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Introducing non-linear dynamics to the two-regime market model: Evidence☆

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Abstract

The existing two-regime asset-pricing models do not reach a consensus, either in the definition of bull and bear market conditions or in the modelling of beta non-stationarity. We apply a logistic smooth transition regression model to address the beta non-stationarity issue. Using eight different definitions of bull and bear market conditions, we intend to ascertain the most appropriate definition with which to capture the non-linear dynamics of security returns. We find, through a series of linearity tests, that the Logistic Smooth Transition Market (LSTM) model provides an adequate description of the data generating process. Further, we explore the adequacy of a duration dependent description of market conditions in our model. Often we find that the 4-month lagged yield spread is a more appropriate definition of market condition than is a coincident economic indicator, excess market returns and a moving average of excess market returns. We also find duration dependence in market conditions.

JEL classification: G12; G14; C50; C51

Keywords: Beta; Bull and bear markets; Duration dependence; Logistic smooth transition market model; Non-linearity

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1. Introduction

Recent papers by Maheu and McCurdy (2000a) and Lunde and Timmermann (2004) on duration dependence of stock returns in bull and bear market conditions and by McQueen and Thorley (1994) on duration dependence of rational bubbles have raised doubts about the Fabozzi and Francis (1977), Kim and Zumwalt (1979) and Chen (1982) dual-beta asset-pricing models. In particular, Lunde and Timmermann (2004) found that the longer the duration, the higher is the volatility of a securities return if in a bear market and the lower is the volatility if in a bull market. They relate this evidence to the investors' changing risk preferences with the amount of time spent in a particular market condition. So, the length of time spent in the bull or bear market state may be a key determinant to explain the risk/return trade-off relationship of risky assets. These new findings reflect the importance of the duration of a particular market condition while measuring risk.

Contradicting the existing single-regime market model is the evidence of non-linearities in stock prices and the evidence of asymmetric regime cycles found by various researchers. The non-linear behaviour of stock prices has been related to various behavioural dynamics of the investors. Some prominent behavioural dynamics discussed in the recent papers are: (1) heterogenous objectives due to different risk profiles and different investment horizons by Peterson (1994) and Guillaume et al. (1995); (2) herd behaviour by Lux (1995); and (3) heterogenous beliefs on the market conditions by Brock and LeBaron (1998) and Brock and Hommes (1998). Almost all research to date has modelled the transition from bull to bear market as a discrete jump. Even the latest markov-switching model by Maheu and McCurdy (2000a) assumes the switch between regimes to be abrupt. Such a definition may contradict recent evidence of heterogeneous beliefs among investors. The transition is said to be abrupt when investors have homogeneous beliefs and they collectively switch from one market condition to another, as they share the same information. The homogeneous beliefs theory is hard to accept unless one believes in the existence of a strong form of Efficient Market Theory.

Apart from the duration dependence and non-linearity issues discussed above, the definition of bull and bear market conditions itself has drawn many controversies. There has been a substantial divergence in the literature in the definition of bull and bear market conditions. For a range of definitions used in other papers, see Fabozzi and Francis (1977), Bhardwaj and Brooks (1993), Lunde and Timmermann (2004) and Pagan and Sossounov (2003).

The discussion up to this point highlights the need for a new method of modelling systematic risk that addresses:

(1) the duration dependence of market conditions controversy;
(2) the non-linearities in stock returns issue and heterogeneous versus homogeneous beliefs hypothesis (abrupt versus smooth transition between regimes);
(3) the market condition definition controversy.

The direct implication of the findings by Lunde and Timmermann (2004) for an asset-pricing model is that the market condition alone does not determine the pricing of risky assets. Rather, the market condition combined with the duration of being in that particular market state determines the risk premium for pricing risky assets. Our proposed market model allows us to assess the degree of duration dependence of market conditions in systematic risk.

Further, the simple dichotomisation of the market into bull and bear markets with no allowance for the degree of market condition, assumed in the discrete jump models such as the dual-beta market (DBM) used in other studies, is unrealistic. For stock markets with many agents switching at different times and with heterogeneous beliefs and differing investment horizons smooth transition may be more appropriate. See Brock and Hommes (1998) and the discussion in Section 3.1 for justification of smooth transition. We propose a model that will enable us to assess the rate of transition between regimes thereby helping to interpret investor reaction to changes in market conditions. For example, abrupt/smooth transition may indicate that investors have homogeneous/heterogeneous beliefs as they switch from one regime to the next instantly/gradually as the transition variable crosses the threshold. Abrupt/gradual switch may be due to the flow of symmetric/asymmetric information in terms of the efficient market hypothesis.

Finally, given that definitions of market conditions have been arbitrarily assigned in prior research, a robust definition of market conditions will be ascertained. Therefore, the purpose of this paper is to propose and test a new model, which takes these issues into account. We address all three issues by applying a Logistic Smooth Transition Market (LSTM) model to monthly returns on a sample of 50 heavily traded securities in Australia over the period April 1986 to December 2001. We also apply the LSTM model to a US airlines industry composite returns index, since this industry has plenty of firms and is expected to be effected by oil price shocks. Teräsvirta and Anderson (1992) indicate that non-linearities are an important implication of large negative shocks such as oil price shocks. By using US data, we also generalize our results.

Rather than simply using the excess market return to characterise the state of the market, as has been done in most other studies, we use a rolling 6-month moving average of this variable, the Westpac coincident economic indicator and the 4-month lagged yield spread as transition variables in our model. These series are much smoother than the excess market return itself and therefore better capture the long-run dependencies and drift in the data. In addition, unlike others, we include some duration dependent definitions of market conditions in our analysis.

The LSTM model represents an improvement over the threshold dual-beta market model used in other studies. The indicator function, which is used to model a discrete jump between regimes in the DBM is replaced with a logistic smooth transition function that allows for smooth and continuous transition between the two states. For stock markets with many agents switching at different times and with heterogeneous beliefs and differing investment horizons the LSTM model, which allows for both the constant risk and DBM models as special cases, is more appropriate. Our estimated models will enable us to determine whether transition between regimes is gradual or abrupt. Such a finding will help to understand the heterogeneous beliefs theory. Model selection procedures are employed to ascertain a "best" choice of transition variable and to determine the efficacy of the inclusion of duration dependence in the model. A logistic smooth transition auto-regression (LSTAR) model has been used to characterize business cycles in Teräsvirta and Anderson (1992). However, this is the first application of the logistic smooth transition regression model in a bull and bear market asset-pricing context.

The parameter estimates of the LSTM model offer interesting results. First, we find that transition between bull and bear markets for most non-linear firms in Australia and for the airline industry in the US is smooth rather than abrupt. This result is consistent with the heterogeneous beliefs among investor theory by Brock and LeBaron (1998) and Brock and Hommes (1998). Second, in contrast to the Fabozzi and Francis (1977), Kim and Zumwalt (1979) and Chen (1982) findings, we find that for most firms in Australia and for the airline industry in the US beta does vary in response to changing market conditions. Further, when a duration dependent definition of market condition has been employed as the transition variable in the LSTM model, the results indicate that the duration of the market condition is an important component with which to characterise risk. Fourth, the results vary with the definition of bull and bear market conditions employed. Among the four definitions tested, the 4-month lagged yield spread performs better than the excess market return ($R_{mt} - R_{ft}$), the Westpac coincident indicator, a coincident indicator of economic conditions in Australia, and the 6-month rolling moving average of the excess market returns. However, the results have to be interpreted with caution as they are based on 50 Australian firms and the US airline industry. The same inference cannot be drawn with regard to the market in general.

