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THEORETICAL INVESTIGATION OF CORNER STRESS IN CONCRETE PAVEMENTS USING DIMENSIONAL ANALYSIS

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ABSTRACT: Since corner breaks are one of the major structural distresses in jointed concrete pavements, this research study focuses on the determination of the critical bending stresses at the corner of the slab due to the individual and combination effects of wheel loading and thermal curling. A well-known slab-on-grade finite element program (ILLI-SLAB), developed over the past 15 years at the University of Illinois, was used for the analysis. Based on the principles of dimensional analysis, the dominating mechanistic variables were carefully identified and verified. The resulting ILLI-SLAB corner stresses were compared to theoretical Westergaard solutions. Adjustment factors (R) were introduced to account for this theoretical discrepancy. Prediction models were developed and could be used as an alternative to the very time-consuming and complicated F.E. analysis to estimate stresses for design purposes with efficiency and sufficient accuracy. A practical design example showing the use of the models was also provided.

1 INTRODUCTION

Recently, Portland cement concrete (PCC) has gradually recognized as an alternative pavement material in our highway pavement community due to its high rigidity and superior bearing capability as compared to asphalt concrete (AC). To accomodate our dramatically increasing traffic loadings, constructing PCC (or rigid) pavements in some special areas is definitely the future trend. Thus, the inconvenience induced by the frequently needed maintenance and rehabilitation work of AC (or flexible) pavements, which greatly reduces our highway's transportability, can be minimized. Yet, there still has not been adequate theoretical investigation in the stress analysis of corcrete pavements.

Cracking of jointed concrete pavements (JCP) is often caused by three different critical repeated loading positions: transverse joint, longitudinal joint midway between transverse joints, and at the corner. Given certain design, construction, and loading conditions, any of these load positioins could lead to fatigue cracking of the slab over time.

"**Load repetition combined with loss of support and curling stresses**" are usually recognized as the main causes for corner breaks. Thus, this paper mainly focuses on the determination of the critical bending stresses **at the corner** due to loading and thermal curling.

Two methods can be used to determine the stresses and deflections in concrete pavements: closed-form formulas and finite element computer programs. The formulas originally developed by Westergaard can be applied to a single wheel load based on the assumptions of infinite slab size and full contact between the slab-subgrade interface. To more accurately and realistically account for the effects of a finite slab size as well as possible loss of subgrade support due to a linear temperature differential, finite element (F.E.) computer program should be used. Nevertheless, the difficulties of the required run time and complexity of F.E. analysis often prevent it from being used in practical pavement design.

The main objectives of this research work are to help develop an alternative stress determination process which can be incorporated into existing mechanistic-based design procedures with sufficient accuracy and efficiency for practical pavement designs.

2 CLOSED-FORM SOLUTIONS

2.1 Corner Loading

In the analysis of a slab-on-grade pavement system, Westergaard has presented closed-form solutions for three primary structural response variables, i.e., slab bending stress, slab deflection, and subgrade stress, due to a single wheel load based on medium-thick plate theory. Based on the assumptions of an infinite or semi-infinite slab over a dense liquid foundation (Winkler foundation), Westergaard applied a method of successive approximations and obtained the following equations for a circular corner loading condition (*12*):

$$\sigma_w = \frac{3P}{h^2}\left[1-\left(\sqrt{2}\,\frac{a}{\ell}\right)^{0.6}\right]$$

$$\delta_w = \frac{P}{k\ell^2}\left[1.1-0.88\left(\sqrt{2}\,\frac{a}{\ell}\right)\right] \qquad \textbf{(Eq.1)}$$

Where:

- σ_w = critical corner stress, [FL^{-2}];
- δ_w = critical corner deflection, [L];
- P = total applied wheel load [F];
- h = thickness of the slab [L];
- a = radius of the applied load [L];
- k = modulus of subgrade reaction [FL^{-3}];
- ℓ = radius of relative stiffness of the slab-subgrade system [L];

$$\ell = \sqrt[4]{\frac{Eh^3}{12\left(1-\mu^2\right)k}} \qquad \textbf{(Eq.2)}$$

- **E = modulus of elasticity of the concrete slab [FL^{-2}]**; and
- μ = Poisson's ratio of the concrete.

