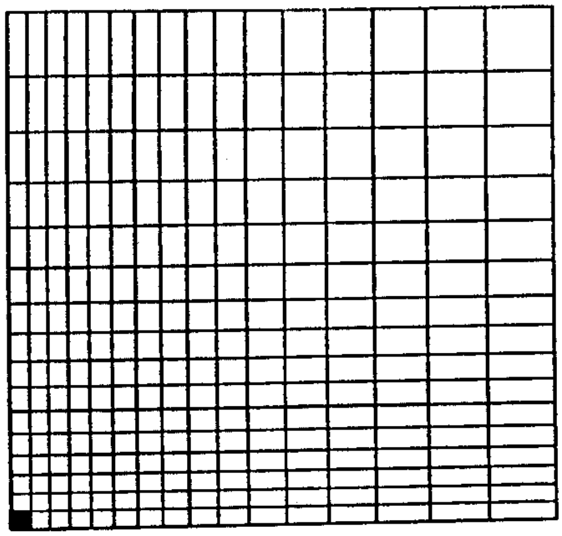
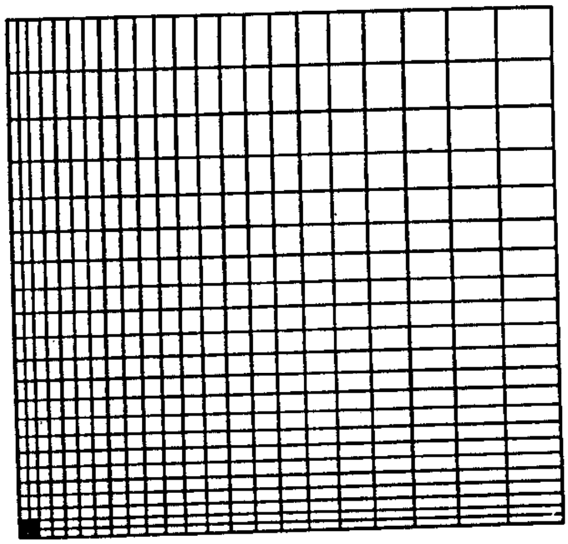
Excerpt from: Kuo, C.-M., "Three-Dimensional Finite Element Analysis of Concrete Pavement," Ph.D. Dissertation, University of Illinois, Urbana, 1994. pp. 50–54.



(a) 1 x 1 mesh



(b) 2 x 2 mesh

Figure 3–4 Finite element meshes used for interior loading (case 1). (Note: one quadrant of grid is shown; symmetry is used to reduce run time)

Table 3–4 Deflections at loading center (x 10^{-3})

		C3D20	C3D20R
1 x 1	δ_{top}	9.272	9.273
	δ_{bottom}	9.214	9.221
	ε_{zz}	9.667	8.667
2 x 2	δ_{top}	9.276	9.273
	δ_{bottom}	9.211	9.215
	ε_{zz}	10.83	9.667

Table 3–5 Stresses at loading center

		C3D20	C3D20R
1 x 1	σ_{top}	–305.8	–308.3
	σ_{bottom}	289.0	299.7
	τ	49.09	43.62
2 x 2	σ_{top}	–292.6	–289.3
	σ_{bottom}	277.9	283.6
	τ	49.54	41.46

developed an equation based on "special theory" to take the shear deformation near a small loaded area into account [7].

- Loading size plays a key role in this situation. Although the length to thickness ratio of this plate is 50, which satisfies the thin plate criterion, a small load area may still violate plate theory in the neighborhood of loaded area. When the load is distributed over a large area, transverse shear should be negligible as assumed in thin plate theory .

Performance of 3–D Elements Versus 2–D Elements

Another interesting issue in developing a 3–D model is how 3–D results compare with 2–D results. As mentioned previously, it is not expected that 3–D results and 2–D results will match because some plate responses are neglected in 2–D model formulation. The agreement of two models is expected when the following conditions are satisfied :

1. The plate is thin enough,
2. The load size is large enough, and
3. The finite element meshes meet requirements for accuracy.