The paper is divided into seven sections. Section 2 reviews the existing literature on two-regime market models and addresses the potential problems associated with them. Section 3 offers some theory to support the new LSTM form. In this section, we also develop the LSTM model and discuss the linearity testing methodologies used. Section 4 presents a description of the data. Section 5 provides a discussion of the empirical results based on a sample of 50 heavily traded Australian firms. Section 6 provides a brief discussion of the empirical results for the US airline industry and Section 7 draws concluding remarks.

2. Literature review

It is a well-established fact in the asset-pricing literature that beta varies with changing economic conditions (see Ferson & Schadt, 1996). However, there is no consensus on the definition of these economic conditions, the nature of the investor's risk preferences in relation to these economic conditions, or the methodology with which to capture the behaviour of these economic conditions.

By far the most frequently applied model of the relationship between beta risk and market conditions is the dual-beta market model. In this two-regime model, the economy is divided into two phases based on the upward (bull) or downward (bear) direction of stock market movement. Based on the works of Levy (1974) and Black (1972), Fabozzi and Francis (1977) used a DBM to test for a difference in betas over bull and bear market conditions. They concluded that there is no significant difference in bull and bear market betas. Many researchers extended the work of Fabozzi and Francis (1977) by first offering more sophisticated trend-based definitions of bull and bear market conditions and second by addressing the beta non-stationarity issue using more robust methods of estimation.

2.1. *Definition of bull and bear market conditions*

There is no sacrosanct definition of bear and bull markets. However, all researchers have either compared a market index to some critical threshold value to divide the market into "up" and "down" months, or have defined bull and bear markets using a trend-based scheme. Bhardwaj and Brooks (1993), for example, used the median return on the market portfolio to dichotomise the market into bull and bear months. Wiggins (1992), Chen (1982) and Fabozzi and Francis (1977) used the most common up and down market scheme, which involves separating up from down market months by comparing the excess market return to zero.

Lunde and Timmermann (2004) point out that the definition of market condition should reflect long-term dependencies in stock prices and it should incorporate information about the trend in the stock price level. Cognizant of this fact, several studies have used a trend-based approach in their analysis of market conditions. Fabozzi and Francis (1977, 1979), for example, used the dates published in Cohen, Zinbarg, and Zeikel (1973, 1987) to define bull and bear markets. A more complex Bry and Boschan (1971) algorithm was used in a recent article by Pagan and Sossounov (2003) to model the turning points between bull and bear markets. Lunde and Timmermann (2004), inspired by Sperandeo's (1990) definition, defined market conditions by dividing price movements into intermediate and long-term highs and lows. They define bull markets as a series of intermediate highs interrupted by a series of intermediate lows when there is a long-term upward movement and bear markets as a series of intermediate lows interrupted by a series of intermediate highs in a long-term downward movement. In other words, they try to smooth the price process from its short-term noise and identify the upward or downward trends.

Maheu and McCurdy's (2000) markov-switching model allowed the data to determine bull and bear markets endogenously. However, Lunde and Timmermann (2004) point out that defining market conditions endogenously by using a market switching process tends to identify turning points too often, since regime switching is in fact a process in which switching occurs after a sequential trend.

In order to capture the enduring periods of growth and contraction that are generally associated with the concepts of bull and bear markets, we use several trend-based definitions of bull and bear markets in our analysis. We offer three different trend-based definitions of market conditions, which are easy to compute, and whose data are readily available. In addition, as a touchstone, we perform our analysis using the traditional but highly erratic excess market return. In this way, we hope to see if our new definitions represent an improvement over existing methods. The definitions used in our analysis are the 6-month rolling moving average of the excess market return; the coincident economic indicator; the 4-month lagged yield spread; and the commonly used excess market return. We also use duration dependent versions of each of these four variables.

2.1.1. Moving average

We use a 6-month rolling moving average of the excess market return to capture the underlying cyclical nature of the stock market. This moving average performs the same smoothing function as described by Lunde and Timmermann (2004) and it is also one of the most widely used tools by analysts to analyse future trends in the stock market (Brown, Goetzmann, & Kumar, 1998). Additional support for our smoothing method is given by Fama (1990) who found that for the USA, the correlation between stock returns and real economic activity is higher for yearly than for monthly returns. Note that larger positive (negative) magnitudes of this variable indicate better (worse) conditions than do smaller magnitudes. Thus, we do not dichotomize the market but instead allow this variable to provide a smooth and continuous characterization of market conditions. Fig. 1 reveals that by using the 6-month moving average of the highly erratic excess market return series, we capture the smooth transition in and out of market phase that is generally expected for the stock market. The excess market return, which has been used by most other researchers, on the other hand, can be seen to imply frequent jumps in and out of market phase.

2.1.2. *Coincident economic indicator*

Seigel (1998) strongly suggests that the business cycle is a key determinant of stock values. Diebold and Rudebusch (1990) found that recessions and expansions of the business cycle are duration dependent. Following this evidence, Lunde and Timmermann (2004) argue that business cycles are also related to bull and bear market conditions, since the duration dependence is similar. With this in mind, we characterize the market through the movement of the detrended Westpac coincident economic indicator, constructed by The Westpac-Melbourne Institute Indices of Economic Activity. This composite economic indicator is a weighted average of six major economic indicators of the Australian economy. The main economic indicators used to construct the index are a retail trade index, an industrial production index, and the civilian employment and unemployment rate. As a weighted average of different economic indicators, it reduces distortion due to measurement error.

2.1.3. *Lagged yield spread*

The causal relationship between the shape of the yield curve and the stage of the business cycle is well established in the economics literature. Chen, Roll, and Ross (1986) found that the slope of the term structure and the industrial production growth rate are both important explanatory variables in explaining historical stock market returns. In a recent paper by Resnick and Shoesmith (2002), the 4-month lagged yield spread between the 10-year US treasury bond and the 3-month treasury bill was used as an explanatory variable to time forth coming bear markets. They found that the yield spread is an important variable to predict bear markets 4 months in advance. We use the 4-month lagged yield spread of Resnick and Shoesmith (2002) as our third definition of market condition. We characterize the state of the market by the continuous path of the yield spread around its mean value. Large extreme positive (negative) deviations from the mean indicate extreme bull (bear) market conditions. The lagged yield spread serves as an indicator of market conditions, since it provides a short-term view of the future state of the market. For example, conditions will become more optimistic (pessimistic) when the short-term rate is much larger (smaller) than the long-term rate.

2.1.4. *Excess market return*

Finally, we use the conventional excess market return ($R_{mt} - R_{ft}$) as our fourth definition to test the significance of betas in bull and bear regimes.