Note that primary dimensions are represented by [F] for force and [L] for length. The distance to the point of maximum stress along the corner angle bisector was found to be roughly:

$$X_1 = 2\sqrt{\sqrt{2}\,a\ell} \cong 2.38\sqrt{a\ell} \qquad \textbf{(Eq.3)}$$

The above stress and deflection equations were derived using a simple approximate process and has been debated and led to numerous revisions such as those proposed by Bradbury, Kelly, Teller and Sutherland, Spangler, and Pickett over the years (*5*). Despite this argument, Ioannides et al. (*4*) later has indicated that the ILLI-SLAB F.E. results closely fall between those predicted by Westergaard and Bradbury. The ILLI-SLAB stresses are the values of the **minor principal (tensile) stress** occurring at the top fiber of the slab. This similarity indicated that Westergaard's approximation was still fairly good.

2.2 Thermal Curling

Considering curling stresses caused by a linear temperature differential on a concrete slab over a dense liquid foundation, Westergaard (*13*) developed equations for three slab conditions (i.e., infinite, semi-infinite, and an infinite long strip). The interior stress for an infinite slab is:

$$\sigma_0 = \frac{E\alpha\Delta T}{2(1-\mu)} \qquad \textbf{(Eq.4)}$$

Where:

- σ_0 = curling stress of the slab, $[FL^{-2}]$;
- α = thermal expansion coefficient of the slab, $[T^{-1}]$;
- ΔT = a linear temperature differential through the slab thickness, [T];

Primary dimensions are represented by [F] for force, [L] for length, and [T] for temperature.

Bradbury (*1*) later expanded Westergaard's bending stress solutions for a slab with finite dimensions in both transverse and longitudinal directions. The edge and interior curling stresses can be determined by:

$$\pm_{ce} = \frac{CE\sim\Delta T}{2} = \frac{E\sim\Delta T}{2}\left[1-\frac{2\cos a \cosh a}{\sin 2a+\sinh 2a}(\tan a+\tanh a)\right]$$

$$\pm_{ci} = \frac{E\sim\Delta T}{2}\left[\frac{C_1 + «C_2}{1-«^2}\right]$$

$$a = \frac{B}{\mathrm{a}\sqrt{8}} \qquad \textbf{(Eq.5)}$$

Where:

- σ_{ce}, σ_{ci} = edge and interior curling stresses of the slab, $[FL^{-2}]$;
- B = finite slab width or length, [L];
- C_1, C_2 = curling stress coefficients for the desired and perpendicular directions.

However, there exists no explicit closed-form corner stress solutions for thermal curling alone.

2.3 Loading Plus Thermal Curling

Considering the combined effect of loading plus curling, Bradbury further analyzed the curling stress on a diagonal corner section located at or near the section at which the maximum loading stress occurs, i.e. the location determined by (Eq.3). Consequently, Bradbury derived the following approximate corner curling stress:

$$\pm_{ct} = \frac{E\sim\Delta T}{3(1-«)}\sqrt{\frac{a}{\mathrm{a}}} \qquad \textbf{(Eq.6)}$$

Where:

- σ_{ct} = maximum curling stress to be combined with maximum stress induced by load at the corner, $[FL^{-2}]$.

Even though Westergaard and Bradbury all suggested that this effect could be treated as "a simple mater of addition" in most cases, many investigators have indicated that such an action may not always be conservative (*4, 9*) due to the possible loss of subgrade support and violation of full contact assumptions. This problem will be further investigated in the subsequent sections.

3 F.E. COMPUTER PROGRAM

The analysis of finite slab length and width effect was not possible until the introduction of finite element models. The basic tool for this analysis is the ILLI-SLAB F.E. computer program which was originally developed in 1977 and has been continuously revised and expanded at the University of Illinois over the years. The ILLI-SLAB model is based on classical medium-thick plate theory, and employs the 4-noded 12-degree-of-freedom plate bending elements. The Winkler foundation assumed by Westergaard is modeled as a uniform, distributed subgrade through an equivalent mass foundation. Curling analysis was not implemented until versions after June 15, 1987.