Study Case 1 – Interior Loading

Slab size = 20 ft × 20 ft ,	Slab thickness = 6 in,
PCC E = 4,000,000 psi,	$\mu = 0.15$
P=9720 lb,	k = 200 pci

To compare the accuracy of the elements for various mesh fineness and loading sizes, a series of runs was made using different elements and changing loading area (Figure 3–5). The results are given in Table 3–6 and 3–7, and plotted as Figure 3–6 and 3–7.

Figure 3–7 shows that deflections from all elements are very consistent, but Westergaard's equation diverges when the loaded area spreads. This is more significant in the stress comparison as shown in Figure 3–6. This confirms the statement by Timoshenko [41] that the Westergaard's equations apply only when the loaded area is small in comparison with ℓ. However, the transverse shear effect may become significant when the loaded area approaches to a point load. Consequently, as shown in Figure 3–6, Westergaard's equation agrees with 3–D model within an intermediate range of load area size. Detailed research concerning load size effects on 2–D finite element results has been conducted by Ioannides [42] and shown the same conclusion.

The curves could be divided into two groups. Thin plate elements including ILLI–SLAB, FINITE and STRI3 are a little apart from the elements which consider the transverse shear effect, especially in the case of small loading size. This is expected because the transverse shear stresses are significant when the load is more concentrated.

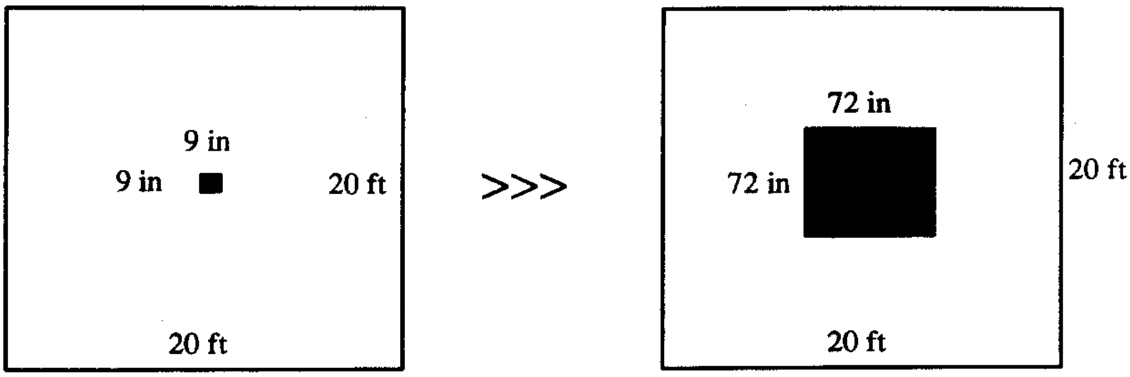


Figure 3–5 Change of load size in interior loading of case 2

Table 3–6 Maximum stress comparison

	ILLI–SLAB	FINITE	ABAQUS (STRI3)	ABAQUS (S4R)	ABAQUS (S8R)	ABAQUS (C3D27R)	Westergaard
9*9	345.129	345.141	339.5	275.8	344.8	346.1	324.79
18*18	228.168	228.176	228.0	212.2	227.3	227.5	223.56
27*27	168.186	168.204	168.4	161.6	167.5	167.7	166.61
36*36	128.584	128.601	128.7	125.1	128.0	128.2	128.39
45*45	99.597	99.975	100.1	97.92	99.47	99.61	100.99
54*54	78.321	78.350	78.42	77.07	77.92	78.04	80.92
63*63	61.582	61.602	61.65	60.78	61.25	61.34	66.29
72*72	48.426	48.449	48.48	47.91	48.15	48.23	55.98

Table 3–7 Maximum deflection comparison

	ILLI–SLAB	FINITE	ABAQUS (STRI3)	ABAQUS (S4R)	ABAQUS (S8R)	ABAQUS (C3D27R)	Westergaard
9*9	9.86	9.847	9.779	10.132	10.186	10.144	9.807
18*18	9.43	9.417	9.370	9.600	9.642	9.600	9.370
27*27	8.892	8.880	8.848	9.012	9.042	9.014	8.867
36*36	8.300	8.288	8.263	8.386	8.408	8.389	8.296
45*45	7.686	7.673	7.655	7.752	7.764	7.751	7.718
54*54	7.070	7.058	7.044	7.119	7.127	7.119	7.164
63*63	6.471	6.459	6.449	6.507	6.512	6.507	6.659
72*72	5.897	5.885	5.876	5.923	5.926	5.923	6.224

