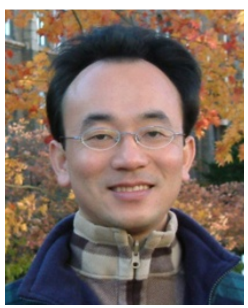
Tamkang University

Data Mining
資料探勘

關連分析
(Association Analysis)

1022DM02
MI4
Wed, 6,7 (13:10-15:00) (B216)



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2014-02-26

課程大綱 (Syllabus)

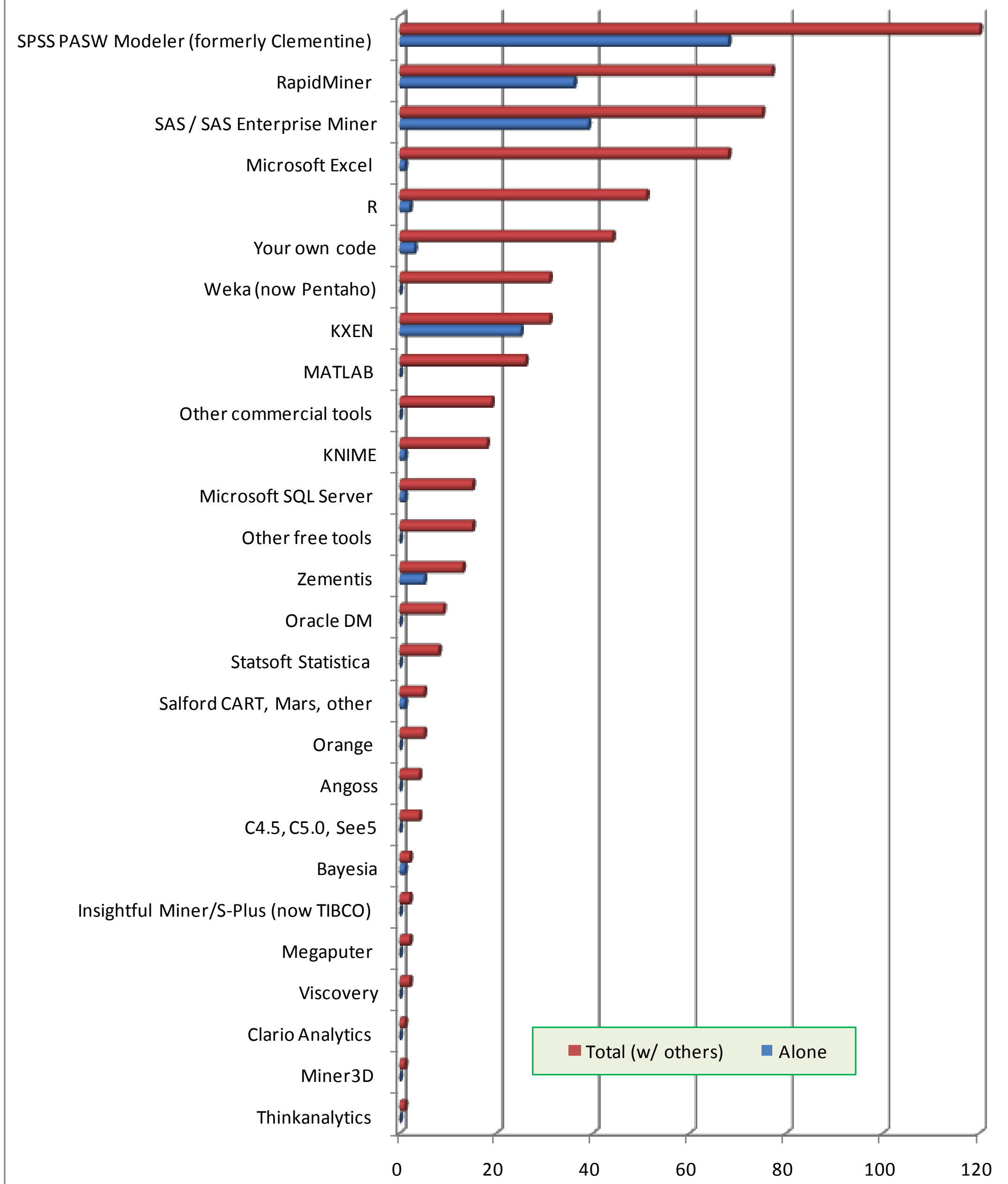
週次 (Week)	日期 (Date)	內容 (Subject/Topics)
1	103/02/19	資料探勘導論 (Introduction to Data Mining)
2	103/02/26	關連分析 (Association Analysis)
3	103/03/05	分類與預測 (Classification and Prediction)
4	103/03/12	分群分析 (Cluster Analysis)
5	103/03/19	個案分析與實作一 (SAS EM 分群分析)： Case Study 1 (Cluster Analysis – K-Means using SAS EM)
6	103/03/26	個案分析與實作二 (SAS EM 關連分析)： Case Study 2 (Association Analysis using SAS EM)
7	103/04/02	教學行政觀摩日 (Off-campus study)
8	103/04/09	個案分析與實作三 (SAS EM 決策樹、模型評估)： Case Study 3 (Decision Tree, Model Evaluation using SAS EM)

