6th Int. DUT-Workshop on Fundamental Modelling of Design and Performance of Concrete Pavements

Application of Modern Regression Techniques and Artificial Neural Networks on Pavement Prediction Modeling

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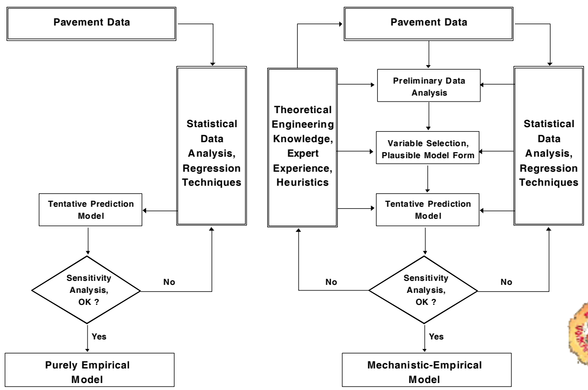
INTRODUCTION

- Prediction Models: (pavement analysis, design, rehabilitation, PMS)
- Model Development Using Purely Empirical and Mechanistic-Empirical Approaches
- Systematic Statistical and Engineering Approach (Lee, 1993)

Model Development Using Purely Empirical or Mechanistic-Empirical Concept (Lee, 1993)



Previous Work on Pavement Prediction Modeling

- **Application of Modern Regression Techniques**
  - Using conventional "parametric" linear and nonlinear & several "robust" and "nonparametric" regression techniques (Lee, 1993; etc.)
  - Developed pavement performance and structural response prediction models
- **Application of Artificial Neural Network (ANN) Techniques**
  - Pavement structural evaluation for simulated data:
    - Often use original input parameters to generate the training and testing data.
    - Some parameters were fixed to certain prescribed values to reduce the database size. Result in limiting the inference space of the resulting model.
    - Nevertheless, some literature also illustrated the advantages of using the principles of dimensional analysis when generating the data.
  - Some built-in functions including learning rate and momentum term which form key neural network algorithm were not adequately investigated (Attoh-Okine, 1994;1999)
  - Adding many hidden layers gets the network to learn faster and the mean square error becomes a little smaller, but the generalization ability of the network reduces. (Sorsa et al., 1991)

Previous Work on Pavement Prediction Modeling (continue …)

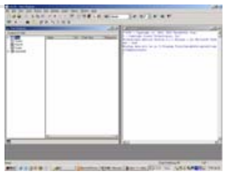
- Ripley (1993) discussed many statistical aspects of neural networks and tested it with several benchmark examples against traditional and modern regression techniques and concluded that **in one sense neural networks are little more than non-linear regression and allied optimization methods**.
- "That two-layer networks can approximate arbitrary continuous functions does not change the validity of more direct approximations such as **statistical smoothers, which certainly 'learn' very much faster**" (Ripley, 1993).
- **Statistical and subject-related knowledge** can be used to guide modeling in most real-world problems and so enable much more convincing generalization and explanation, in ways which can never be done by **'black-box' learning systems** (Ripley, 1993).

OBJECTIVES

- To illustrate the benefits of incorporating the principles of dimensional analysis, subject-related knowledge, and statistical knowledge into pavement prediction modeling process
  - Using local regression & ANN techniques
- Case Studies:
  - To improve the prediction accuracy of simulated pavement deflections (using factorial 2-D and 3-D finite element runs and BISAR runs for different pavement systems)

Modern Regression & ANN Techniques

- Projection Pursuit Regression
  - Revised Two-Step Modeling Approach Using PPR
- Locally-Weighted Regression (loess)
  - Concept of loess k-d tree algorithm
- Statistical Software Used
  - S-PLUS 6.1
  - LOCFIT Program
- Artificial Neural Networks
  - QNET2000 Program



S-PLUS 6.1

QNET2000 Program

Projection Pursuit Regression *(PPR)*

$$y = \bar{y} + \sum_{m=1}^{M_0} \beta_m \, \phi_m (a_m^T x) + \varepsilon$$

$$E\left[\phi_m (a_m^T x)\right] = 0 \, , \; E\left[\phi_m^2 (a_m^T x)\right] = 1$$

Minimizing the mean squared residuals:

$$E\left[r^2\right] = E\left[y - \bar{y} - \sum_{m=1}^{M_0} \beta_m \phi_m (a_m^T x)\right]^2$$