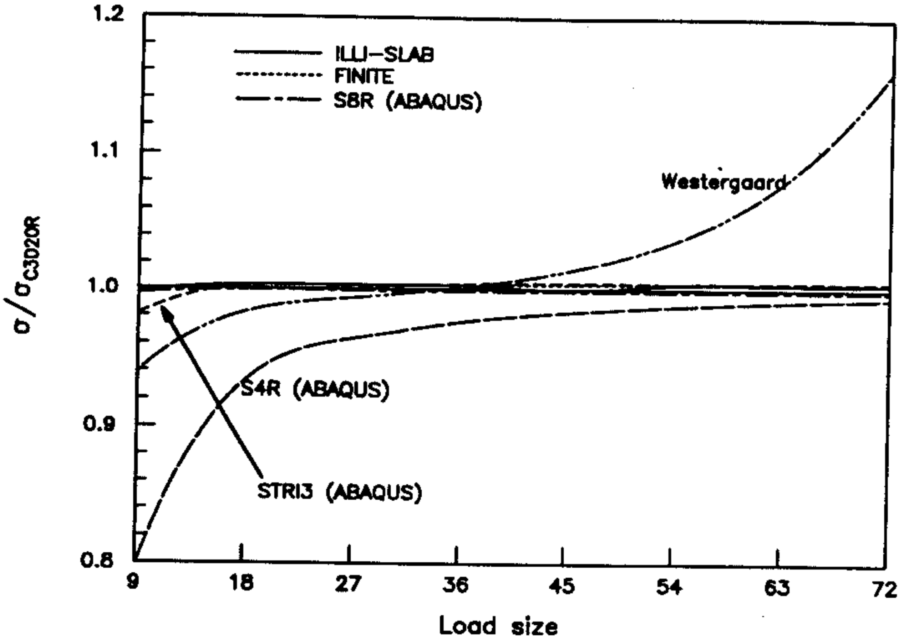


Figure 3–6 Stress comparison of interior loading

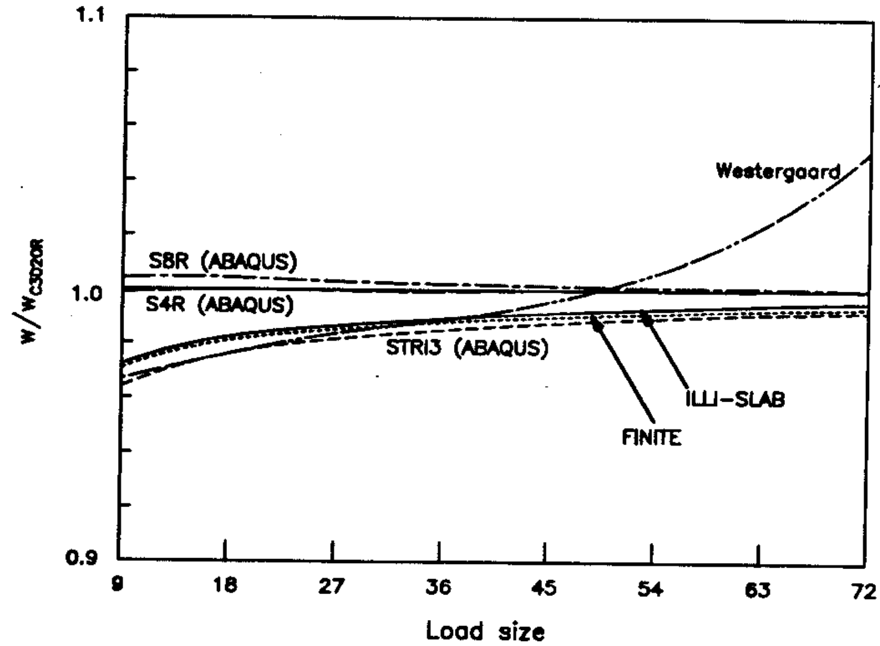


Figure 3–7 Deflection comparison of interior loading