Thus, we have a total of eight definitions of bull and bear market conditions, the above four definitions and their four duration dependent counterparts. The duration dependent

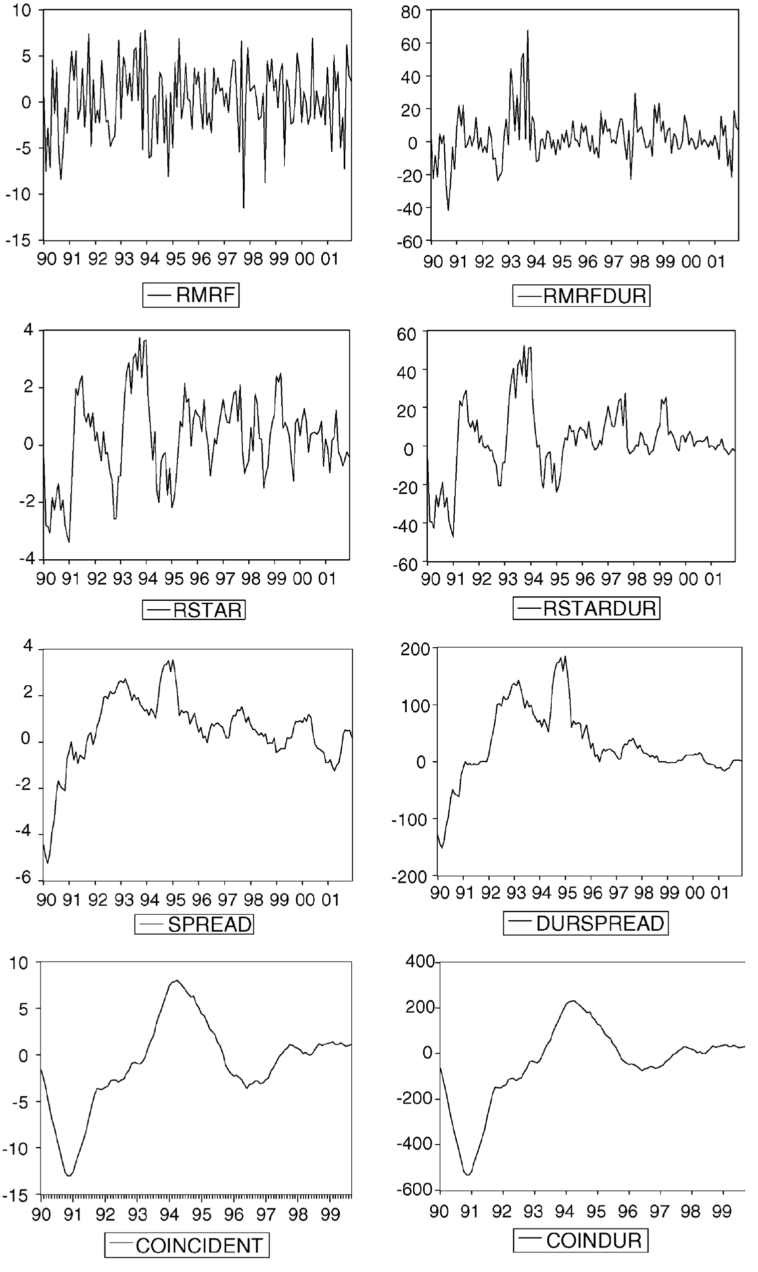


Fig. 1. Time graphs of the eight selected transition variables. Note that the horizontal axis provides the dates (year and month) of the observed value of the indicator. Also note that the first column of graphs is the excess market return, the 6 month moving average of the excess market return, the 4-month lagged yield spread and the coincident indicator, respectively, and that the second column of graphs is the duration dependent version of the first column variables.

definitions are obtained from each of the four definitions above by simply multiplying the value of the transition variable at each point in time by the contemporaneous duration of the state the market is in at that particular point in time. All eight definitions are applied separately as transition variables in the LSTM model. The findings will reflect the most appropriate of the eight characterizations of bull and bear market conditions. Fig. 1 represents the time series of market conditions as defined by all eight definitions. Among the eight definitions, the coincident index smooths the data to the greatest extent, as it takes the average of six economic indicators that represent different components of economic activity. The 4-month lagged yield spread is the next smoothest series and describes well the predominant trends in market conditions as can be seen by the way the peaks and troughs correspond to expansions and contractions that have occurred in the Australian economy. Among the given definitions, the conventional excess market return ($R_{mt} - R_{ft}$) provides the poorest representation of underlying trends in market conditions. As mentioned before, this series is too noisy and therefore does not reveal the underlying trends or cyclical nature of the data. These graphs clearly show the smoother nature of the three new definitions over the conventional excess market return, ($R_{mt} - R_{ft}$) definition. In all the definitions, when duration is introduced, the trend is even more pronounced.

3. Theory, model development and statistical methodology

3.1. Theory

While the threshold DBM model is appealing because it reflects the intuition that investors respond to excess market returns differently depending on whether they are negative or positive, the implied single threshold of zero unnecessarily abstracts from the complications of reality. In fact, investors are likely to have differing thresholds due to differing investment horizons, risk preferences and transaction costs. Taking the differing transactions costs argument alone, investors are unlikely to have common thresholds. The time spent gathering the information and organizing the transactions, which are necessary to adjust ones portfolio in response to changing market conditions, vary from one individual to the next. Add to this, the varying tax and brokers fees across individuals and we have individual specific transaction cost thresholds. Therefore, a threshold of zero does not divide aggregate behavioural regimes, but instead a model that smooths transition between regimes is a more appropriate description of the process. One way of modelling these considerations is to let C_{ij} measure the perceived transaction cost threshold associated with trading stock j for individual i. Then individual i will only want to respond to excess market returns that are greater than this value. The C_{ij} differ across individuals $i \in I$ as discussed above. Therefore, one can think of C_{ij} as a single draw from a random variable C_i, which measures transaction cost thresholds associated with the trading of security i. Assume $-\infty < C_i < \infty$ and let $F_\gamma(C_i)$ be the cumulative density function of C_i. Given an excess market return of Z_t, the cumulative density evaluated at Z_t then measures the proportion of individuals who would find it profitable to react to this value Z_t. Aggregate adjustment will depend on investor response, measured by $F_\gamma(Z_i)$. Thus, we offer the market model $Y_t = \alpha_1 + \beta X_t + \beta^U F_\gamma(Z_t) X_t + \varepsilon_t$. The discussion in the following section addresses the heterogeneity issue using the logistic smooth transition model.

3.2. The logistic smooth transition market model

An unconditional beta for any asset or portfolio can be estimated using the constant risk market model (CRM) regression:

$$R_{it} = \alpha_i + \beta_i R_{mt} + \varepsilon_{it}, \tag{1}$$

where R_{it} is the return, or excess return, on asset i for period t, R_{mt} is the return, or excess return, on the market index for period t, $\beta_i = \text{cov}(R_{it}, R_{mt})/\sigma_{mt}^2$ and ε_{it} is the disturbance term which has zero mean and is assumed to be serially independent and homoscedastic. Under this specification, α_i and β_i are constant with respect to time.

A dual-beta market model can be specified as:

$$R_{it} = \alpha_i + \beta_i R_{mt} + \beta_i^U D_t R_{mt} + \varepsilon_{it} \tag{2}$$

where D_t is a dummy variable defining up and down markets by taking the value 1 if the return, or excess market return, R_{mt} (or any other market characterizing transition variable R_t^*) exceeds some critical value c and zero otherwise. Notice that in this specification, the difference between the up and down market value of the slope coefficient is β_i^U.