課程大綱 (Syllabus)

週次 (Week)	日期 (Date)	內容 (Subject/Topics)
9	103/04/16	期中報告 (Midterm Project Presentation)
10	103/04/23	期中考試週 (Midterm Exam)
11	103/04/30	個案分析與實作四 (SAS EM 迴歸分析、類神經網路)：Case Study 4 (Regression Analysis, Artificial Neural Network using SAS EM)
12	103/05/07	文字探勘與網頁探勘 (Text and Web Mining)
13	103/05/14	海量資料分析 (Big Data Analytics)
14	103/05/21	期末報告 (Final Project Presentation)
15	103/05/28	畢業考試週 (Final Exam)

Data Mining Software

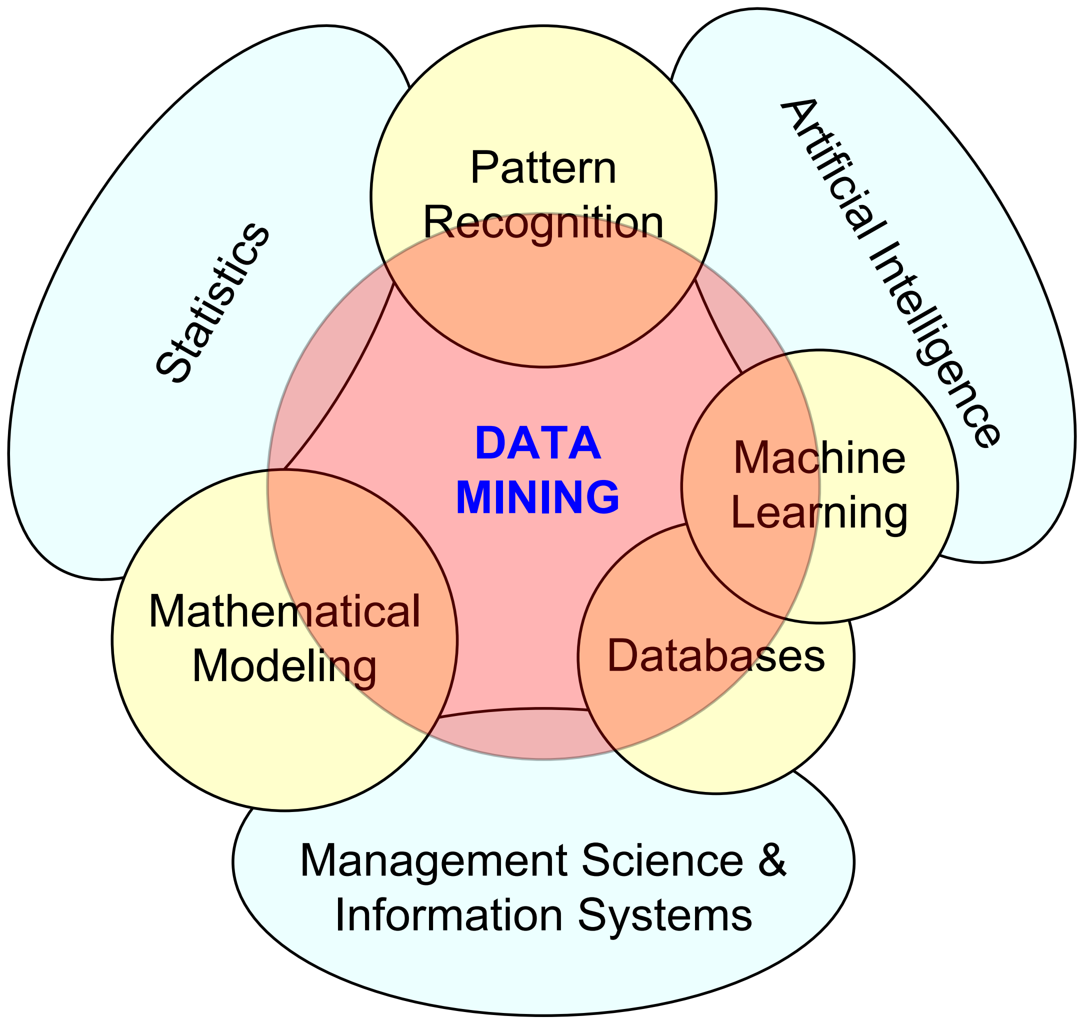
- Commercial
  - SPSS - PASW (formerly Clementine)
  - SAS - Enterprise Miner
  - IBM - Intelligent Miner
  - StatSoft – Statistical Data Miner
  - … many more
- Free and/or Open Source
  - Weka
  - RapidMiner…



