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Rational bubbles in the US stock market? Further evidence from a nonparametric cointegration test

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In this study, we revisit the issue as to the presence of rational bubbles in the US stock market during the 1871 to 2002 period using both the Johansen cointegration and the Bierens (1997) nonparametric cointegration tests. The results from the conventional Johansen cointegration test fully support the existence of rational bubbles, whereas those from the Bierens nonparametric cointegration test attest to the absence of rational bubbles. On account of the superiority of the nonparametric method to detect cointegration when the error–correction mechanism is nonlinear, we firmly believe that the results from the nonparametric cointegration test are considerably more reliable than those derived from the conventional Johansen approach.

I. Introduction

The purpose of this study is to re-investigate whether rational bubbles were present in the US stock market during the period 1871 to 2002. The occurrence of rational bubbles signifies that no long-run relationships exist between stock prices and dividends. In pursuit of determining whether or not stock prices and dividends are cointegrated, empirical studies have, for the most part, employed cointegration techniques. Among the most notable of these is the widely employed Johansen cointegration test (Johansen, 1988; Johansen and Juselius, 1990) which is based on the linear autoregressive model and, as such, assumes that the underlying dynamics are in a linear form. A wealth of empirical evidence that has caused some researchers to flatly reject such a linear paradigm has, however, been reported. From a theoretical perspective, there is no sound reason to assume that economic systems are intrinsically linear (see, Barnett and Serletis, 2000). In fact, numerous studies have empirically demonstrated that financial time series, such as stock prices, exhibit nonlinear dependencies (see, Hsieh, 1991; Abhyankar et al., 1997). Besides this, substantive evidence from the Monte Carlo simulations in Bierens (1997), in fact, has indicated that inherent to the conventional Johansen cointegration framework is a misspecification problem when the true nature of the adjustment process is nonlinear and that the speed of adjustment varies with the magnitude of the disequilibrium. The work of Balke and Fomby (1997) as well as that of Enders and Granger (1998) also pointed out a potential loss of power in conventional cointegration tests under the threshold autoregressive data generating process (DGP).

Motivated by the aforementioned considerations, in this study, we re-examine the issue of rational bubbles in the US stock market during the period 1871 to 2002, using the powerful nonparametric...
cointegration test, as developed by Bierens (1997), and for comparison, the conventional Johansen cointegration test. The results from the former, i.e. the Bierens nonparametric cointegration test, are expected to confirm the absence of rational bubbles. Contrast this with the results from the conventional Johansen cointegration which, it is anticipated, should likely uphold the notion of the existence of rational bubbles.

The remainder of this article is organized as follows: Section II describes the data used in this study; Section III briefly presents the methodology and discusses the empirical results, and Section IV reviews the conclusions we draw.

II. Data

We analyse the annual US Standard and Poor’s stock price index and dividend data over the period 1871 to 2002 which we take from Shiller’s Web site http://aida.econ.yale.edu/~shiller. The data begin from 1871 since data for both series are available from this period. A description of the time series can be found in Shiller (2001).

III. Methodology and Empirical Results

Unit root tests

Recently, a general consensus has been emerging in support of the likelihood that stock price data exhibit nonlinearities and that such conventional tests for stationarity as the ADF unit root test have too low of a power to be able to detect the mean-reverting tendency of a series. It follows, then that stationary tests must be applied in a nonlinear framework. To this end, in this study, we use the nonlinear stationary test advanced by Kapetanios et al. (2003) (henceforth, the KSS test).

The purpose of the KSS test is to detect the presence of nonstationarity against a nonlinear but globally stationary exponential smooth transition autoregressive (ESTAR) process. The model is given by

\[ \Delta Y_t = \gamma Y_{t-1} \{1 - \exp(-\theta Y_{t-1}^2)\} + \nu_t \] (1)

where \( Y_t \) is the data series of interest, \( \nu_t \) is an i.i.d. error with a zero mean and constant variance, and \( \theta \geq 0 \) is the transition parameter of the ESTAR model and governs the speed of transition. Under the null hypothesis \( Y_t \) follows a linear unit root process, but under the alternative, \( Y_t \) follows a nonlinear stationary ESTAR process. One shortcoming of this framework is that the parameter \( \gamma \) is not identified under the null hypothesis. Kapetanios et al. (2003) have used a first-order Taylor series approximation for \( \{1 - \exp(-\theta Y_{t-1}^2)\} \) under the null hypothesis \( \theta = 0 \) and have then approximated Equation (1) by using the following auxiliary regression:

\[ \Delta Y_t = \xi + \delta Y_{t-1}^3 + \sum_{i=1}^{k} b_i \Delta Y_{t-i} + \nu_t, \]

\[ i = 1, 2, \ldots, T \] (2)

Under this framework, the null hypothesis and the alternative hypothesis are expressed as \( \delta = 0 \) (nonstationarity) against \( \delta < 0 \) (nonlinear ESTAR stationarity). Table 1 presents the KSS nonlinear stationary test results, and they clearly indicate that both the stock prices and dividends series are integrated of order one.

