The 8th REAAA Conference

# Theoretical Investigation of Corner stress in Concrete Pavements Using Dimensional Analysis

Speaker:
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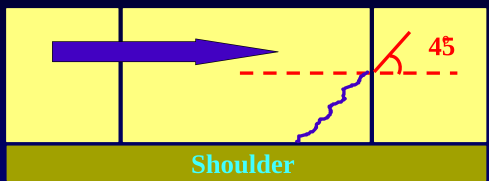
## OBJECTIVES

- Determine the critical bending stresses at the slab corner due to loading & curling
  - Free corner loading combined with negative temperature gradient (upward curling)
- Develop an alternative stress determination process that could be utilized in a spreadsheet or PC program for rapid calculation purposes
- May become a part of mechanistic design procedure and encourage use by practitioners

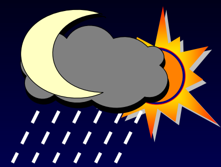
## Corner Break

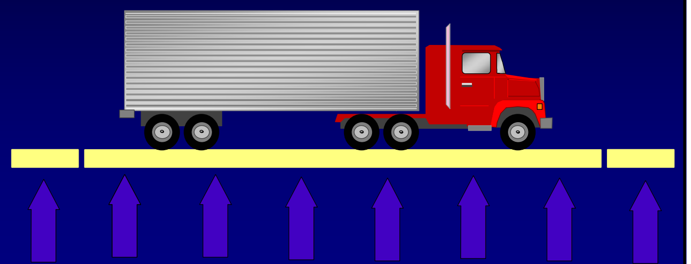



- Load stress - located at free corner
- curling stress - negative temp. gradient (nighttime)
- combined stress

## Main Causes

- *Load repetition combined with loss of support and curling stresses*



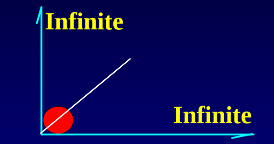


## Research Approach

- Compare Theoretical Westergaard/Bradbury closed-form solutions & F.E. computer program (ILLI-SLAB)
- Identify the dominating mechanistic variables using the principle of dimensionless analysis
- Investigation the characteristics of pavement structural response due to loading & curling
- Conduct factorial F.E. runs
- Develop stress prediction models using the PPR technique

## Corner loading only

Westergaard/Bradbury




$$\sigma_w = \frac{3P}{h^2}\left[1-\left(\frac{a}{l}\right)^{0.6}\right]$$

$$\frac{\sigma h^2}{P} = f\left(\frac{a}{l},\frac{W}{l},\frac{L}{l}\right)$$

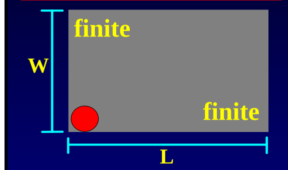
$$l = \sqrt[4]{\frac{Eh^3}{12(1-\mu^2)k}}$$

Critical stress locations along the corner bisector

$X_1 = 2.38\sqrt{al}$

## Corner loading only

ILLI-SLAB Model




$$\frac{\sigma h^2}{P} = f\left(\frac{a}{l},\frac{W}{l},\frac{L}{l}\right)$$

The ILLI-SLAB stress are the values of the minor principal (tensile) stress occurring at the top fiber of the slab

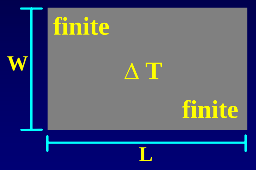
## Thermal curling

Westergaard/Bradbury


No explicit closed-form coner stress solutions

# Thermal curling

## ILLI-SLAB Model




$$\frac{\sigma}{E} = f\left(\frac{a}{l}, \frac{W}{l}, \frac{L}{l}, D_r\right)$$

$$D_r = \frac{\gamma h^2}{k l^2}$$

# Loading plus thermal curling

## Westergaard/Bradbury


$$\sigma_T = \sigma_w + \sigma_{ct}$$

$$\sigma_{ct} = \frac{E\alpha\Delta T}{3(1-\mu)} \sqrt{\frac{a}{l}}$$

# Loading plus thermal curling

## ILLI-SLAB Model

$$\frac{\sigma h^2}{P} = f\left(\frac{a}{l}, \frac{W}{l}, \frac{L}{l}, D_r, D_p\right)$$

