

CORNER LOADING AND CURLING STRESS ANALYSIS OF CONCRETE PAVEMENTS

混凝土鋪面版之角隅荷重與翹曲應力分析

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ABSTRACT

Since corner breaks are one of the major structural distresses in jointed concrete pavements, this research study focuses on the determination of the critical bending stresses at the corner of the slab due to the individual and combination effects of wheel loading and thermal curling. A well-known slab-on-grade finite element program (ILLI-SLAB) was used for the analysis. Based on the principles of dimensional analysis, the dominating mechanistic variables were carefully identified and verified. The resulting corner stresses were compared to theoretical Westergaard solutions. Adjustment factors were introduced to account for this discrepancy. Prediction models were developed as an alternative to the very time-consuming and complicated F.E. analysis to estimate stresses for design purposes with sufficient accuracy.

INTRODUCTION

Cracking of jointed concrete pavements (JCP) is often caused by three different critical repeated loading positions: transverse joint, longitudinal joint midway between transverse joints, and at the corner. Given certain design, construction, and loading conditions, any of these load positions could lead to fatigue cracking of the slab over time. "**Load repetition combined with loss of support and curling stresses**" are usually recognized as the main causes for corner breaks. Thus, this paper focuses on the determination of critical bending stresses **at the slab corner** due to loading and curling.

CLOSED-FORM SOLUTIONS

For a slab-on-grade pavement system, Westergaard has presented closed-form solutions for three primary structural response variables, i.e., slab bending stress,

deflection, and subgrade stress, due to a single wheel load based on medium-thick plate theory. By assuming an infinite or semi-infinite slab over a dense liquid (Winkler) foundation, Westergaard obtained the following equation for a circular corner load [1]:

$$\tau_w = \frac{3P}{h^2} \left[1 - \left(\sqrt{2} \frac{a}{\lambda} \right)^{0.6} \right] , \quad u_w = \frac{P}{k\lambda^2} \left[1.1 - 0.88 \left(\sqrt{2} \frac{a}{\lambda} \right) \right] \quad (\text{Eq.1})$$

Where σ_w is the critical corner stress, [FL⁻²]; δ_w is the critical corner deflection, [L]; P is the total applied wheel load [F]; h is the thickness of the slab [L]; a is the radius of the applied load [L]; $\lambda = [Eh^3 / (12 * (1 - \mu^2) * k)]^{0.25}$ is the radius of relative stiffness of the slab-subgrade system [L]; k is the modulus of subgrade reaction [FL⁻³]; E is the modulus of elasticity of the concrete slab [FL⁻²]; and μ is the Poisson's ratio of the concrete. Note that primary dimensions are represented by [F] for force and [L] for length. The distance to the point of maximum stress along the corner angle bisector was roughly:

$$X_1 = 2\sqrt{\sqrt{2}a} \cong 2.38\sqrt{a} \quad (\text{Eq.2})$$

The above stress and deflection equations were derived using a simple approximate process and has been debated and led to numerous revisions such as those proposed by Bradbury, Kelly, Teller and Sutherland, Spangler, and Pickett over the years [2]. Despite this argument, Ioannides et al. [3] later has indicated that the ILLI-SLAB F.E. results closely fall between those predicted by Westergaard and Bradbury. The ILLI-SLAB stresses are the **minor principal (tensile) stresses** occurring at the top fiber of the slab corner. Thus, Westergaard's approximation was still fairly good.

Considering curling stresses caused by a linear temperature differential on a concrete slab over a dense liquid foundation, Westergaard [4] developed equations for three slab conditions (i.e., infinite, semi-infinite, and an infinite long strip). The interior stress for an infinite slab is:

$$\tau_0 = \frac{Er\Delta T}{2(1 - \mu)} \quad (\text{Eq.3})$$

Where σ_0 is the interior curling stress, [FL⁻²]; α is the thermal coefficient, [T⁻¹]; and ΔT is the temperature differential through the slab thickness, [T]. Primary dimensions are represented by [F] for force, [L] for length, and [T] for temperature. Bradbury [5] later expanded Westergaard's bending stress solutions for a slab with finite dimensions. However, there exists no explicit closed-form corner stress solutions.