* capable of modeling variable interactions (Friedman and Stuetzle, 1981)

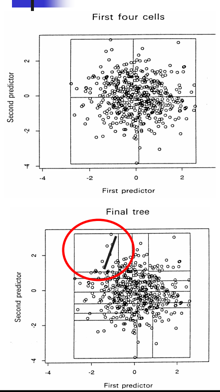
Revised Two-Step Modeling Approach Using PPR (Lee & Darter, 1994)

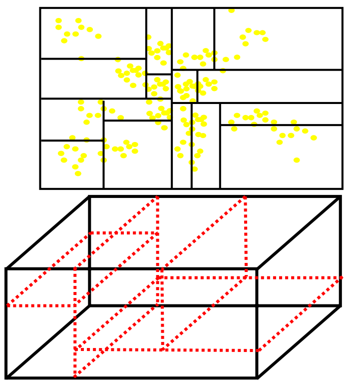
- Step 1:
  - **Use Projection Pursuit Regression (PPR)**
  - Model the multi-dimensional response surface as a sum of several smooth projected curves, graphically representable in 2-D.
- Step 2:
  - Plausible functional forms and applicable boundary conditions may then be easily identified and specified.
  - **Traditional linear, piecewise-linear, and nonlinear regressions** are then utilized to model each projected curve.
- Revised Step 2:
  - **Regression spline algorithm** was adopted here to assure smooth junctions at the change points.

Application of Locally-Weighted Regression (LOESS) Technique

- **An approach to regression analysis by local fitting (Cleveland & Devlin ,1988; Cleveland & Grosse, 1991)**
- **A particular data structure called k-d tree is used for partitioning space by recursively cutting cells in half by a hyperplane orthogonal to one of the coordinate axes.**
- **Use a smoothing technique for fitting a nonlinear curve to the data points locally, so that any point of the curve depends only on the observations at that point and some specified neighboring points.**
- **Provide much greater flexibility in fitting a multi-dimensional response surface as a series of many sub-divided regions with single smooth functions of all the predictors.**

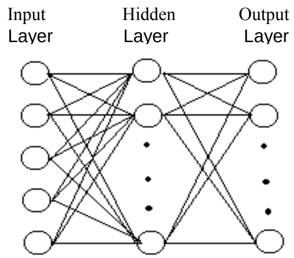
Illustration of loess k-d tree algorithm (Cleveland & Grosse, 1991)





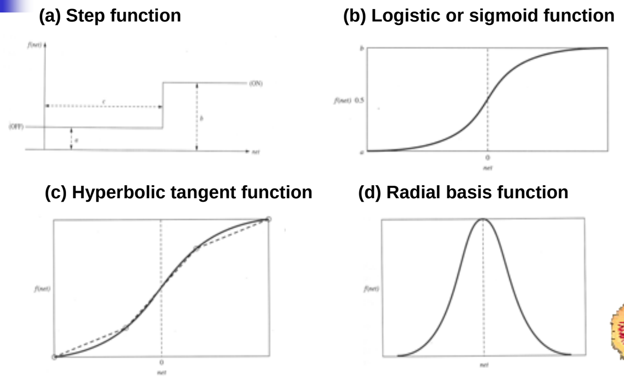
Artificial Neural Networks

- A flexible way to generalize linear regression functions (but with so many parameters)
- Commonly using generalized delta rule or the steepest descent of gradient method (Back Propagation Network, BPN)
- Training & Testing Data (Over Learning Problem)



$$Min. \frac{1}{2}\sum_{p}\left\| y^{p} - c^{p} \right\|^{2}$$

Various Activation (or Transfer) Functions



13

Applications of ANN & Modern Regression Techniques

- Rigid Pavement Deflection Prediction Models
  - Case I: 2-D Infinite slab
  - Case II: 2-D Finite slab
  - Case III: 3-D Finite slab

$$R_1 = \frac{\delta_{FEM}}{\delta_{max}} = f_1\left(\frac{a}{\ell}, \frac{r}{\ell}\right)$$

$$R_2 = \frac{\delta_{FEM}}{\delta_{max}} = f_2\left(\frac{a}{\ell}, \frac{r}{\ell}, \frac{L}{\ell}, \frac{W}{\ell}\right)$$