Dr: $\frac{\gamma h^2}{k l^2}$

Dr: $\frac{P h^2}{k l^4}$

# Now, We are ......

- ✓ Compare Theoretical Westergaard/Bradbury closed-form solutions & F.E. computer program (ILLI-SLAB)
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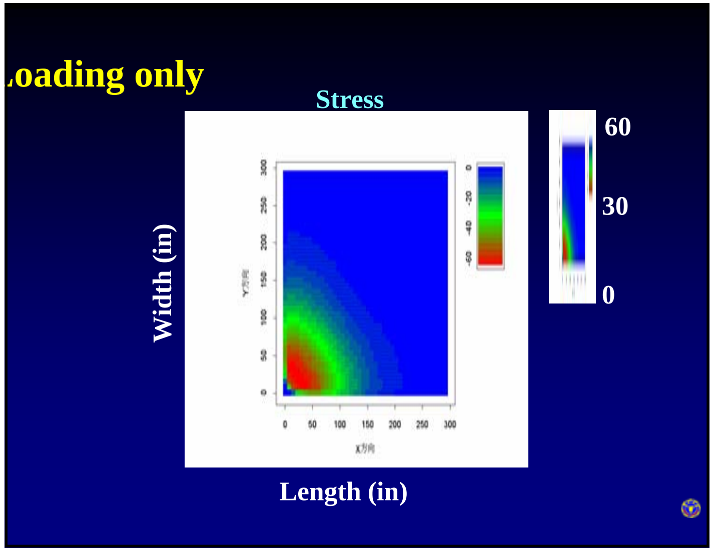
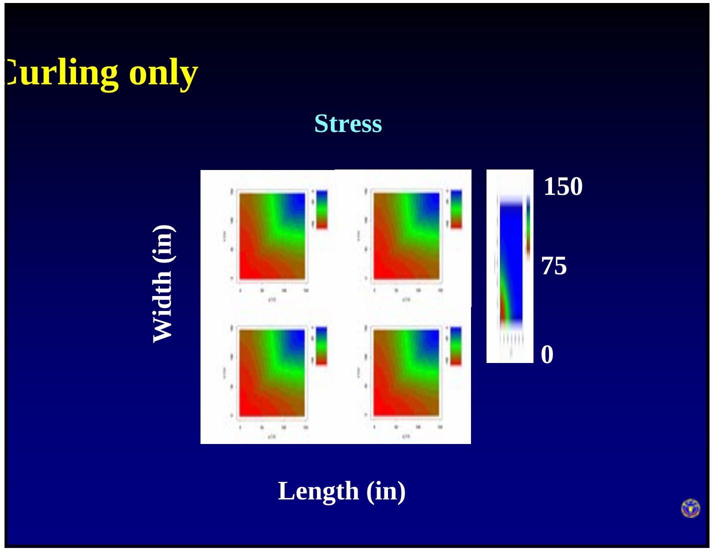
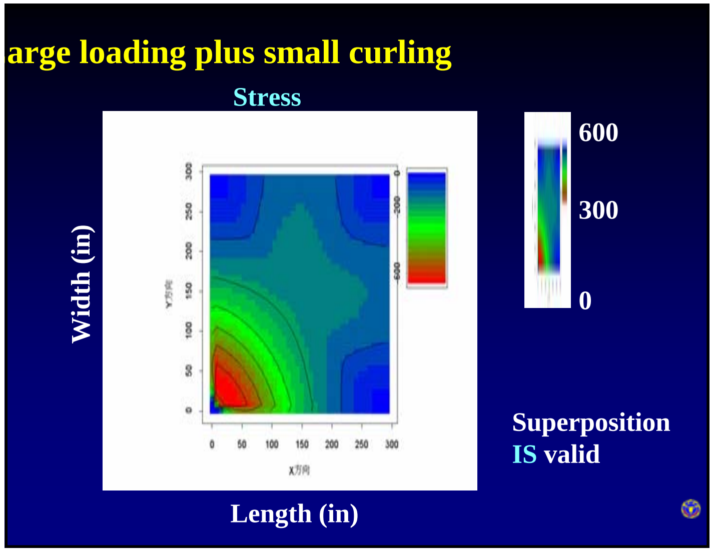
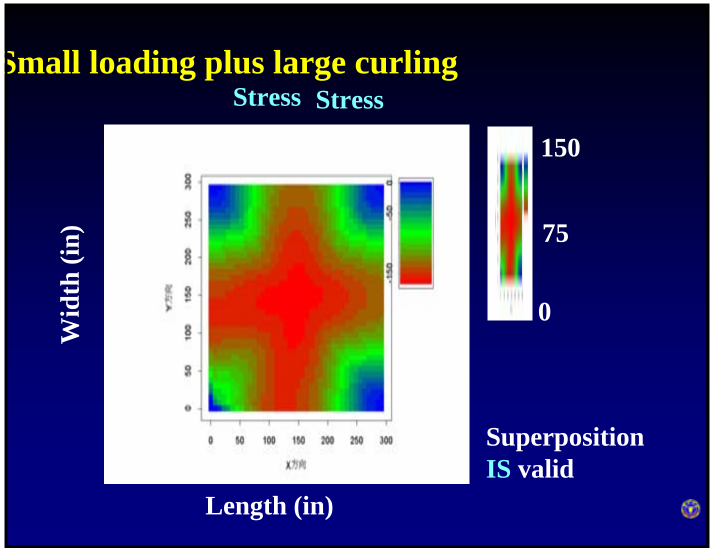