Even though Westergaard and Bradbury all suggested that the combined effect of loading plus curling could be treated as "a simple matter of addition" in most cases,

many investigators have indicated that such an action may not always be conservative [3, 6] due to the possible loss of subgrade support and violation of full contact assumptions.

F.E. COMPUTER PROGRAM

The analysis of finite slab length and width effect was not possible until the introduction of finite element models. The ILLI-SLAB F. E. program developed at the University of Illinois since 1977 was used for the analysis. The present version (March 15, 1989) [6] was successfully compiled on Unix-based workstations of the Civil Engineering Department at Tamkang University. With some modifications to the original codes, a micro-computer version of the program was also developed using Microsoft FORTRAN PowerStation [7].

CHARACTERISTICS OF CORNER STRESSES

A preliminary analysis of the structural response characteristic of a slab has indicated that the location of the maximum combined stress due to loading plus curling varies from case to case. Thus, unlike the analysis of interior or edge stresses where the maximum stresses occur at the same critical center or mid-slab location, the analysis of corner stresses is probably the most difficult one among these three cases. Assume a slab with the following characteristics: $L/\lambda = 7$, $W/\lambda = 7$, $\lambda = 41.86$ in., $h = 12$ in., $k = 240$ pci, $E = 5$ Mpsi, $\gamma = 0.087$ pci, $\mu = 0.15$, $a/\lambda = 0.1$, $c = 7.5$ in., $\Delta T = -20$ °F, and $\alpha = 5.5E-06$ /°F, as illustrated in Figure 1 (a).

The individual and combined stress contour plots of all cases were shown in Figure 1 (b) - (f), respectively. In summary, if the temperature differential is relatively small combined with a large corner load, the critical stress location is very close to Westergaard's maximum load stress location (Eq.2). However, if the temperature differential is very large along with a very small corner load, the critical stress location may shift toward and up to the center of the slab. For the combined effects of medium loading and medium curling, the maximum stress location falls

between them. Thus, the location of the maximum combined stresses due to loading plus curling will fall within the Westergaard's location and the center of the slab along the corner angle bisector as illustrated in the above figure. Furthermore, the corner stress along the line of a 1/4 circle centered at the very corner of the slab also shows about the same magnitude at most locations. This may help to explain the mechanism of the development of corner breaks as well.

Consequently, necessary modifications were made to the existing ILLI-SLAB codes to facilitate the search of critical stresses and locations along the corner angle bisector or the diagonal nodes up to the center of the slab for the remaining analyses.

IDENTIFICATION OF DIMENSIONLESS VARIABLES

Investigators [8] have demonstrated that F.E. solutions for three primary structural responses due to a single wheel load can be concisely defined by the following expression for a constant Poisson's ratio (usually $\mu \approx 0.15$) using dimensional analysis:

$$\frac{\sigma h^2}{P}, \frac{\delta k^2}{P}, \frac{q^2}{P} = f_1\left(\frac{a}{L}, \frac{L}{W}\right) \quad (\text{Eq.4})$$

Where σ , q are slab bending stress and vertical subgrade stress, [FL⁻²]; δ is the slab deflection, [L]; f_1 is a function of a/L , and W/L ; and L , W are finite slab length and width, [L]. Also Note that variables in both sides of the expression are all dimensionless. The dependent variables are $\sigma h^2/P$, $\delta k^2/P$ and q^2/P , which are only dominated by the normalized load radius (a/L), and the normalized slab length and width (L/W) rather than the other input parameters, such as E , h , k , a , etc.