Now consider the logistic smooth transition regression (LSTR) model, henceforth called the logistic smooth transition market model, which has (1) and (2) as special limiting cases:

$$R_{it} = \alpha_i + \beta_i R_{mt} + \beta_i^U F(R_t^*) R_{mt} + \varepsilon_{it} \tag{3}$$

with

$$F(R_t^*) = (1 + \exp[-\gamma R_t^*])^{-1}, \quad \gamma > 0. \tag{4}$$

The superscript U signifies an up market differential value of the parameter β, F is the logistic smooth transition function with transition variable R_t^* and the critical threshold value for the demeaned R_t^* series is 0 and ε_{it} $niid(0, \sigma_i^2)$. Note that in our case, R_t^* is one of the eight definitions of market conditions. Clearly, beta in the state dependent model (3) changes monotonically with the independent variable R_t^* as (4) in (3) is a smooth continuous increasing function of R_t^* and takes a value between 0 and 1, depending on the magnitude of R_t^*. When $R_t^* = 0$, the value of the transition function is 0.5 and the current regime is half way between the two extreme upper and lower regimes. When R_t^* is large and positive R_{it} is effectively generated by the linear model $R_{it} = \alpha_i + (\beta_i + \beta_i^U)R_{mt} + \varepsilon_{it}$, while when R_t^* is large and negative R_{it} is virtually generated by $R_{it} = \alpha_i + \beta_i R_{mt} + \varepsilon_{it}$. Intermediate values of R_t^* give a mixture of the two extreme regimes. Note that the DBM obtains as a special case since when γ approaches infinity in (4), FR_t^* becomes an indicator function with $F(R_t^*) = 1$ for all values of R_t^* greater than 0 and $F(R_t^*) = 0$ otherwise. Also, notice that the constant risk market model is a special case since as the smoothness parameter, γ, approaches zero, (3) becomes the constant risk market (CRM) model.

3.3. Tests of LSTM against linearity

As mentioned in section 3.2, when γ approaches zero, (3) becomes the CRM, implying that the constant risk market model is nested in the LSTM model. Therefore, our first step in specifying the model is to test for linearity against the LSTM form. If the null of linearity cannot be rejected, we shall conclude that the constant risk market model adequately represents the data generating process. On the other hand, if linearity is rejected, we go on to estimate the highly non-linear LSTM form using the non-linear least squares (NLS) method.

From (3) and (4), it can be seen that testing $H_0{:}\gamma = 0$ is a non-standard testing problem, since (3) is identified only under the alternative $H_1{:}\gamma \neq 0$. Thus, standard t- and F-testing methods are not appropriate steps to arrive at a model choice. Therefore, following Luukkonen et al. (1988), we replace FR_t^* by either a first order or a third order Taylor series linear approximation in a version of (3) that allows the intercept to vary as well as the slope coefficient and expand to form an auxiliary model with which to test the equivalent null hypothesis that both α_i^U and β_i^U are not zero or $\gamma \neq 0$ in Eq. (3).

When a third order Taylor series approximation is used, the expanded and reparameterized equation is:

$$R_{it} = \phi_0 + \phi_1 R_{mt} + \phi_2 R_t^* + \phi_3 (R_t^*)^2 + \phi_4 (R_t^*)^3 + \phi_5 R_{mt} R_t^* + \phi_6 R_{mt} (R_t^*)^2 + \phi_7 R_{mt} (R_t^*)^3 + u_{it} \tag{5}$$

where in this reparameterized form the null hypothesis is: $H_0{:}\phi_j = 0 (j = 2, \ldots, 7)$ The test is then carried out as follows:

(i) Regress R_{it} on $\{1, R_{mt}\}$, form the residuals $\hat{\varepsilon}_{it}(t = 1, \ldots, T)$ and the residual sum of squares $\text{SSE}_0 = \sum \hat{\varepsilon}_{it}^2$.
(ii) Regress $\hat{\varepsilon}_{it}$ on $\{1, R_{mt}, R_t^*, (R_t^*)^2, (R_t^*)^3, R_{mt} R_t^*, R_{mt}(R_t^*)^2, R_{mt}(R_t^*)^3\}$, form the residuals $\hat{\eta}_{it}(t = 1, \ldots, T)$ and $\text{SSE}_2 = \sum \hat{\eta}_{it}^2$.
(iii) Compute the test statistic $S_3 = (T - 8)/6(\text{SSE}_0 - \text{SSE}_3)/\text{SSE}_3$.

Under H_0, S_3 is approximately F distributed. When a first order Taylor series is used, the test statistic is denoted S_1 and is derived similarly. In this case, the test regressors are $\{1, R_{mt}, R_t^*, R_{mt} R_t^*\}$. An S_1^* test statistic with test regressors $\{1, R_{mt}, R_t^*, R_{mt} R_t^*, R_{mt} R_t^{*3}\}$ will also be used. Because S_1, S_1^* and S_3 are Lagrange multiplier type test statistics, they can be expected to have reasonable power. Further, both Luukkonen et al. (1988) and Petruccelli (1990) have shown that these tests are powerful in small samples when the true alternative is either the smooth transition regression or the abrupt regime switch form. Thus, we can expect that in our case, there will be reasonable power against the DBM as well. In this paper, we will use the S_1, S_1^* and S_3 statistics. Though S_3 is not as powerful as S_1 or S_1^* when the up market and down market intercept terms are the same, it is generally more powerful if that assumption does not hold.

We also use Tsay's (1989) test of non-linearity. This procedure involves sorting the bivariate observations $(R_{it} - R_{mt})$ in ascending or descending order based on the ranked order of the corresponding threshold variable R_t^*. A sequence of OLS regressions is then conducted starting with the first b ranked bivariate observations. Then OLS is again performed for the first $b + 1$ observations and so on until we come to the last ordered pair. The standardized one-step ahead predictive residuals $\hat{e}$ are then regressed on the corresponding (reordered) regressor R_{mt}

$$\hat{e}_t = \omega_0 + \omega_1 R_{mt} + \varepsilon_t \tag{6}$$

and the associated F-statistic $F(2, n-b-2) = \{[(\sum \hat{e}_t - \sum \varepsilon'^2_t)/2]/(\sum \varepsilon'^2_t/(n-b-2)\}$ is calculated. The power of this test comes from the fact that the sequential OLS estimates are consistent estimates of the lower regime parameters as long as the last bivariate observation used in the regression does not belong to the upper regime and there are a sufficient number of observations to estimate the parameters of the lower regime. In this case, the predictive residuals are orthogonal to the corresponding regressor R_{mt}. However, for the residuals corresponding to R_t^* greater than the unknown threshold value c, the predictive residuals are biased because of the model change at this unknown change point.

4. Data description

The data used in this study was obtained from four sources. The Securities Research Centre of the Asia/Pacific (SIRCA) was the source for the Australian stock price data. The Reserve Bank of Australia was the source for the 10-year treasury bond rate and the 3-month treasury bill rate, which were used to calculate the yield spread. Further, the Melbourne Institute of the University of Melbourne was the source for the Westpac coincident economic indicator. The S&P 500 US airline industry price index series and the S&P 500 composite market index were obtained from Datastream, while the long-term bond yields and the T-bill returns were obtained from the St. Louis Federal Reserve Bank Website.

We selected the top 50 heavily and continuously traded Australian firms representing different industries. However, the sample has some concentration towards materials/mining and financial services industries. A high level of trading activity for a firm is an important factor with which to measure investor interests and their information utilization process. Observations are monthly ranging from April 1986 to December 2001. We use monthly rather than daily data because daily data is expected to have ARCH effects. ARCH type non-linearities may confound the testing for non-linearity in the conditional mean. The linearity tests we use also have power in detecting ARCH type non-linearities. For the US airline industry data observations are monthly ranging from January 1990 to September 2003.

A continuously compounded percentage return series for each security, the airline index and the market indices were calculated as the difference of the log of the prices.