Source: KDNuggets.com, May 2009

Source: Turban et al. (2011), Decision Support and Business Intelligence Systems

Data Mining at the Intersection of Many Disciplines



Source: Turban et al. (2011), Decision Support and Business Intelligence Systems

A Taxonomy for Data Mining Tasks

Data Mining	Learning Method	Popular Algorithms
Prediction	Supervised	Classification and Regression Trees, ANN, SVM, Genetic Algorithms
Classification	Supervised	Decision trees, ANN/MLP, SVM, Rough sets, Genetic Algorithms
Regression	Supervised	Linear/Nonlinear Regression, Regression trees, ANN/MLP, SVM
Association	Unsupervised	Apriory, OneR, ZeroR, Eclat
Link analysis	Unsupervised	Expectation Maximization, Apriory Algorithm, Graph-based Matching
Sequence analysis	Unsupervised	Apriory Algorithm, FP-Growth technique
Clustering	Unsupervised	K-means, ANN/SOM
Outlier analysis	Unsupervised	K-means, Expectation Maximization (EM)

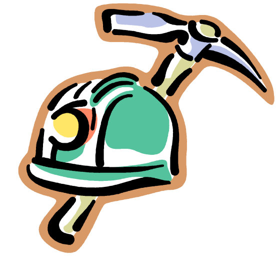
Source: Turban et al. (2011), Decision Support and Business Intelligence Systems

Why Data Mining?

- More intense competition at the global scale
- Recognition of the value in data sources
- Availability of quality data on customers, vendors, transactions, Web, etc.
- Consolidation and integration of data repositories into data warehouses
- The exponential increase in data processing and storage capabilities; and decrease in cost
- Movement toward conversion of information resources into nonphysical form

Source: Turban et al. (2011), Decision Support and Business Intelligence Systems

Definition of Data Mining



- The nontrivial process of identifying valid, novel, potentially useful, and ultimately understandable patterns in data stored in structured databases.
  - Fayyad et al., (1996)
- <u>Keywords in this definition</u>: Process, nontrivial, valid, novel, potentially useful, understandable.
- Data mining: a misnomer?
- Other names:
  – knowledge extraction, pattern analysis, knowledge discovery, information harvesting, pattern searching, data dredging,…