Furthermore, according to recent research by Lee and Darter [8] for the stress analysis at the very edge of the slab, concise relationships have been proposed and numerically validated through a series of F.E. runs. The dimensionless variables due to the effects of curling alone and loading plus curling for a constant Poisson's ratio are:

$$\begin{aligned} \frac{\tau}{E}, \frac{u h}{k^2}, \frac{q h}{k^2} &= f_2\left(r \Delta T, \frac{L}{W}, \frac{\lambda h^2}{k^2}\right) \\ \frac{\tau}{E}, \frac{u h}{k^2}, \frac{q h}{k^2} &= f_3\left(\frac{a}{L}, r \Delta T, \frac{L}{W}, \frac{\lambda h^2}{k^2}, \frac{p h}{k^4}\right) \\ D_x &= \frac{\lambda h^2}{k^2}, D_p = \frac{p h}{k^4} \end{aligned} \quad (\text{Eq.5})$$

Where τ is the unit weight of the concrete slab, [FL⁻³]; and f_2 , f_3 are functions for curling alone and curling plus loading, respectively. Also note that D_x was defined as

the relative deflection stiffness due to self-weight of the concrete slab and the possible loss of subgrade support, whereas D_p was the relative deflection stiffness due to the external wheel load and the loss of subgrade support. Conceptually, the above relationship should be applicable to any given loading conditions.

CORNER STRESS PREDICTION MODELS

A series of F. E. factorial runs were performed based on the dominating mechanistic variables identified [9]. Several BASIC programs were written to automatically generate the F. E. input files and summarize the desired outputs. The F. E. mesh was generated according to the guidelines established in earlier studies [2]. As proposed by Lee and Darter [10], the projection pursuit regression (PPR) introduced by Friedman and Stuetzle [11] was used for the development of the following three stress prediction models. This algorithm is available in the S-PLUS statistical package [12].

CASE I: Loading Only

In CASE I, a single wheel load was applied at the slab corner alone. The following factorial F.E. runs were conducted:

$$a/}: 0.05, 0.1, 0.2, 0.3; \quad L/}: 2, 3, 4, 5, 6, 7; \quad W/}: 2, 3, 4, 5, 6, 7 \quad (L/} \quad W/})$$

Since $L/}$ and $W/}$ are analogous, a total of 84 runs were only necessary if slab length was chosen to be greater than slab width. The resulting corner stresses were compared to Westergaard solution. The following prediction model was developed:

$$R = \frac{f_{i(CASE I)}}{f_w} = 1.030 + 0.030\Phi_1 + 0.045\Phi_2$$

$$\Phi_1 = 92.415 - 149.276(A1) + 59.747(A1)^2$$

$$\Phi_2 = \begin{cases} -6.034 + 23.128(A2) - 22.022(A2)^2 & \text{if } A2 \leq 0.6 \\ -0.117 + 0.375(A2) & \text{if } 0.6 < A2 \end{cases}$$

$$A1 = 0.8272.x1 - 0.1219.x2 + 0.0002.x3 + 0.5485.x4$$

$$A2 = -0.9034.x1 + 0.2973.x2 - 0.0118.x3 - 0.3088.x4$$

$$X1 = [x1, x2, x3, x4] = \left[\frac{a}{}, \frac{L}{}, \frac{W}{}, \frac{L}{} \times \frac{W}{}, \sqrt{\frac{L}{}} + \sqrt{\frac{W}{}} \right] \quad (\text{Eq.6})$$

Statistics and Limits:

$$N = 84, R^2 = 0.980, SEE = 0.0081, CV = 0.79\%, 0.05 \leq a/} \leq 0.3, 2 \leq L/} \leq 7, W/} \leq L/}$$

Note that N is the number of data points, R^2 is the coefficient of determination, SEE is the standard error of estimates, and CV is the coefficient of variation. This

prediction model is also applicable to a larger slab when the upper bound value of 7.0 is used for the normalized slab length or width ($L/\}$, $W/\}$).