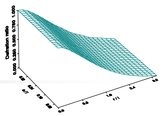
$$\frac{1}{R} = \frac{\delta_{Westergaard}}{\delta_{3D}} = f_3\left(\frac{a}{\ell}, \frac{L}{\ell}, \frac{h}{a}\right)$$

$$\ell = \sqrt[4]{\frac{Eh^3}{12(1-\mu^2)k}}$$

- Flexible Pavement Deflection Prediction Models
  - Case IV: BISAR Runs

14

Case I: Data Preparation



- ILLI-SLAB FE Program
- Input parameters:
  - P=40 kN,
  - p=0.62 MPa,
  - E= 13.78~48.23 GPa,
  - k=13.5~175.5 MN/m^3,
  - h= 15.2~76.2 cm
  - r determined by mesh

  (N=12,329)

- Using Dimensional Analysis
  - a/ℓ : 0.05~0.4 (step 0.01)
  - L/ℓ=W/ℓ=8
  - r/ℓ : 0~3.2 determined by mesh generation

  (N=494)

15

Case I: Comparing ANN Models

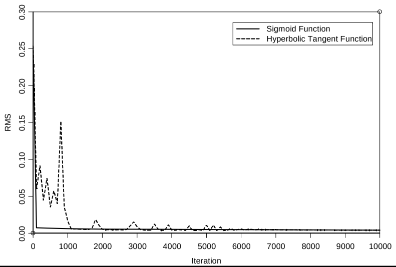
ANN Type	NET1	NET2
Outputs	R	R
Inputs	E, k, h, r	*a/ℓ, r/ℓ*
Data Points	Training: 11,329 Monitoring: 1,000	Training: 394 Monitoring: 100
Hidden Layer (s)	2	1
Neurons in Each Hidden Layer	12-12	6
Learning Cycle	30,000	10,000
Learning Rate	0.5	0.1
Modeling Time	6 hrs 43 min.	42 min.
RMS	Training: 0.00290 Monitoring: 0.00420	Training: 0.00377 Monitoring: 0.00360
Coefficient of Determination, R^2	0.999	0.9999

Note: Benefit of Using Dimensional Analysis (smaller & faster)

16

Case I: Convergence Characteristics

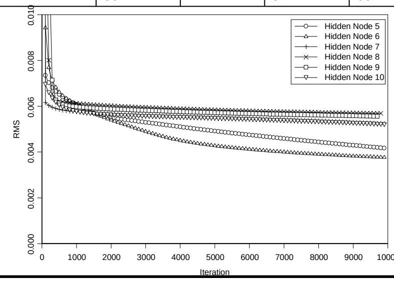
Function	Logistic or sigmoid	Hyperbolic tangent	Radial basis / Step
Training RMS	0.00416	0.00405	Cannot converge
Monitoring RMS	0.00384	0.00411	NA
R-Squared	0.9999	0.9999	NA
Time	35"	60"	NA



Note: Convergence characteristics of various transfer functions

Case I: Convergence Characteristics
(continue …)

Neurons in hidden layer	5	6	7	8	9	10
Training RMS	0.00416	0.00377	0.00524	0.00569	0.00554	0.00520
Monitoring RMS	0.00384	0.00360	0.00492	0.00529	0.00520	0.00490
R-Squared	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
Time	35"	42"	52"	60"	67"	82"

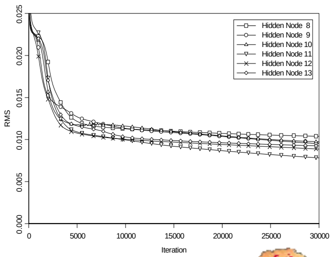


Note:
Increase the # of neurons does NOT necessarily improve the fit.

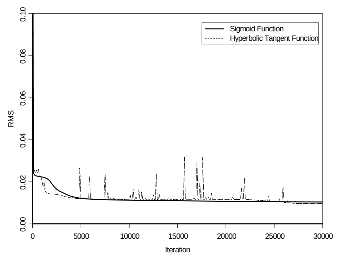
Case II: Data Preparation

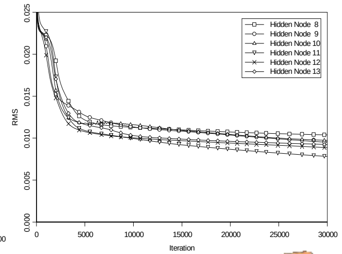
- ILLI-SLAB FE Program
- Using Dimensional Analysis
  - a/ℓ : 0.05~0.4
  - L/ℓ : 2~7 (Step 1)
  - W/ℓ : 2~7 (Step 1)
  - r/ℓ : 0~3.2 determined by mesh generation