We calculated the duration of market conditions based on each of the four different definitions of bull and bear markets for the 50 Australian firms. For the US composite airline industry portfolio, we only considered the duration dependent lagged yield spread. The duration of a particular market is calculated as the consecutive number of uninterrupted up or down months relative to the threshold value. The threshold value for the excess market return, the 6-month moving average of that variable and the coincident index is zero. Thus, an uninterrupted sequence of say five positive (negative) values of the transition variable indicates that that particular bull (bear) market had a duration of 5 months. For the lagged yield spread, we used the mean value of this variable as the threshold.

5. Empirical results for 50 Australian firms

5.1. Linearity tests

As mentioned in the methodology section, we begin by testing for non-linearity using the Luukonen and Tsay test statistics. If non-linearity is detected, we then go on to estimate the non-linear LSTM model using non-linear least squares (NLS). Although not reported, the statistics and their p-values were based on White's (1980) heteroscedasticity consistent standard error estimates. For the 50 heavily traded Australian securities, we tested for significant non-linearity using the eight different definitions of bull and bear markets with and without duration of these market conditions incorporated in the model. We rejected the null constant risk model if at least one of the Luukonen and Tsay test statistics is significant at the 10% level. Table 1 summarizes the linearity test results. The number of securities found to be non-linear are 15, 13, 13 and 21 for the excess market return, the 6-month moving average of the excess market return, the Westpac coincident index and the 4-month lagged yield spread, respectively. The number of securities found to be non-linear for the duration dependent versions of these variables is 16, 15, 16 and 21, respectively (See the second row of Table 1). Since there is a consensus among academics and practitioners that beta is in fact time-varying, the application of a transition variable that gives more non-linearity across firms is most appropriate. We found that the 4-month lagged yield spread or the duration dependent version of this transition variable captures the non-linear dynamics in the market model better than do any of the other transition variables tested. Additionally, between these two variables, the duration dependent model frequently offers a better fit, in terms of the mean squared errors (MSE) of the residuals criterion, thereby providing significant evidence of duration dependence in stock market conditions. We discuss the comparison of fit below.

Luukonen and Tsay tests were designed to have good power properties in detecting the LSTM and DBM forms. Therefore, the fact that for the lagged yield spread and duration dependent lagged yield spread variables, non-linearity was detected in 21 out of the 50 cases suggests that the lagged yield spread and the duration dependent lagged yield spread capture

Table 1
Proportion of firms found to be non-linear for each transition variable

Transition variable	$(R_m - R_f)$	ave$(R_m - R_f)$	Coincident	Spread
Proportion found non-linear	15/50	13/50	13/50	21/50

Transition variable	$(R_m - R_f) \times$ duration	ave$(R_m - R_f) \times$ duration	Coincident $\times$ duration	Spread $\times$ duration
Proportion found non-linear	16/50	15/50	16/50	21/50

Table 2
Evidence favoring Durspread over spread using mean square error (MSE) criterion

Probability of selecting **Durspread** over **Spread** 16 times or more for the 21 **firms** that were non-linear for both variables assuming a Bernoulli process with success probability 0.5	$P(X) \geq 16 \mid n = 21; \pi = 0.5) = 0.013$

the non-linear dynamics in the market model more than any of the other variables analysed. Furthermore, Table 2 provides significant evidence that the duration dependent version of the lagged yield spread is the more frequent choice when compared to the lagged yield spread. As the table indicates, between these two variables, the duration dependent transition variable was selected using the MSE residuals criterion in 16 out of the 21 cases for which non-linearity was detected. If the duration dependent variable is generally not favoured over its non-duration dependent counterpart and the selection of the transition variables are independent across firms, we would expect to select a duration dependent variable for about half of the firms. Therefore, assuming a Bernoulli process with success probability 0.5, the probability of observing 16 or more firms favouring the duration dependent variable choice would be 0.013 indicating strong and significant support for the superiority of the duration dependent variable. Thus, using linearity tests in combination with the minimum mean square error criterion, we find strong evidence to support duration dependence in market condition systematic risk. Thus, we find, in the context of asset pricing, that the market condition alone does not fully determine the pricing of risky assets. Instead the market condition combined with the duration of being in that particular state determines the risk premium for pricing risky assets.

Although not reported, we found the mean squared errors of the residuals of the spread based models to be smaller than the mean squared errors of the other variables across almost all firms. Therefore, we feel justified in focusing our attention on only the spread based models. In addition, since we found evidence of duration dependence, we report and discuss only the estimation results of the models that use the duration dependent lagged yield spread as transition variable. Note that although not reported the parameter estimates using the lagged yield spread are very similar to those using the duration dependent lagged yield spread variable that are reported in Table 3.

5.2. *Results of logistic smooth transition market model estimation*

5.2.1. *Interpretation of the shape of the time-varying beta process*

The estimates of β and β^U in Table 3 cannot be interpreted in quite the same way as in the dual-beta market model representation. Since $F(X_t)$ is a monotonic increasing function of the market condition variable X_t and ranges from 0 to 1, β, the linear and constant portion of risk can be interpreted as the extreme down market risk of the security in question. $\beta^U F(X_t)$ on the other hand captures the time-varying non-linear portion of risk. When β^U is positive (negative), the risk of the security increases (decreases) with improving market conditions and the extreme up market risk $\beta + \beta^U$ is greater (less) than the extreme down market risk β. The transition between the two extreme conditions is characterized by the transition function $F(X_t)$. The smoothness parameter γ governs the rate of transition between regimes

Table 3
(Spread × duration) parameter estimates for Logistic Smooth Transition models using spread × duration as transition variable

Firm (Industry)	α	β	β^U	γ	MSE
(1) Amcor (Industrials)	−0.212 (0.542)	0.636 (0.000)	0.335 (0.070)	559.027	21.487
(2) Argo Investments (Finance-Investment Management Company)	0.299 (0.410)	1.368 (0.000)	−1.442 (0.000)	0.438	24.671
(3) ANZ (Finance-Banking)	0.067 (0.876)	0.694 (0.000)	0.291 (0.204)	5.391	31.013
(4) Aust. Gas and Lighting (Energy)	0.030 (0.952)	1.118 (0.000)	−0.546 (0.022)	1601.820	43.480
(5) BHP Billiton (Mining)	0.066 (0.862)	0.732 (0.000)	0.250 (0.150)	22.294	25.274
(6) Brambles Industries (Industrials)	0.376 (0.306)	0.933 (0.000)	−0.247 (0.101)	1240.471	25.701
(7) General Pr. Tst. (Finance-Real Estate)	−0.324 (0.220)	0.315 (0.000)	0.257 (0.062)	75.622	13.095
(8) Goodman Fielder (Consumer-Staples)	−0.871 (0.056)	1.053 (0.000)	−0.459 (0.060)	67.136	40.716
(9) Gud Holdings (Industrials)	−0.105 (0.854)	0.696 (0.000)	−0.470 (0.076)	5399.380	59.363
(10) Hardie James (Construction Equipment)	−0.131 (0.782)	0.528 (0.000)	0.483 (0.020)	1902.611	41.052
(11) Lend Lease Corporation (Finance-Real Estate)	−0.332 (0.408)	1.240 (0.000)	−0.250 (0.103)	2.245	29.465
(12) MEM Group (Mining)	0.281 (0.880)	−2.451 (0.000)	5.742 (0.000)	0.707	628.783
(13) MIM (Mining)	−1.385 (0.038)	1.531 (0.000)	−0.426 (0.294)	4.499	81.826
(14) News Corporation (Media)	0.079 (0.914)	2.155 (0.000)	−1.180 (0.008)	15.063	100.156
(15) PBL (Media)	0.718 (0.282)	1.278 (0.004)	−0.145 (0.754)	74303	58.980
(16) QBE Insurance Group (Finance-Insurance)	0.233 (0.672)	0.631 (0.000)	0.686 (0.044)	115.850	49.503
(17) South Corporation (Consumer Staples)	0.309 (0.498)	0.749 (0.000)	0.063 (0.714)	5.691	38.155
(18) Striker Resources (Mining)	−2.611 (0.092)	−1.486 (0.000)	5.169 (0.000)	0.564	486.125
(19) Suncorp (Finance-Banking and Insurance)	1.043 (0.005)	0.864 (0.000)	−0.137 (0.062)	20.875	24.734
(20) Westpac Banking (Finance-Banking)	−0.152 (0.688)	0.634 (0.000)	0.484 (0.002)	2.466	25.831
(21) WMC (Mining)	−0.428 (0.442)	1.422 (0.000)	−0.210 (0.538)	519.385	59.770