CASE II: Loading Plus Curling, but $UT=0$

In CASE II, the combination effect of a single wheel load and a linear temperature differential (ΔT) at the slab corner was considered. **But, UT was assumed to be zero or very close to zero.** Therefore, the ILLI-SLAB program was modeled to allow partial contact between the slab-subgrade interface. The following F.E. runs were conducted:

$$a/\}: 0.05, 0.1, 0.2, 0.3; L/\}: 2, 3, 4, 5, 7, 9, 11, 13, 15; W/\} = L/\}; \alpha\Delta T=0.$$

Note that a square slab up to a maximum normalized slab length ($L/\}$) of 15, which may satisfy Westergaard's infinite slab assumption for thermal curling analysis. Furthermore, to account for D_x and D_p effects without increasing the number of F.E. runs, the above factorial runs were randomized by these two factors for different $a/\}$ values [9]. The following predictive model was developed:

$$R = \frac{f_{i(\text{CASE II})}}{f_w} = 0.9949 + 0.17037\Phi_1 + 0.03020\Phi_2$$

$$\Phi_1 = \begin{cases} -0.85525 + 15.53557(A1) + 1.71139(A1)^2 & \text{if } (A1 \leq 0.1) \\ 0.24816 + 0.28387(A1) - 0.06692(A1)^2 & \text{if } (A1 > 0.1) \end{cases}$$

$$\Phi_2 = \begin{cases} -0.93998 + 3.72027(A2) + 11.13839(A2)^2 & \text{if } (A2 \leq 0.18) \\ -2.93892 + 16.93742(A2) & \text{if } (A2 > 0.18) \end{cases} \quad (\text{Eq.7})$$

$$A1 = -0.95810x1 + 0.03604x2 + 0.28368x3 - 0.00231x4 - 0.00033x5 - 0.00236x6 - 0.00144x7 + 0.01621x8$$

$$A2 = 0.99699x1 - 0.02358x2 + 0.05534x3 - 0.00265x4 + 0.00055x5 + 0.01611x6 - 0.00041x7 - 0.04602x8$$

$$X = [x1, x2, \dots, x8]$$

$$= \left[\frac{a}{\}, \frac{L}{\}, \frac{a}{\} \times \frac{L}{\}, \frac{a}{\} \times \frac{L}{\}, \frac{a}{\} \times DP, DP, DG, \frac{DP}{DG}, \frac{a}{\} \times DG \right]$$

Statistics and Limits:

$$N = 108, R^2 = 0.962, \text{SEE} = 0.0096, 0.05 \leq a/\} \leq 0.3, 2 \leq L/\} \leq 15, \\ W/\} = L/\}, 1 \leq DG \leq 10, 2 \leq DP \leq 130, DG = D_x \times 10^5, DP = D_p \times 10^5$$

CASE III: Loading Plus Curling, but $UTM0$

In CASE III, UT was assumed to be different from zero. The factorial F.E. runs for CASE II with different ΔT values: ΔT : -10, -20, -30, -40 °F ($\alpha=5.5E-06$ /°F) was selected for this case. Thus, a total of 432 factorial F.E. runs were conducted for this analysis. The following predictive model for the adjustment factor was developed:

$$R = \frac{f_{i(\text{CASE III})} - f_{i(\text{CASE II})}}{f_o} = 0.2548 + 0.3076\Phi_1 + 0.1058\Phi_2 + 0.05934\Phi_3$$

$$\Phi_1 = \begin{cases} -0.28987 + 0.02840(A1) & \text{if } A1 \leq 0 \\ -0.22478 + 0.40575(A1) & \text{if } A1 > 0 \end{cases}$$

$$\Phi_2 = \begin{cases} -1.46318 + 0.39571(A2) + 0.00231(A2)^2 - 0.00155(A2)^3 & \text{if } A2 \leq 15 \\ 5.06880 - 0.32371(A2) & \text{if } A2 > 15 \end{cases}$$

$$\Phi_3 = \begin{cases} 0.73250 + 0.74738(A3) & \text{if } A3 \leq 0 \\ 0.68128 + 0.25940(A3) & \text{if } A3 > 0 \end{cases}$$

$$A1 = -0.04291x1 + 0.56894x2 - 0.43915x3 + 0.05771x4 \\ -0.12609x5 + 0.02591x6 + 0.01885x7 - 0.15518x8 \\ + 0.50270x9 - 0.01229x10 + 0.31315x11 - 0.00903x12 \\ + 0.00649x13 + 0.28839x14 - 0.04413x15 + 0.03329x16 - 0.00002x17$$