The present version (March 15, 1989) (*9*) was successfully complied on available Unix-based workstations of the Civil Engineering Department at Tamkang University. With some modifications to the original FORTRAN codes, a micro-computer version of the program was also successfully developed using Microsoft FORTRAN PowerStation (*10*) under this study.

4 IDENTIFICATION OF DOMINATING MECHANISTIC VARIABLES

4.1 Principles of Dimensional Analysis

When there exist no closed-form solutions for the selected theoretical analysis tools or when analyzing most empirical but practical engineering problems, the use of the principles of dimensional analysis is often guaranteed. The principles of dimensional analysis treate a theoretical equation in non-dimensional form, which is comprised by a set of many dimensionless paremeters representing a concise interrelationship among any complicated combinations of all input variables with dimensions. Thus, the number of parameters and data analysis time and costs may be reduced dramatically. This approach has also been widely accepted for engineering research.

4.2 Dimensionless Mechanistic Variables

Through the use of the principles of dimensional analysis, earlier investigators (*6*) have demonstrated that theoretical Westergaard solutions and F.E. solutions for three primary structural responses due to a single wheel load can be concisely defined by the following expression for a constant Poisson's ratio (usually $\mu \approx 0.15$):

$$\frac{\sigma h^2}{P}, \frac{\delta k \ell^2}{P}, \frac{q \ell^2}{P} = f_1\left(\frac{a}{\ell}, \frac{L}{\ell}, \frac{W}{\ell}\right) \qquad \textbf{(Eq.7)}$$

Where:

σ, q = slab bending stress and vertical subgrade stress, respectively, [FL^{-2}];

δ = slab deflection, [L];

f_1 = function of a/ℓ, L/ℓ, and W/ℓ; and

L, W = finite slab length and width, [L].

Note that variables in both sides of the expression are all dimensionless. The dependent variables are $\sigma h^2/P$, $\delta k \ell^2/P$ and $q \ell^2/P$, which are only dominated by the normalized load radius (a/ℓ), and the normalized slab length and width (L/ℓ and W/ℓ) rather than the other input parameters, such as E, h, k, a, etc.

Furthermore, according to recent research by Lee and Darter (*5, 6*) for the stress analysis at the very edge of the slab, concise relationships have been proposed and numerically validated through a series of F.E. runs. The dimensionless mechanistic variables due to the effects of thermal curling alone and loading plus curling for a constant Poisson's ratio are:

$$\frac{\sigma}{E}, \frac{\delta h}{\ell^2}, \frac{qh}{k\ell^2} = f_2\left(\alpha \Delta T, \frac{L}{\ell}, \frac{W}{\ell}, \frac{\gamma h^2}{k\ell^2}\right)$$

$$\frac{\sigma}{E}, \frac{\delta h}{\ell^2}, \frac{qh}{k\ell^2} = f_3\left(\frac{a}{\ell}, \alpha \Delta T, \frac{L}{\ell}, \frac{W}{\ell}, \frac{\gamma h^2}{k\ell^2}, \frac{ph}{k\ell^4}\right)$$

$$D_\gamma = \frac{\gamma h^2}{k\ell^2}, \; D_p = \frac{ph}{k\ell^4} \qquad \textbf{(Eq.8)}$$

Where:

γ = unit weight of the concrete slab, [FL^{-3}];

f_2, f_3 = functions for curling alone and curling plus loading, respectively.

Note that D_γ was defined as the relative deflection stiffness due to self-weight of the concrete slab and the possible loss of subgrade support, whereas D_p was the relative deflection stiffness due to the external wheel load and the loss of subgrade support.

Conceptually, the above relationship should be applicable to any given case of loading conditions. To numerically validate the above relationships for the individual and combined corner stresses due to loading and thermal curling in this study, several series of factorial F.E. runs were performed. While keeping the dominating mechanistic variables constant but changing any other individual input variables to different values, the F.E. results have indicated that the aforementioned relationships also hold for the corner condition as expected. Detailed summary tables of this analysis can be found in Reference (*8*).

5 FACTORIAL F.E. RUNS

A series of F. E. factorial runs were performed based on the dominating mechanistic variables identified. Several BASIC programs were written to automatically generate the finite element input files for future routine analyses. The F. E. mesh was generated according to the guidelines established in earlier studies (*3*). The desired results were also automatically summarized to reduce the possibility of untraced processing errors.