  (N=2,227)



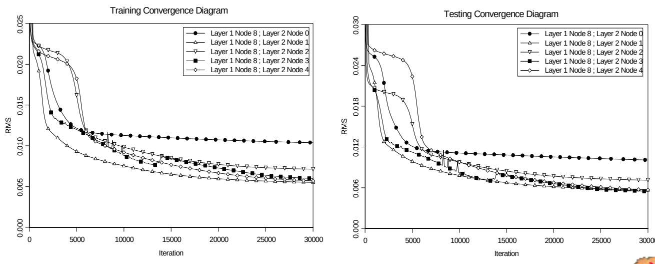
Case II: Convergence Characteristics





Note: due to (a) various transfer function; (b) different # of neurons (with only 1 hidden layer)

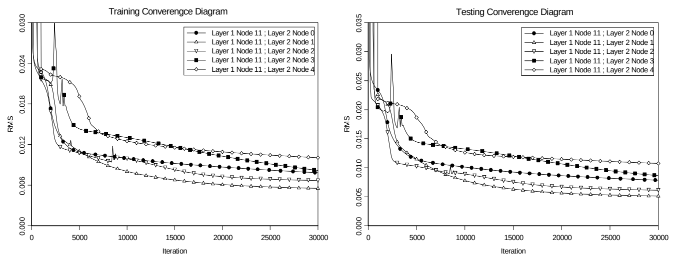
Case II: Convergence Characteristics
(Continue …)



Note: Two hidden layer (with 8 neurons in layer 1) converges ok!

21

Case II: Convergence Characteristics
(Continue …)

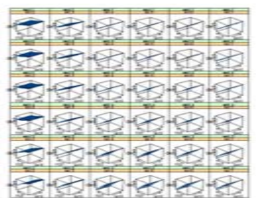


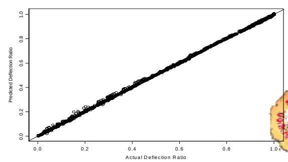
Note: Using higher # of hidden layers and neurons sometimes lead to even worse fit, i.e., indication of over training to be avoided.

22

Case II: Loess Model

- Smoothing Parameter:
  - span=0.1
  - cell=0.01
- Regression Statistics:
  - N = 2,227
  - equivalent number of parameters = 31.9
  - SEE = 0.006376
  - R-squared = 1
- $R_2 = \frac{\delta_{FEM}}{\delta_{\max}} = f_2\left(\frac{a}{\ell}, \frac{r}{\ell}, \frac{L}{\ell}, \frac{W}{\ell}\right)$



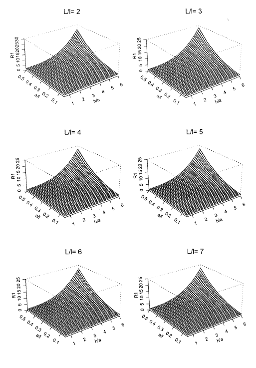


23

Case III: Data Preparation

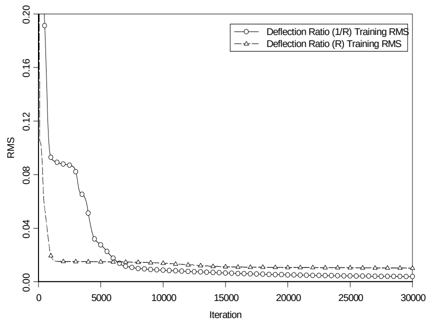
- ABAQUS 3-D FE Program
- Using Dimensional Analysis
  - a/ℓ：0.05, 0.1~0.5 (step 0.1)
  - L/ℓ=W/ℓ：2~8 (step 1)
  - h/a：0.5~6 (step 0.5)
  - Maximum deflection (r=0)

(N=504)



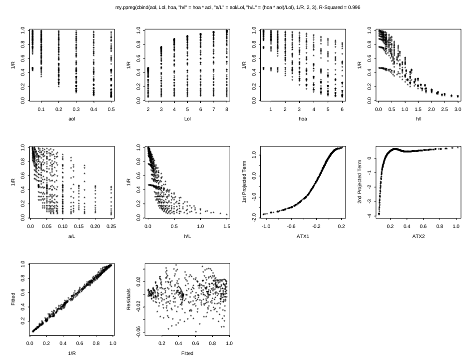
24

Case III: Comparing ANN Models



Note: Incorporating Subject Related Knowledge (1/R is better)

