\ell$  conditions have better convergence characteristics
- Larger  $h/a$  & larger  $a/\ell$  → More difficult to converge
- Recommendations: Horizontal=3, Vertical=3 (layers), Zone II=3\*C, C3D20 or C3D27 elements

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## DISCUSSIONS & CONCLUSIONS<sup>(3)</sup>

- Identified and verified an additional parameter ( $h/a$ ) for 2-D shell and 3-D solid elements (from Burmister's layered theory and thick elastic plates)
- Developed an automated analysis program for ABAQUS batch processing & creating databases
- Developed deflection adjustment prediction models
- Implications to future applications & validations

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**Questions?**  
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## RESEARCH SCOPE

- Single slab, dense liquid foundation, 3 loading conditions (interior, edge, corner), ABAQUS and ILLI-SLAB, stress and deflection databases

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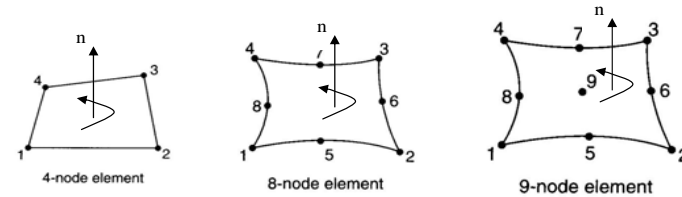
## ILLI-SLAB Program

- Originally Developed by Tabatabaie, 1977
- Continuously Revised by Wong, Conroyd, Ioannides, 1980-1985
- Included Curling Analysis by Korovesis, 1986-1989
- Re-Compiled by Lee, 1995 (Microsoft FORTRAN PowerStation)

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## 2-D Shell Elements (ABAQUS)

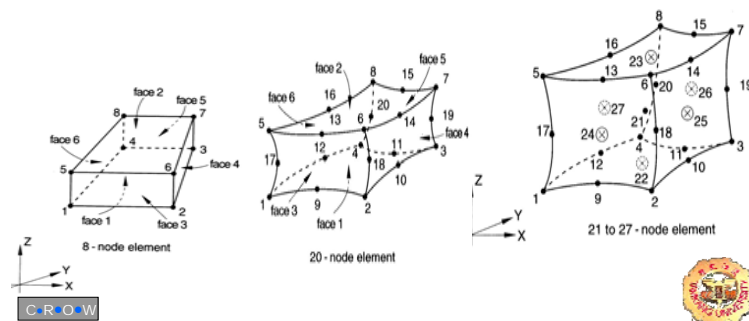


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## 3-D Solid Elements (ABAQUS)



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## 2-D Shell and 3-D Solid Elements

Name	ILLI-SLAB	ABAQUS 3-D SHELL ELEMENTS					
		S4	S4R	S4R5	S8R	S8R5	S9R5
Dimension	2-D	3-D					
Nodes	4	4	4	4	8	8	9
D.O.F.	3	6	6	5	6	5	5
Reduced Integration	No	No	Yes	Yes	Yes	Yes	Yes
Gauss Points	4	4	1	1	4	4	4
Restriction	Thin	General	General	Thin	Thick	Thin	Thin

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## Deflection Ratio Predictive Model

■  $L/\ell = W/\ell$

$$\frac{1}{RI} = 0.57628 + 0.29988 \Phi_1 + 0.03984 \Phi_2$$

$$\Phi_1 = \begin{cases} 1.15062 + 3.59112(A1) + 1.41207(A1)^2 + 0.16542(A1)^3 & \text{if } (A1) \leq 0 \\ 1.00125 + 1.81296(A1) - 2.35892(A1)^2 - 6.28127(A1)^3 & \text{if } (A1) > 0 \end{cases}$$

$$\Phi_2 = \begin{cases} -14.76436 + 25.89010(A2) - 13.77861(A2)^2 + 2.28462(A2)^3 & \text{if } (A2) \leq 2 \\ 9.85184 - 9.96245(A2) + 3.28991(A2)^2 - 0.36233(A2)^3 & \text{if } (A2) > 2 \end{cases}$$

$$A1 = 0.57228x1 + 0.02624x2 - 0.02631x3 - 0.81921x4$$

$$A2 = -0.64426x1 + 0.18742x2 + 0.04724x3 + 0.73998x4$$

$$X = [x1, x2, x3, x4] = \left[ \frac{a}{\ell}, \frac{L}{\ell}, \frac{h}{a}, \frac{h}{\ell} \right]$$

■ Limits:  $0.05 < \frac{a}{\ell} < 0.5$ ,  $0.5 < \frac{h}{a} < 6$ ,  $3 < \frac{L}{\ell} < 7$

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