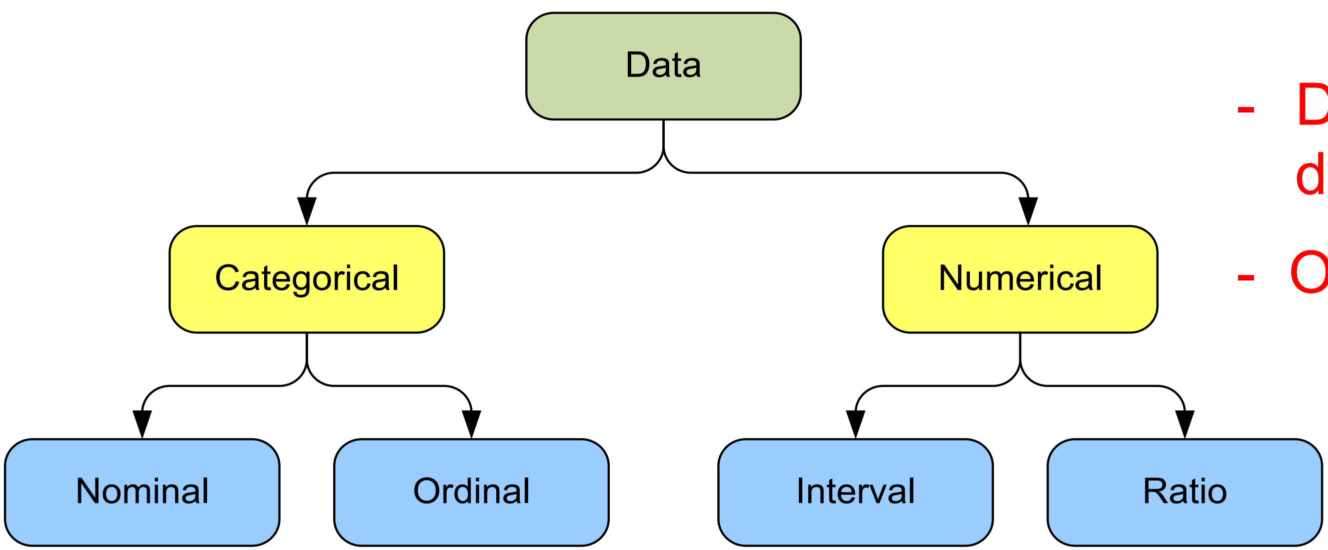
Data Mining Characteristics/ Objectives

- Source of data for DM is often a consolidated data warehouse (not always!)
- DM environment is usually a client-server or a Web-based information systems architecture
- Data is the most critical ingredient for DM which may include soft/unstructured data
- The miner is often an end user
- Striking it rich requires creative thinking
- Data mining tools' capabilities and ease of use are essential (Web, Parallel processing, etc.)

Source: Turban et al. (2011), Decision Support and Business Intelligence Systems

Data in Data Mining

- Data: a collection of facts usually obtained as the result of experiences, observations, or experiments
- Data may consist of numbers, words, images, …
- Data: lowest level of abstraction (from which information and knowledge are derived)



- DM with different data types?
- Other data types?

Source: Turban et al. (2011), Decision Support and Business Intelligence Systems

What Does DM Do?

- DM extract patterns from data
  - Pattern?
    A mathematical (numeric and/or symbolic) relationship among data items

- Types of patterns
  - Association
  - Prediction
  - Cluster (segmentation)
  - Sequential (or time series) relationships

Source: Turban et al. (2011), Decision Support and Business Intelligence Systems

Data Mining Applications

- Customer Relationship Management
  - Maximize return on marketing campaigns
  - Improve customer retention (churn analysis)
  - Maximize customer value (cross-, up-selling)
  - Identify and treat most valued customers
- Banking and Other Financial
  - Automate the loan application process
  - Detecting fraudulent transactions
  - Optimizing cash reserves with forecasting

Source: Turban et al. (2011), Decision Support and Business Intelligence Systems

Data Mining Applications (cont.)

- Retailing and Logistics
  - Optimize inventory levels at different locations
  - Improve the store layout and sales promotions
  - Optimize logistics by predicting seasonal effects
  - Minimize losses due to limited shelf life
- Manufacturing and Maintenance
  - Predict/prevent machinery failures
  - Identify anomalies in production systems to optimize the use manufacturing capacity
  - Discover novel patterns to improve product quality

Source: Turban et al. (2011), Decision Support and Business Intelligence Systems

Data Mining Applications (cont.)

- Brokerage and Securities Trading
  - Predict changes on certain bond prices
  - Forecast the direction of stock fluctuations
  - Assess the effect of events on market movements
  - Identify and prevent fraudulent activities in trading
- Insurance
  - Forecast claim costs for better business planning
  - Determine optimal rate plans
  - Optimize marketing to specific customers
  - Identify and prevent fraudulent claim activities

Source: Turban et al. (2011), Decision Support and Business Intelligence Systems

Data Mining Applications (cont.)

- Computer hardware and software
- Science and engineering
- Government and defense
- Homeland security and law enforcement
- Travel industry
- Healthcare
- Medicine
- Entertainment industry
- Sports
- Etc.