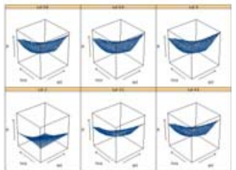
Case III: Proposed PPR Model

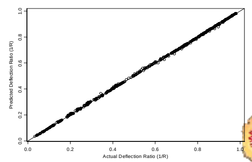


Note: With regression spline (R^2=0.9942, SEE=0.02241)

Case III: Loess Model

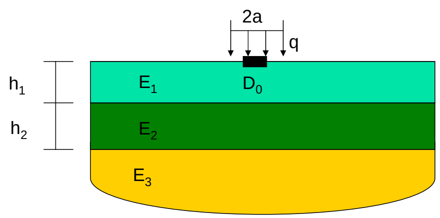
- Smoothing Parameter:
  - span=0.1
  - cell=0.1
- Regression Statistics:
  - N = 504
  - equivalent number of parameters = 56.6
  - SEE = 0.004784
  - R-squared = 1
- $\frac{1}{R} = \frac{\delta_{Westergaard}}{\delta_{3D}} = f_3\left(\frac{a}{\ell}, \frac{L}{\ell}, \frac{h}{a}\right)$





Case IV: Data Preparation

- Factorial BISAR Runs (Flexible Pavement Deflection)
  - a/h_2: 0.2, 0.4, 0.8, 1.2, 1.8, 2.4
  - h_1/h_2: 0.5, 1.0, 1.5, 2.0, 4.0, 5.0
  - $E_2/E_3, E_1/E_2$: 0.5, 1.0, 2.0, 5.0, 10, 30, 50, 90, 140, 170
  - Training Data = 3,600, Testing Data = 1,728



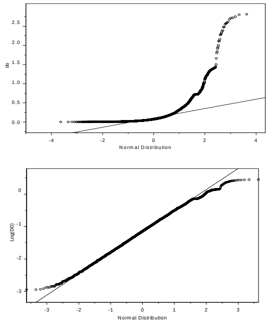
Case IV: Comparing ANN Models

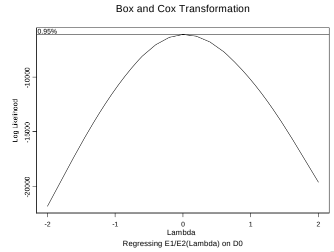
ANN Type	NET1	NET2	NET3
Outputs	D_0	Log(D_0)	Log(D_0)
Inputs	E_1/E_2, E_2/E_3, h_1/h_2, a/h_2	E_1/E_2, E_2/E_3, h_1/h_2, a/h_2	log(E_1/E_2), log(E_2/E_3), h_1/h_2, a/h_2
Hidden Layer(s)	3	3	2
Neurons in Each Hidden Layer	20-10-5	15-10-5	12-6
Learning Cycle	Cannot converge	200,000	27,000
Modeling Time	> 24 hrs	10 hrs	26 min
RMS	---	Training: 0.0048 Monitoring: 0.0045	Training: 0.0040 Monitoring: 0.0039

Note: Benefit of Incorporating Statistical Knowledge (Power Transformation)