$$A2 = -0.02058x1 + 0.83621x2 - 0.36689x3 + 0.25029x4 \\ -0.16713x5 + 0.04484x6 + 0.07580x7 + 0.03647x8 \\ -0.09497x9 + 0.00207x10 - 0.04534x11 - 0.00721x12 \\ + 0.0007x13 + 0.23382x14 + 0.01217x15 + 0.01038x16 - 0.00016x17$$

$$A3 = 0.04637x1 - 0.44327x2 + 0.39157x3 + 0.47010x4 \\ -0.12200x5 - 0.00537x6 - 0.00851x7 - 0.01246x8 \\ -0.48078x9 + 0.00443x10 + 0.01520x11 + 0.00322x12 \\ -0.00293x13 + 0.42430x14 + 0.01628x15 - 0.01370x16 + 0.00007x17$$

$$X = [x1, x2, \dots, x17] = \begin{bmatrix} a/\}, L/\}, ADT, (a/\})*(L/\}, \\ (a/\})*ADT, (L/\})*ADT, \\ (a/\})*(L/\})*ADT, DP, DG, \\ DP*DG, DP*(a/\}, DP*(L/\}, \\ DP*ADT, DG*(a/\}, DG*(L/\}, \\ DG*ADT, ADT*(L/\})*(a/\})*DP*DG \end{bmatrix} \quad (\text{Eq.8})$$

Statistics and Limits:

$$N = 432, R2 = 0.951, SEE = 0.09, 0.05 \leq a/\} \leq 0.3, 2 \leq L/\} \leq 15, W/\} = L/\}, \\ 1 \leq DG \leq 10, 2 \leq DP \leq 130, 5.5 \leq ADT \leq 22, DG = D_g \times 10^5, DP = D_p \times 10^5, ADT = -r \times \Delta T \times 10^5$$

Where σ_i is the combined maximum corner stress, [FL⁻²]; and σ_0 is defined by (Eq.3).

VALIDATION OF STRESS PREDICTIONS

To further validate the applicability of the proposed prediction models for all three cases, a totally separate set of database was created using the following input parameters: E = 3.0, 5.5, 8.0 Mpsi; k = 50, 250, 500 pci; L = 120, 240, 360 in.; h = 8, 12, 16 in.; $\Delta T = 0, -20, -30, -40$ °F ($\alpha = 5.5E-06$ /°F). Note that the other pertinent input parameters are: c = 10 in., a = 5.642 in., P = 9000 lbs, p = 90 psi, $\mu = 0.15$, $\gamma = 0.087$ pci, and W = L.

This will result in a total of 81 ILLI-SLAB runs with the ranges of $a/\} = 0.07 \sim 0.21$, $L/\}$ and $W/\} = 1.4 \sim 15.9$ for Case I and Case II, and a total of 243 ILLI-SLAB runs for Case III. When the values are outside the specified limits of the prediction

model, the upper or lower bounds were applied for the analysis. The predicted stresses were plotted against the resulting ILLI-SLAB stresses [9]. All the plots showed very good agreements and thus further verified the applicability of the prediction models.

A NUMERICAL EXAMPLE

Consider a pavement slab with the following characteristics: $E = 3$ Mpsi, $k = 400$ pci, $L = 141$ in., $W = 141$ in., $h = 9.97$ in., $\nu = 0.224$ pci, $\mu = 0.15$, and $\alpha = 5.5E-06 / ^\circ\text{F}$. A single wheel load of 7,624 lbs with a loaded rectangle of the size of 10×10 in² is applied at the slab corner. A linear temperature differential of -10 °F (night-time curling) exists through the slab. Determine the critical corner stresses due to loading alone, and loading plus curling. (Note: 1 psi = 6.89 kPa, 1 pci = 0.27 MN/m³, 1 in. = 2.54 cm, 1 °F = $(F - 32) / 1.8$ °C, 1 lb = 4.45 N.)