6 STRESS PREDICTION MODELS

With the incorporation of the principles of dimensional analysis, a series of finite element factorial runs over a wide range of pavement designs were carefully selected and conducted. The resulting corner stresses due to loading and curling are compared to the theoretical Westergaard solutions. Adjustment or multiplication factors (R) were introduced to account for this theoretical discrepancy.

As proposed by Lee and Darter (*7*), the projection pursuit regression (PPR) introduced by Friedman and Stuetzle (*2*) was utilized to assist in the proper selection of functional forms. Through the use of local smoothing techniques, the PPR attempts to model a multi-dimensional response surface as a sum of several nonparametric functions of projections of the explanatory variables. The projected terms are essentially two-dimensional curves which can be graphically represented, easily visualized, and properly formulated. Nonlinear or piece-wise linear regression technique can then be utilized to obtain the parameter estimates for the specified functional forms of the predictive models. This regression algorithm is available in S-PLUS statistical package (*11*).

6.1 Predictive Models for Loading Only

Based on previous investigation (*4*), Westergaard's inifinte slab assupmtion may be achieved if the normalized slab length (L/*l*)
is equal to 5.0 or more. Thus, a more conservative value of 7.0 for both L/*l* and W/*l* was selected to ensure infinite slab condition. The following factorial F.E. runs were conducted:

a/*l*: 0.05, 0.1, 0.2, 0.3
L/*l*: 2, 3, 4, 5, 6, 7
W/l: 2, 3, 4, 5, 6, 7 (L/*l* ≧ W/*l*)

Since L/*l* and W/*l* are analogous, a total of 84 runs were only necessary if slab length was chosen to be greater than slab width. The resulting maximum corner stresses ($\pm_i$) were obtained and compared to the Westergaard soultion ($\pm_w$) as shown in (Eq.1).

By using the aforementioned PPR modeling approach, the following predictive model for the adjustment factor (R_L) was developed:

$$R_L = \frac{\pm_i}{\pm_w} = 1.030 + 0.030\Phi_1 + 0.045\Phi_2$$

$$\Phi_1 = 92.415 - 149.276(A1) + 59.747(A1)^2$$

$$\Phi_2 = \begin{cases} -6.034 + 23.128(A2) - 22.022(A2)^2 & if \quad A2 \le 0.6 \\ -0.117 + 0.375(A2) & if \quad 0.6 < A2 \end{cases}$$

$$A1 = 0.8272x1 - 0.1219x2 + 0.0002x3 + 0.5485x4$$

$$A2 = -0.9034x1 + 0.2973x2 - 0.0118x3 - 0.3088x4$$

$$X = [x1, x2, \ldots, x4] = \left[\frac{a}{a}, \frac{L}{a} + \frac{W}{a}, \frac{L}{a} * \frac{W}{a}, \sqrt{\frac{L}{a}} + \sqrt{\frac{W}{a}}\right]$$

(Eq.9)

Statistics:

N=84, R^2=0.980, SEE= 0.0081, CV=0.79%

Limits:

$$0.05 \le a/ª \le 0.3, 2 \le L/ª \le 7, W/ª \le L/ª$$

Note that N is the number of data points, R^2 is the coefficient of determination, SEE is the standard error of estimates, and CV is the coefficient of variation. This prediction model is also applicable to a larger slab when the upper bound value of 7.0 is used for the normalized slab length or width (L/*l*, W/*l*).

6.2 Predictive Models for Loading Plus Curling

The combination effect of loading plus curling cannot be adequately described using the principle of superposition due to the violation of full contact assumption between slab and subgrade. Since night-time (negative ΔT) curling condition will result in additional tensile stress at the top fiber of the slab, this study is only limited to the most critical case of corner loading plus night-time curling.

Unlike the analysis of interior or edge stresses where the maximum stresses occur in the same specified position, the analysis of corner stresses is probably the most difficult one among them. A preliminary corner stress analysis of the ILLI-SLAB program has clearly indicated that the location of maximum combined stress due to loading plus curling varies from case to case. Generally speaking, if temperature differentials are relatively small combined with a large corner load, the critical stress location is very close to Westergaard's maximum load stress location (Eq.3). However, if temperature differentials are very large along with a very small corner load, the critical stress location may shift toward and up to the center of the slab.