29

Benefit of Incorporating Statistical Knowledge



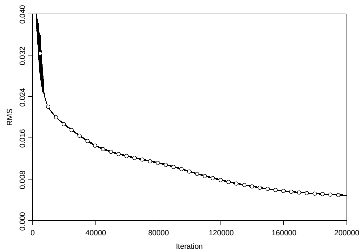


Note: Normality test (Q-Q plot) & Box-Cox Power Transformation

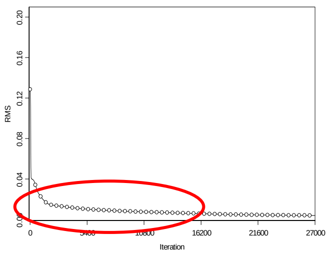
30

Comparing Convergence Characteristics

NET2



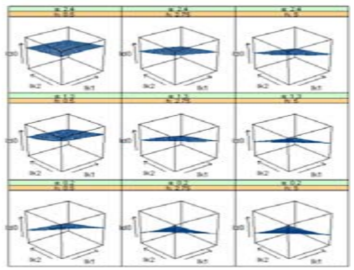
NET3



31

Case IV: Loess Model

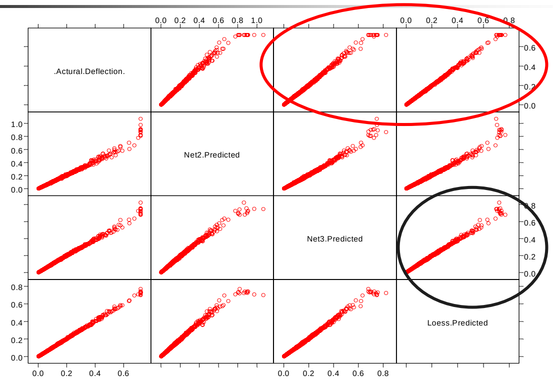
- Smoothing Parameter:
  - span=0.1
  - cell=0.1
- Regression Statistics:
  - N = 3,600
  - equivalent number of parameters = 31.9
  - SEE = 0.02792
  - R-squared = 1



- $$\log(D_0) = f_4\left(\log(\frac{E_1}{E_2}),\quad \log(\frac{E_2}{E_3}),\quad \frac{h_1}{h_2},\quad \frac{a}{h_2}\right)$$

32

Model Comparison (Testing Data)



Concluding Remarks

- Illustrated the benefits of incorporating
  - the principles of dimensional analysis,
  - subject-related knowledge, and
  - statistical knowledge

  into pavement prediction modeling process
- Proved to have higher accuracy
- Required smaller data and less network training time
- Increasing the complexity of ANN models does NOT necessarily improve the fit

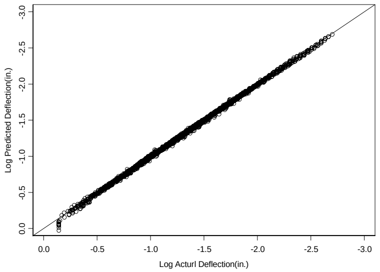
Concluding Remarks (Continue ...)

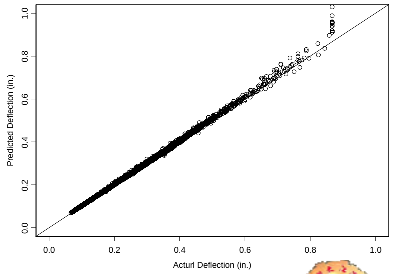
- Using higher # of neurons and hidden layers sometimes lead to even worse fit (indication of over training to be avoided)
- Reasonable good predictions can be achieved using both ANN and modern regression techniques
- Statistical and subject-related knowledge can be used to guide modeling and so enable much more convincing generalization and explanation, in ways which can never be done by "black-box" learning systems (Ripley, 1993)

Questions?

THANKS FOR YOUR ATTENTION

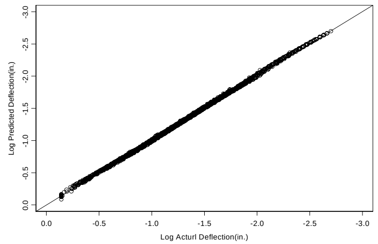
NET2 Goodness of Fit (NET2)

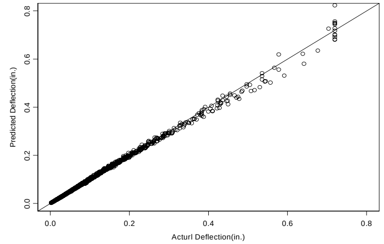




Predicting Log (D_0) Predicting D_0

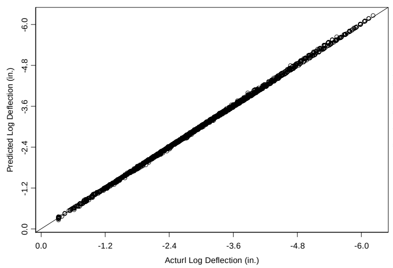
NET3 Goodness of Fit

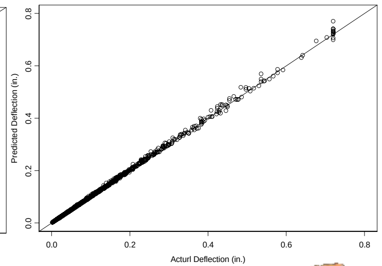




Predicting Log (D_0) Predicting D_0

Loess Model: Goodness of Fit





Predicting Log (D_0) Predicting D_0