Note: *p*-values, based on heteroscedasticity consistent standard errors, are in parentheses beneath the parameter estimates. A na entry means convergence was not achieved.

with respect to the transition variable X_t. If γ is relatively small (large) transition is gradual (abrupt). Therefore, the smoothness parameter γ may help to interpret investor reaction to changes in market conditions. For instance, an abrupt switch, as indicated by a large value of γ, may represent the fact that investors have homogeneous beliefs as they switch from one regime to the next instantly as the transition variable crosses the threshold value. Abrupt switch may be due to the flow of symmetrical information in terms of the efficient market hypothesis. A relatively small value of γ on the other hand may indicate that investors have heterogeneous beliefs and that information flow is asymmetric. See Section 3 for a theoretical explanation of this process. In Fig. 2, we provide graphs of the transition functions $F(X_t)$ for each of the 21 firms for which non-linearity was detected and convergence was achieved. The shape of the transition function $F(X_t)$ is smooth (s-shape) for small γ values and abrupt (Z-shaped) for large γ values. Fig. 2 (Panel A) exhibits the small γ smooth transition cases while Fig. 2 (Panel B) exhibits the large γ abrupt switching firms. The γ values indicate that the transition varies from firm to firm. However, there is some evidence of commonality when the firms are sorted into their respective industries. For instance, all banking companies with the exception of WMC, and all mining companies exhibit smooth transition. While, on the other hand, all industrial equipments companies exhibit abrupt transition. However, the sample size is not sufficient to draw firm conclusions. Further research based on a larger dataset may provide more interesting insights. While the reasons behind such industry specific results is beyond the scope of this paper they may be related to information hypotheses. One can argue that abrupt transition is the result of a general consensus in the valuation of a firm, which may be attributed to uniformity in the information being shared in the market. This sort of symmetrical information has been attributed to disclosure practices and corporate governance in the literature. Although our findings do not provide a direct link to explain the information asymmetry found for some firms, the γ value is a good representation of the market's reaction to changing economic conditions.

5.2.2. Interpretation of the time-varying LSTM betas

The beta values reported in Table 3 provide some interesting results. At the 10% level of significance, out of the 21 companies reported, 15 companies have a significant β^U. This indicates that risk significantly differs between the two extreme regimes. However, the sign and the magnitude of the β^U coefficients vary across the fifteen companies. For 7 out of the 15 companies, the estimated coefficient β^U is positive while for the remaining 8, the coefficient is negative. A positive (negative) β^U coefficient indicates that systematic risk will increase (decrease) with improving market conditions. In other words, unlike the DBM with two betas, we have a continuum of beta values where beta is represented as a monotonic increasing (decreasing) function of the state of the market. For 8 of the 15 companies, risk decreases in response to improving market conditions. The remaining seven companies exhibit the opposite effect. Our findings do not offer much support for Kim and Zumwalt's (1979) findings suggesting that investors expect a higher risk premium for being in a bear market. However, the problem with the Kim and Zumwalt (1979) and other related works to date is that they define the market as being in either a bull or a bear state at any given point in time with no allowance for the magnitude of market condition. Our findings support Maheu and McCurdy (2000a)

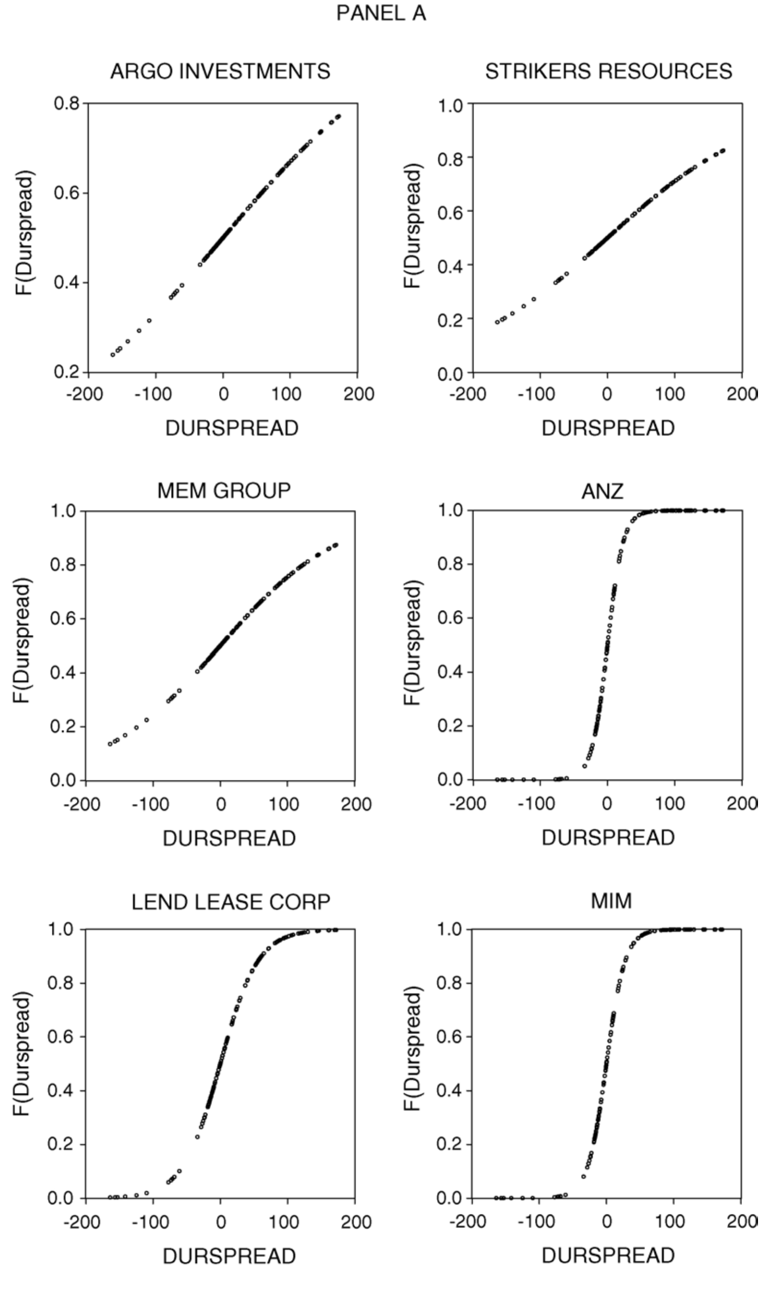


Fig. 2. Graphs of transition function $F_{\gamma}(R^{*})$ with R^{*} = spread × duration as transition variable. Panel A (Australian Industry Smooth Transition Firms); Panel B (Australian Industry Abrupt Transition Firms); Panel C (US Airline Industry Portfolio). Note that Durspread on the horizontal axis is the demeaned duration dependent 4-month lagged yield spread series.