oading only
Stress
Width (in)
Length (in)
60
30
0


urling only
Stress
Width (in)
Length (in)
150
75
0

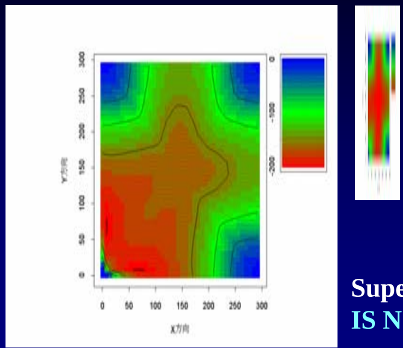

arge loading plus small curling
Stress
Width (in)
Length (in)
600
300
0
Superposition
IS valid


mall loading plus large curling
Stress Stress
Width (in)
Length (in)
150
75
0
Superposition
IS valid

## Medium loading plus Medium curling

Stress

Width (in)



200

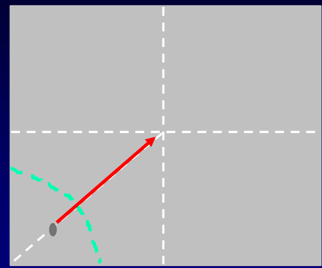
100

0

Length (in)

Superposition IS NOT valid

## Location of Maximum Stress



## Now, We are……

- Compare Theoretical Westergaard/Bradbury closed-form solutions & F.E. computer program (ILLI-SLAB)
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## Stress prediction model for loading only

### Factor F.E. runs

a/*l*: 0.05,0.1,0.2,0.3
L/*l*:2,3,4,5,6,7
W/*l*:2,3,4,5,6,7(L/l>=W/l)
Total of 84 runs
$\sigma_i = R_L \times \sigma_w$

$$R_L = f\left(\frac{a}{l}, \frac{W}{l}, \frac{L}{l}\right)$$

### Statistics

N=84,R =0.980,
SEE=0.0081,CV=0.79%

### Limits

$0.05 \leq a/l < 0.3$
$2 \leq L/l \leq 7$
$W/l \leq L/l$

## Stress prediction model for loading plus curling

- **Factor F.E. runs**

**$a/l$: 0.05,0.1,0.2,0.3**
**$L/l$:2,3,4,5,7,9,11,13,15**
**($L/l=W/l$)**
**Total of 432 runs**

$\sigma_i=\sigma_L+R_T \times \sigma_o$

$$R_T = \frac{\sigma_i-\sigma_L}{\sigma_o}$$

| a/l | (DG,DP) |
|---|---|
| 0.05 | (1,2) (10.30) (7,130) |
| 0.10 | (4,30) (7,70) (4,130) |
| 0.20 | (4,2) (7,30) (10,70) |
| 0.30 | (1,2) (10,70),1,130) |

## Stress prediction model for loading plus curling

- **Statics**

**N=432,**

**R =0.97,**

**SEE=0.051**

- **Limits**

$0.05 < a/l < 0.3$
$2 < L/l < 15$
$W/l = L/l$
$5.5 < ADT < 22$
$1 \leq DG \leq 10$
$2 \leq DP \leq 130$

## Validation of the models

## Conclusions

- **Developed alternative stress prediction models**
- **Investigation the characteristics of pavement structural response due to loading & curling**
- **Identified the dominating mechanistic variables using the principle of dimensionless analysis**
- **very accurate representation of the predict model**

## Acknowledgments

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## *THANKS FOR YOUR ATTENTION!*

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