For comparison, we also incorporate the Augmented Dickey and Fuller (1981, ADF), the Phillips and Perron (1988, PP) and the Kwiatkowski et al. (1992, KPSS) tests into our study. Table 2 shows the results from the nonstationary tests for the stock prices and dividends using the ADF, PP and the KPSS tests. Again, the test results further indicate that the stock prices and dividends are integrated of order one, \( I(1) \). In light of these results, we proceed to test whether there were rational bubbles in the US stock market during the sample period, and to this end, we employ both the conventional Johansen cointegration test and the Bierens (1997) nonparametric cointegration approach.

Johansen cointegration test

Following Johansen (1988) and Johansen and Juselius (1990), we construct a \( p \)-dimensional \((2 \times 1)\) vector autoregressive model with Gaussian errors which is expressed by its first-differenced error

<table>
<thead>
<tr>
<th>Variable</th>
<th>KSS statistic</th>
<th>1%</th>
<th>5%</th>
<th>10%</th>
</tr>
</thead>
<tbody>
<tr>
<td>lp</td>
<td>0.674838(3)</td>
<td>-2.82</td>
<td>-2.22</td>
<td>-1.92</td>
</tr>
<tr>
<td>ld</td>
<td>0.175361(1)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Notes: Simulated critical values are from Table 1 in Kapetanios et al. (2003). The number in parentheses indicates the selected lag order of the testing model. Lags are chosen based on Campbell and Perron (1991).
correction form:

\[
\Delta Y_t = \Gamma_1 \Delta Y_{t-1} + \Gamma_2 \Delta Y_{t-2} + \cdots + \Gamma_{k-1} \Delta Y_{t-k+1} - \Pi Y_{t-1} + \mu + \epsilon_t \tag{3}
\]

Here \(Y_t\) represent the stock prices and dividends that we study; \(\epsilon_t\) is i.i.d. \(N(0, \Sigma)\); \(\Gamma_i = -I + A_1 + A_2 + \cdots + A_k\) for \(i = 1, 2, \ldots, k-1\); and \(\Pi = I - A_1 - A_2 - \cdots - A_k\). The \(\Pi\) matrix conveys information about the long-run relationships between the \(Y_t\) variables, and the rank of \(\Pi\) is the number of linearly independent and stationary linear combinations of the variables under study. Testing for cointegration involves testing for the rank, \(r\), of the \(\Pi\) matrix by examining whether the eigenvalues of \(\Pi\) are significantly different from zero.

Johansen (1988) and Johansen and Juselius (1990) proposed two test statistics to test the number of cointegrating vectors (or the rank of \(\Pi\)): the Trace \((T_r)\) and the maximum eigenvalue \((L\text{-max})\) statistics. Table 3 presents the results from the Johansen (1988) and Johansen and Juselius (1990) cointegration tests. As shown, both the Trace statistic and \(L\text{-max}\) statistic demonstrate that the null hypothesis of no cointegration cannot be rejected. What this means is that the rational bubbles did indeed exist in the US stock market during the period 1871 to 2002.

Nonlinear test of the error–correction term

As mentioned earlier, the evidence from the Monte Carlo simulations in Bierens (1997) indicates that the conventional Johansen cointegration framework has a misspecification problem when the true nature of the adjustment process is nonlinear and the speed of adjustment varies with the magnitude of the disequilibrium. Bearing this in mind, we follow Granger and Teräsvorta (1993) by employing a nonlinear test on our error–correction term.\(^1\) Table 4 gives the

\(^1\) The detailed procedures are not presented here due to space constraints, but are available upon request.

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**Table 2. Conventional unit root test results**

<table>
<thead>
<tr>
<th></th>
<th>ADF Intercept</th>
<th>Trend</th>
<th>PP Intercept</th>
<th>Trend</th>
<th>KPSS Intercept</th>
<th>Trend</th>
</tr>
</thead>
<tbody>
<tr>
<td>A. Level</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lp</td>
<td>1.2692(0)</td>
<td>-1.3368(0)</td>
<td>2.0097[11]</td>
<td>-1.0495[7]</td>
<td>1.3069[9]**</td>
<td>0.3156[9]**</td>
</tr>
<tr>
<td>Ld</td>
<td>0.8164(2)</td>
<td>-2.8123(1)</td>
<td>1.1726[15]</td>
<td>-1.9937[15]</td>
<td>1.3273[9]**</td>
<td>0.3053[9]**</td>
</tr>
<tr>
<td>B. First difference</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lp</td>
<td>-10.2672(0)**</td>
<td>-9.6395(1)**</td>
<td>-10.2263[6]**</td>
<td>-10.8291[12]**</td>
<td>0.4661[6]**</td>
<td>0.0465[11]</td>
</tr>
</tbody>
</table>