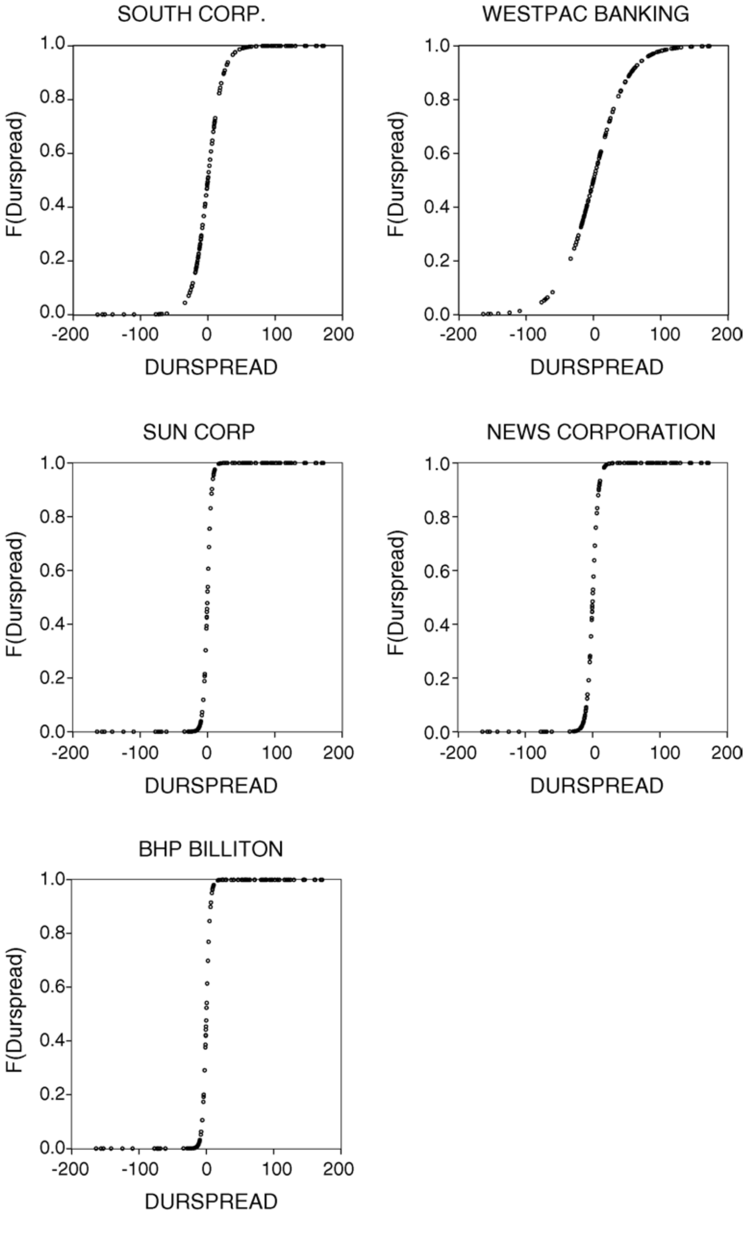


Fig. 2. (*Continued*)

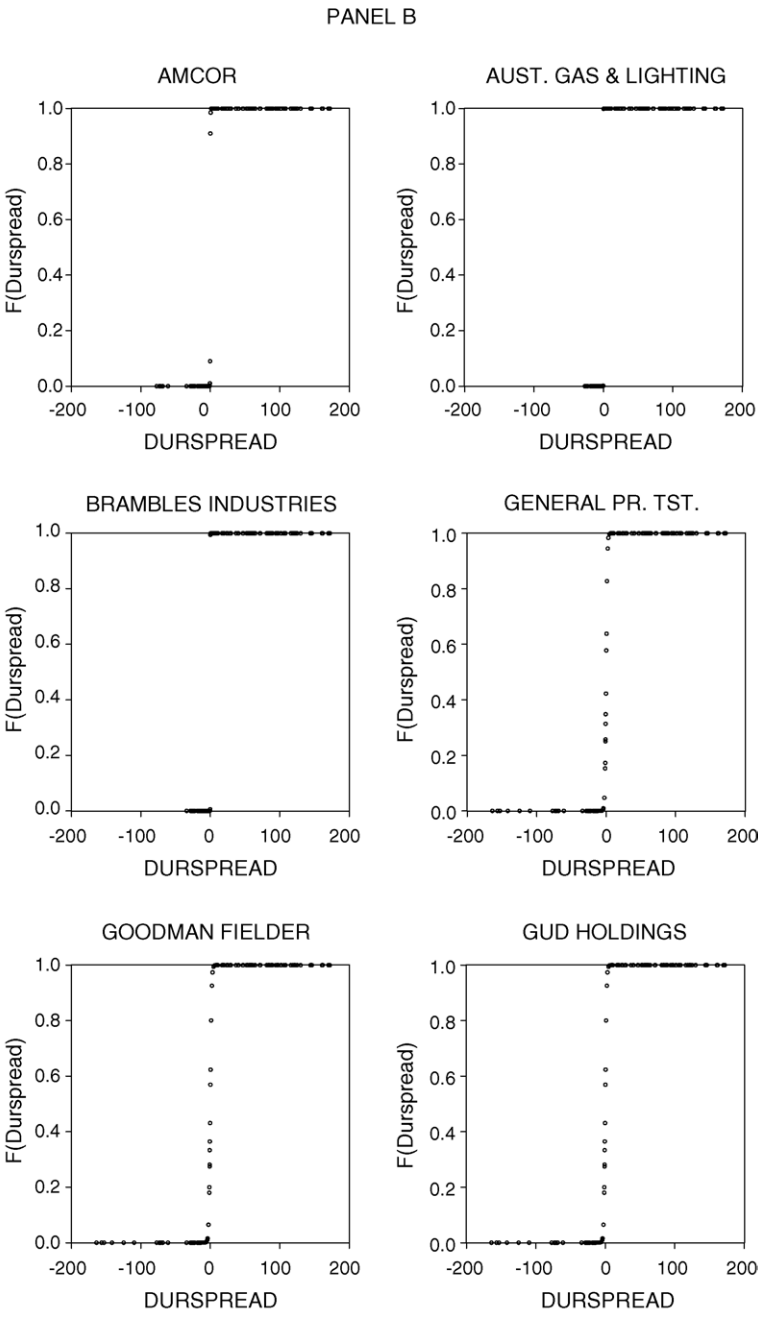
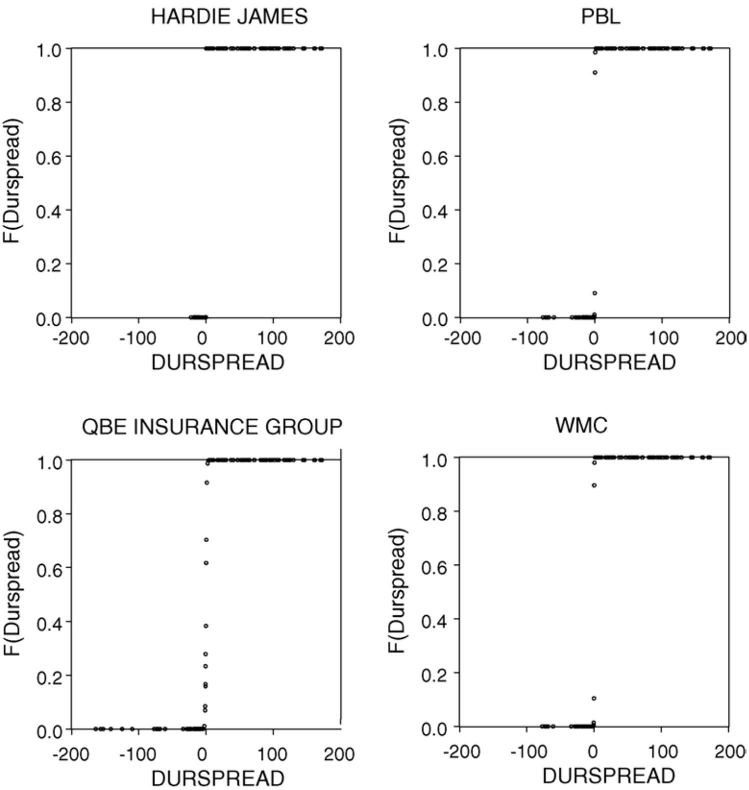