Research continues with special attentions to this different critical stress location problem. Consequently, necessary modifications were made to the existing ILLI-SLAB codes to facilitate the search of critical stresses and locations alone the corner angle bisector or the diagonal nodes up to the center of the slab.

A complete full factorial of all the six dimensionless parameters which requires a tremendous amount of computer time is not feasible. Thus, to minimize the total number of runs for the analysis, the following factorial of ILLI-SLAB runs was performed:

a/*l*: 0.05, 0.1, 0.2, 0.3
L/*l*: 2, 3, 4, 5, 7, 9, 11, 13, 15 (L/*l* = W/*l*)
ΔT: 0, -10, -20, -30, -40
($\sim = 5.5E-06/^oF$)

A square slab with a constant thermal expansion coefficient was assumed. To account for D_γ and D_p effects without increasing the number of F.E. runs, the above factorial runs were randomized by these two factors for different a/*l* values. The corresponding values are given below:

a/l	(DG, DP)
0.05	(1, 2) (10, 30) (7, 130)
0.10	(4, 30) (7, 70) (4, 130)
0.20	(4, 2) (7, 30) (10, 70)
0.30	(1, 2) (10, 70) (1, 130)

Note: $DG = D_{¥} * 10^5$, $DP = D_p * 10^5$

The following adjustment factor (R_T) **was introduced to quantify the difference between stresses due to loading and curling alone.**

$$\pm_i = \pm_L + R_T * \pm_0$$

$$R_T = \frac{\pm_i - \pm_L}{\pm_0} \qquad \textbf{(Eq.10)}$$

Where:

$\pm_i$ = **combined maximum F.E. corner stress, [FL**$^{-2}$];

$\pm_L$ = **F.E. corner stress due to loading alone (**ΔT**=0), which may also be estimated by (**$R_L * \pm_w$**), [FL**$^{-2}$]; and

$\pm_0$ = **as defined by (Eq.4).**

By using the PPR algorithm, the following model for adjustment factor R$_T$ was developed:

$$R_T = 0.255 + 0.310\Phi_1 + 0.102\Phi_2 + 0.078\Phi_3$$

$$\Phi_1 = \begin{cases} -0.267 + 0.045(A1) + 0.00057(A1)^2 & \textit{if} \quad A1 \le 0.58 \\ -0.519 + 0.446(A1) & \text{if} \quad 0.58 < A1 \end{cases}$$

$$\Phi_2 = \begin{cases} -2.107 + 0.234(A2) + 0.057(A2)^2 + 0.013(A2)^3 & \textit{if} \quad A2 \le 2 \\ -2.566 + 0.634(A2) & \text{if} \quad 2 < A2 \le 4 \\ -4.428 + 1.661(A2) - 0.160(A2)^2 + 0.0048(A2)^3 & \text{if} \quad 4 < A2 \le 12 \\ 2.846 - 0.187(A2) & \text{if} \quad 12 < A2 \end{cases}$$

$$\Phi_3 = \begin{cases} 2.458 + 0.892(A3) & \text{if} \quad A3 \le -3.5 \\ 0.290 - 0.180(A3) - 0.125(A3)^2 & \text{if} \quad -3.5 < A3 \le -0.5 \\ 0.444 + 0.132(A3) & \text{if} \quad -0.5 < A3 \end{cases}$$

$$\begin{aligned} A1 = & -0.031x1 + 0.582x2 - 0.327x3 + 0.162x4 \\ & - 0.161x5 + 0.017x6 + 0.010x7 - 0.140x8 \\ & + 0.442x9 - 0.013x10 + 0.340x11 - 0.007x12 \\ & + 0.0048x13 + 0.410x14 - 0.039x15 + 0.032x16 \end{aligned}$$

$$\begin{aligned} A2 = & -0.017x1 + 0.838x2 - 0.220x3 + 0.397x4 \\ & - 0.059x5 + 0.030x6 + 0.023x7 + 0.051x8 \\ & - 0.095x9 + 0.0013x10 - 0.097x11 - 0.0058x12 \\ & - 0.00036x13 + 0.256x14 - 0.012x15 + 0.0064x16 \end{aligned}$$