Notes: *, ** and *** denote significance at the 10%, 5% and 1% level, respectively. The number in parentheses indicates the selected lag order of the ADF model. Lags are chosen based on Campbell and Perron (1991). The number in brackets indicates the lag truncation for the Bartlett kernel, as suggested by the Newey–West (1987) test.

**Table 3. Johansen cointegration test results**

<table>
<thead>
<tr>
<th></th>
<th>lp-ld Trace test</th>
<th>5% critical value</th>
<th>L-max test</th>
<th>5% critical value (VAR lag = 4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ho: (r \leq 0)</td>
<td>8.056</td>
<td>15.41</td>
<td>7.857</td>
<td>14.07</td>
</tr>
<tr>
<td>Ho: (r \leq 1)</td>
<td>0.199</td>
<td>3.76</td>
<td>0.199</td>
<td>3.76</td>
</tr>
</tbody>
</table>

**Table 4. Nonlinear test of the error–correction term**

<table>
<thead>
<tr>
<th></th>
<th>1 (126)</th>
<th>2 (126)</th>
<th>3 (126)</th>
<th>4 (126)</th>
<th>5 (126)</th>
<th>6 (126)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ho F Sta</td>
<td>0.520884</td>
<td>1.356851</td>
<td>1.645378</td>
<td>2.351770</td>
<td>2.413087</td>
<td>1.590679</td>
</tr>
<tr>
<td>p-value</td>
<td>0.668681</td>
<td>0.259140</td>
<td>0.182355</td>
<td>0.075553*</td>
<td>0.069973*</td>
<td>0.195167</td>
</tr>
</tbody>
</table>

Notes: *, ** and *** denote significance at the 10%, 5% and 1% level, respectively. The numbers in parentheses indicate the degree of freedom.
results for the different delay parameters; these demonstrate that the true nature of the adjustment process is nonlinear and that the speed of adjustment varies with the magnitude of the disequilibrium.

**Nonparametric cointegration test of Bierens (1997)**

As pointed out by Bierens (1997), one of the major advantages of his nonparametric method lies in its superiority to detect cointegration when the error correction mechanism is nonlinear. Hence, we have full confidence in using this test in our study.

The Bierens nonparametric cointegration test considers the general framework to be:

$$ z_t = \pi_0 + \pi_1 t + y_t $$  \hspace{1cm} (4)

where $\pi_0(q \times 1)$ and $\pi_1(q \times 1)$ are the terms for the optimal mean and trend vectors, respectively, and $y_t$ is a zero-mean unobservable process such that $\Delta y_t$ is stationary and ergodic. Apart from these conditions of regularity, the method does not require further specifications of the DGP for $z_t$, and in this sense, it is completely nonparametric.

The Bierens method is based on the generalized eigenvalues of the matrices $A_m$ and $(B_m + cT^{-2}A_m^{-1})$, where $A_m$ and $B_m$ are defined in the following matrices:

$$ A_m = \frac{8\pi^2}{T} \sum_{k=1}^{m} k^2 \left( \frac{1}{T} \sum_{t=1}^{T} \cos \left( \frac{2\pi k (t - 0.5)}{T} \right) z_t \right) $$

$$ \times \left( \frac{1}{T} \sum_{t=1}^{T} \cos \left( \frac{2\pi k (t - 0.5)}{T} \right) y_t \right)' \hspace{1cm} (5) $$

$$ B_m = 2T \sum_{k=1}^{m} \left( \frac{1}{T} \sum_{t=1}^{T} \cos \left( \frac{2\pi k (t - 0.5)}{T} \right) \Delta z_t \right) $$

$$ \times \left( \frac{1}{T} \sum_{t=1}^{T} \cos \left( \frac{2\pi k (t - 0.5)}{T} \right) \Delta y_t \right)' \hspace{1cm} (6) $$