Fig. 2. (*Continued*)



PANEL C

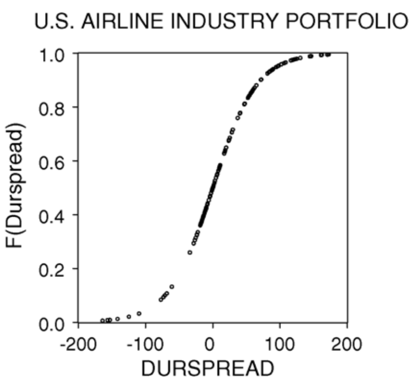


Fig. 2. (*Continued*).

Table 4
p-values of Jarque-Bera normality test, fourth order ARCH test and second, fourth and eighth order Legrange multiplier serial correlation tests of the residuals of the estimated non-linear models

Firm	Jarque-Bera	ARCH(4)	SC(2)	SC(4)	SC(8)
Amcor	0.552	0.017	0.116	0.141	0.301
Argo Investments	0.126	0.107	0.295	0.409	0.173
ANZ	0.137	0.219	0.013	0.029	0.092
Aust. Gas and Lighting	0.536	0.146	0.380	0.071	0.101
BHP Billiton	0.159	0.962	0.106	0.507	0.770
Brambles Industries	0.178	0.497	0.103	0.268	0.262
General Pr. Tst.	0.042	0.193	0.049	0.018	0.083
Goodman Fielder	0.178	0.879	0.091	0.126	0.156
Gud Holdings	0.097	0.058	0.126	0.143	0.049
Hardie James	0.882	0.192	0.311	0.657	0.390
Lend Lease Corporation	0.523	0.302	0.957	0.176	0.675
MEM Group	0.013	0.793	0.001	0.007	0.054
MIM	0.267	0.857	0.116	0.102	0.585
News Corporation	0.084	0.952	0.136	0.279	0.310
PBL	0.027	0.262	0.160	0.352	0.130
QBE Insurance Group	0.231	0.421	0.077	0.191	0.341
South Corporation	0.415	0.520	0.033	0.053	0.023
Striker Resources	0.022	0.484	0.447	0.405	0.620
Suncorp	0.114	0.042	0.232	0.193	0.329
Westpac Banking	0.465	0.068	0.739	0.612	0.452
WMC	0.107	0.115	0.127	0.030	0.150
US Airline Industry	0.056	0.597	0.078	0.011	0.019

Note: All numerical entries are p-values. JB stands for the Jarque-Bera test of normality, ARCH(4) represents the Lagrange multiplier fourth order ARCH test and SC(p) represents the Lagrange multiplier test for pth order serial correlation.

for 8 of the 15 firms. The seven negative β^U values indicate that the overall level of risk, $\beta + \beta^U F_\gamma(R_t^*)$, decreases with improving market conditions. We found that the change in risk is firm-specific. There is no evidence of an influence of industry affiliation on the nature of the relationship between risk and market conditions. Further research on the role of firm-specific characteristics would help to provide a clearer understanding of this phenomenon.

The residual diagnostics, exhibited in Table 4, reveal that both heteroscedasticity and serial correlation are present for some securities. However, the serial correlation is mild and the standard errors have been adjusted for heteroscedasticity using White's (1980) method.

6. Empirical results for US airline industry portfolio

To broaden the scope of our results, we also estimated the LSTM model for the S&P 500 composite Airline Industry Portfolio. We chose this industry because on an a priori basis, we expect non-linearities for this industry and because Teräsvirta and Anderson (1992)

Table 5
Spread × duration parameter estmates for LSTM model using duration dependent 4-month lagged yield spread as transition variable

Industry	α	β	β^U	γ	MSE
(1) US airline industry	−0.8403 (0.104)	0.870 (0.004)	1.159 (0.020)	1.935	41.102

Note: *p*-values, based on heteroscedasticity consistent standard errors, are in parentheses beneath the parameter estimates. A na entry means convergence was not achieved.

found significant non-linearities in economic indicators and attributed this to large negative oil price shocks. Airline industry risk can be expected to be especially sensitive to oil price shocks.

Using the duration dependent 4-month lagged yield spread as transition variable, the Luukkonen et al. (1988) S_1, S_1^*, S_3, Tsay and reverse Tsay test statistics, discussed in Section 3.3 had *p*-values of 0.043, 0.011, 0.027, 0.114 and 0.093, respectively. Therefore, we estimated the duration dependent lagged yield spread based model and the results are in Table 5. Notice that for this industry portfolio, the extreme up market beta is significantly greater than the extreme down market beta at the 5% level of significance. The smoothness parameter is small indicating that transition across regimes is smooth and gradual. See Fig. 2 (Panel C) for a graph of the transition function.

7. Concluding remarks

In this paper, we refined the two-regime dual-beta market model in order to address several new hypotheses put forth in the asset-pricing literature. This research represents a significant contribution to the literature for the following reasons. First, our new LSTM model represents an improvement over the existing DBM in that it allows for the possibility of smooth transition between regimes (a plausible assumption) while including the constant risk model and DBM as special cases. The inclusion of a transition parameter in the LSTM model allowed us to infer that most of the Australian non-linear securities analysed and the US composite airline industry portfolio do exhibit smooth rather than abrupt transition between regimes. This finding provides support for our heterogeneous beliefs hypothesis. Furthermore, this result offers some evidence to contradict the efficient markets hypothesis. Second, our finding of duration dependence of market conditions in betas offers the important implication that the length of time spent in bull and bear markets is a key determinant to explain the risk/return trade-off relationship of risky assets. Furthermore, the parameter estimates obtained using models that ignore this duration dependence fact may suffer from misspecification bias. Finally, the finding that the lagged yield spread represents bull and bear market conditions "better" than all other transition variables considered in this study, including the traditional excess market return, is interesting as it offers a prescription to the selection of securities on the basis of their forecasted betas. Importantly, we found some evidence in favour of the expected result that up markets have less risk than their down market counterparts. Future research on the role that firm characteristics and industry affiliations play on the nature of the risk/market condition relationship is needed.

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