$$\begin{aligned} A3 = & \; 0.078x1 - 0.470x2 + 0.531x3 + 0.115x4 \\ & + 0.223x5 - 0.018x6 - 0.050x7 - 0.082x8 \\ & - 0.600x9 + 0.011x10 + 0.128x11 + 0.00573x12 \\ & - 0.0025x13 + 0.203x14 + 0.013x15 - 0.011x16 \end{aligned}$$

$$\begin{aligned} X = & [x1, x2, \ldots, x16] \\ = & \Big[\frac{a}{a}, \frac{L}{a}, ADT, \frac{a}{a} * \frac{L}{a}, \frac{a}{a} * ADT, \frac{L}{a} * ADT, \\ & \frac{a}{a} * \frac{L}{a} * ADT, DP, DG, DP * DG, DP * \frac{a}{a}, DP * \frac{L}{a}, \\ & DP * ADT, DG * \frac{a}{a}, DG * \frac{L}{a}, DG * ADT \Big] \end{aligned}$$

(Eq.11)

Where: ADT = $\sim\Delta T$ **x 10**^{5}

Statistics:

N=432, R^2=0.97, SEE= 0.051

Limits:

$$0.05 \le a/ª \le 0.3, 2 \le L/ª \le 15, W/ª = L/ª,$$
$$5.5 \le ADT \le 22, 1 \le DG \le 10, 2 \le DP \le 130$$

7 PRACTICAL DESIGN EXAMPLES

Consider a pavement slab with the following characteristics: E = 3 Mpsi, k = 400 pci, L = 141 in., W = 141 in., h = 9.97 in., γ = 0.224 pci, **μ** = 0.15, and **α** = 5.5 x 10^{-6} / ^{o}F. A single wheel load of 7,624 lbs with a loaded rectangle of the size of 10x10 in^2 is applied at the slab corner. A linear temperature differential of -10 ^{o}F (night-time curling) exists through the slab thickness. Determine the critical corner stresses due to loading alone, and loading plus curling. (Note: 1 psi = 6.89 kPa, 1 pci = 0.27 MN/m^3, 1 in. = 2.54 cm, 1 ^oF = (F - 32) / 1.8 ^oC, 1 lb = 4.45 N.)

The equivalent radius of the loaded area is a = 5.64 in. and the radius of relative stiffness of the slab-subgrade system is l = 28.21 in. Therefore, the actual dominating mechanistic variables are a/l = 0.2, L/l = W/l = 5, $\sim\Delta T$ = 5.5E-05, D_γ = 7E-05, and D_p = 30E-05. The theoretical Westergaard solutions based on (Eq.1) and (Eq.4) are $\pm_w$ = 122.2 psi and $\pm_0$ = 97.1 psi for loading and curling alone.

For the case of loading only, the adjustment factor R_L = 1.062 using (Eq.9). Thus, the corner stress determined by the proposed model is 1.062 x 122.2 = 129.8 psi. (Note that the actual ILLI-SLAB stress was 129.1 psi.)

For the case of loading plus curling, the adjustment factor is R_T = 0.188 based on (Eq.11). Thus, the predicted total corner stress determined by the proposed model is 129.8 + 0.188 x 97.1 = 148.1 psi using (Eq.10) . (Note that the actual ILLI-SLAB corner stress was 147.5 psi for this case.)

8 CONCLUSIONS

The corner stress of a concrete slab due to the individual and combination effects of loading and night-time curling was conducted under this study. A linear temperature differential across the slab thickness and a dense liquid foundation were assumed. Based on the principles of dimensional analysis, six dimensionless mechanistic variables which dominate the primary structural responses were used for the analysis. A new modeling procedure was utilized to develop stress prediction models.

The prediction models were properly formulated to satisfy applicable engineering boundary conditions. The models not only cover almost all practical ranges of pavement designs, but they are also dimensionally correct. These models can be implemented as a part of a design procedure to the very time-consuming and complicated F.E. analysis to estimate stresses for design purposes with efficiency and sufficient accuracy. A practical design example showing the use of the models was provided and carefully validated.

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