which are computed as the sums of the outer-products of the weighted means of $z_t$ and $\Delta z_t$, and where $T$ is the sample size. To ensure invariance in the test statistics to drift terms, we recommend using the weighted functions of $\cos(2\pi (t - 0.5)/T)$. Very much like the properties in the Johansen likelihood ratio method are the ordered generalized eigenvalues that we obtain from this nonparametric approach. These serve as the solution to the problem $\text{det}[P_T - \lambda Q_T] = 0$ when we define the pair of random matrices $P_T = A_m$ and $Q_T = (B_m + cT^{-2}A_m^{-1})$. Thus, we can use these to test the hypothesis for the cointegration rank $r$. To estimate $r$, Bierens (1997) proposed two statistics tests. One is the $\lambda_{\min}$ test which corresponds to Johansen’s maximum likelihood procedure, and it tests hypothesis $H_0(r)$ against hypothesis $H_1(r+1)$. The critical values are tabulated in his article. The second set of statistic is determined by the $g_m(r_0)$ test, which is computed from the Bierens’s generalized eigenvalues:

$$ \hat{g}_m(r_0) = \begin{cases} 
\left( \prod_{k=1}^{\lfloor \frac{m}{r} \rfloor} \hat{\lambda}_{k,m} \right)^{-1}, & \text{if } \ldots r_0 = 0 \\
\left( \prod_{k=1}^{\lfloor \frac{m}{r} \rfloor} \hat{\lambda}_{k,m} \right) \left( \prod_{k=\lfloor \frac{m}{r} \rfloor+1}^{n} \hat{\lambda}_{k,m} \right), & \text{if } \ldots n \to 1, \ldots, n - 1 \\
T^{2r} \prod_{k=1}^{n} \hat{\lambda}_{k,m}, & \text{if } \ldots r_0 = n 
\end{cases} $$

This statistic employs the tabulated optimal values (see Bierens, 1997, Table 1) for $m$ when $n > r_0$, provided that we select $m = n$ for $n = r_0$. This verifies that $\hat{g}_m(r_0) = O(n^{-1})$ for $r = r_0$, and in terms of probability, it converges to infinity if $r \neq r_0$. Hence, a consistent estimate of $r$ is given by $\hat{r}_m = \arg\min_{r_0 \leq r \leq n} \hat{g}_m(r_0))$. This statistic is an invaluable tool when double-checking the determination of $r$. Table 5 presents the results from both the

<table>
<thead>
<tr>
<th>Table 5. Bierens nonparametric cointegration test results</th>
</tr>
</thead>
<tbody>
<tr>
<td>5% critical value</td>
</tr>
<tr>
<td>---------------------------------------------------------</td>
</tr>
<tr>
<td>A. $\lambda_{\min}$ test</td>
</tr>
<tr>
<td>$H_0$: $r = 0$</td>
</tr>
<tr>
<td>$H_1$: $r = 1$</td>
</tr>
<tr>
<td>$H_0$: $r = 1$</td>
</tr>
<tr>
<td>$H_1$: $r = 2$</td>
</tr>
<tr>
<td>B. $g_m(r_0)$ test</td>
</tr>
<tr>
<td>Cointegration rank ($r$)</td>
</tr>
<tr>
<td>$r_0 = 0$</td>
</tr>
<tr>
<td>$r_0 = 1$</td>
</tr>
<tr>
<td>$r_0 = 2$</td>
</tr>
<tr>
<td></td>
</tr>
</tbody>
</table>

**Notes:** *, ** and *** denote significance at the 10%, 5% and 1% level, respectively. Both the results of the $\lambda_{\min}$ test and the $g_m(r_0)$ test indicate one cointegration rank.
λ min test and the \( g_m(r_0) \) test. The λ min test results strongly suggest that there are long-run relationships between stock price and dividends. These findings are further supported by the \( g_m(r_0) \) statistics given in Table 5, with the smallest value only appearing in the cointegrating rank of \( r = 1 \). These results reveal that rational bubbles were nonexistent in the US stock market during the period 1871 to 2002. On account of the superiority of the nonparametric method to detect cointegration when the error-correction mechanism is nonlinear, we firmly believe that these results are considerably more reliable than those derived from the conventional Johansen approach. In fact, with regard to the presence of nonlinearity, Ma and Kanas (2000) and Coakley and Fuertes (2001) have found discrepancies between the results from the Johansen approach and those from the Bierens approaches. Beyond this, it is unambiguous that our results are highly consistent with those found in Han (1996), Taylor and Peel (1998) and Caporale and Gil-Alana (2004) in that they confirm the absence of rational bubbles in the US stock market.

IV. Conclusions

In this study, we re-investigate whether rational bubbles existed in the US stock market during the period 1871 to 2002 by using both the Johansen cointegration test and the Bierens nonparametric cointegration test for data covering the same period. The results from the conventional Johansen cointegration test lend credence to the presence of rational bubbles; by stark contrast, the results from the Bierens nonparametric cointegration test indicate that rational bubbles could not have been present in the US stock market in that period.

Acknowledgement

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References