The equivalent radius of the loaded area is $a = 5.64$ in. and the radius of relative stiffness of the slab-subgrade system is $\rho = 28.21$ in. Therefore, the dimensionless mechanistic variables are $a/\rho = 0.2$, $L/\rho = W/\rho = 5$, $ADT = 5.5$, $DG = 7$, and $DP = 30$. The Westergaard solutions are $\sigma_w = 122.3$ psi and $\sigma_0 = 97.1$ psi for loading and curling alone using (Eq.1) and (Eq.3).

For the case of loading only, the adjustment factor $R = 1.054$ using (Eq.6). Thus, the corner stress determined by the proposed model is $1.062 \times 122.3 = 129.9$ psi. (Note that the resulting ILLI-SLAB stress was 129.1 psi.)

For the case of loading plus curling, the adjustment factors for Case II and Case III are $R = 1.054$ and 0.139 using (Eq.7) and (Eq.8), respectively. Thus, the predicted total corner stress determined by the proposed model is $1.054 * 122.3 + 0.139 * 97.1 = 142.4$ psi. (Note that the ILLI-SLAB corner stress was 147.5 psi.)

CONCLUSIONS

The corner stress of a concrete slab due to the individual and combination effects of loading and night-time curling was conducted under this study. A linear temperature differential across the slab thickness and a dense liquid foundation were assumed. Based on the principles of dimensional analysis, six dimensionless mechanistic variables which dominate the primary structural responses were used for the analysis. A new modeling procedure was utilized to develop stress prediction models.

The prediction models were properly formulated to satisfy applicable engineering boundary conditions. The models not only cover almost all practical ranges of pavement designs, but they are also dimensionally correct. These models can be implemented as part of design procedures to the very time-consuming and complicated F.E. analysis to estimate stresses for design purposes with efficiency and sufficient accuracy. A numerical example showing the use of the models was also provided.

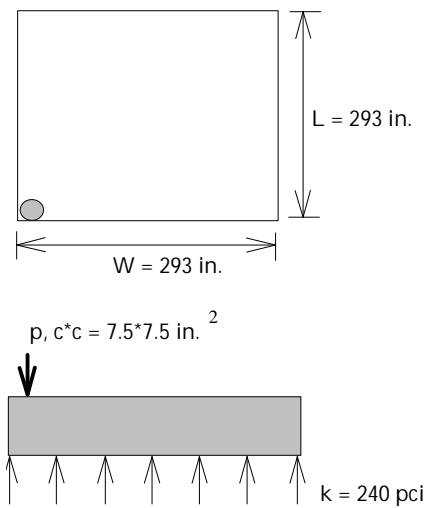
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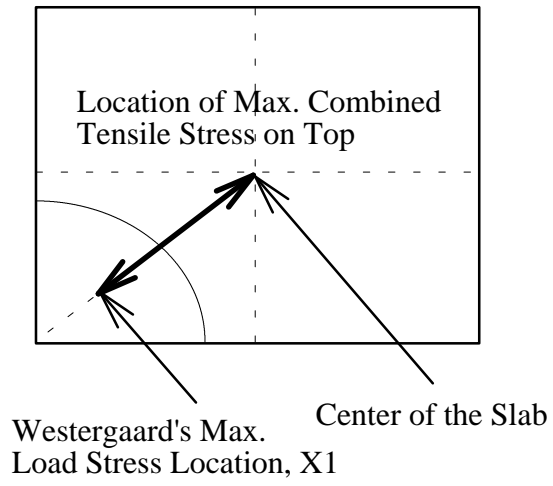
(a) Case Study of Corner Stress Analysis

(b) Loading Only

(c) Curling Only (Night-time Condition) (d) Large Loading Plus Small Curling

(e) Small Loading Plus Large Curling (f) Medium Loading Plus Medium Curling

Figure 1 - Distribution of the Tensile Stresses on the Top of the Slab



Note: This plot to be moved to page 4