Highly popular application areas for data mining

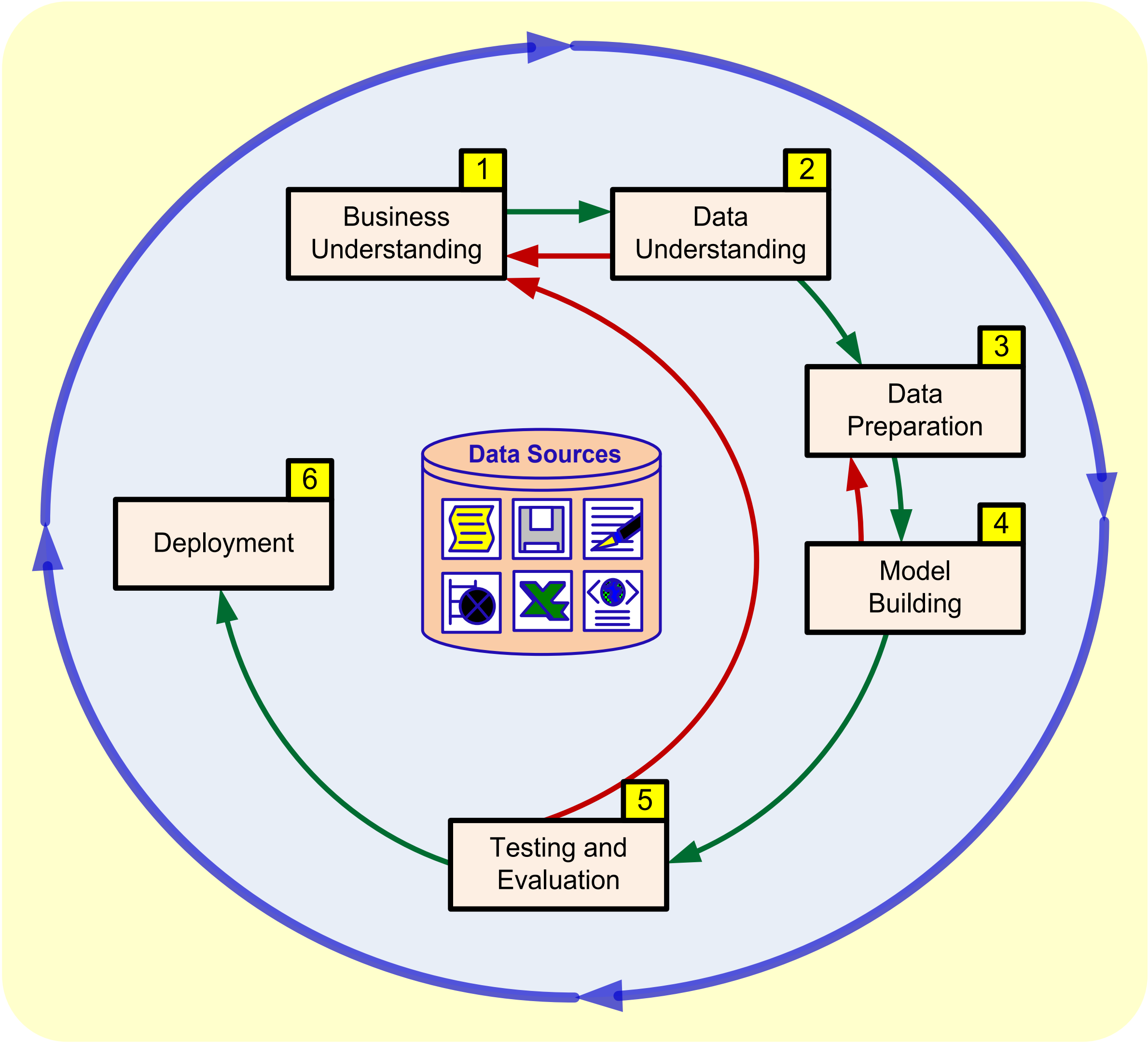
Source: Turban et al. (2011), Decision Support and Business Intelligence Systems

Data Mining Process

- A manifestation of best practices
- A systematic way to conduct DM projects
- Different groups has different versions
- Most common standard processes:
  - CRISP-DM
    (Cross-Industry Standard Process for Data Mining)
  - SEMMA
    (Sample, Explore, Modify, Model, and Assess)
  - KDD
    (Knowledge Discovery in Databases)

Source: Turban et al. (2011), Decision Support and Business Intelligence Systems

Data Mining Process: CRISP-DM



Source: Turban et al. (2011), Decision Support and Business Intelligence Systems

Data Mining Process: CRISP-DM

Step 1: Business Understanding

Step 2: Data Understanding

Step 3: Data Preparation (!)

Accounts for ~85% of total project time

Step 4: Model Building

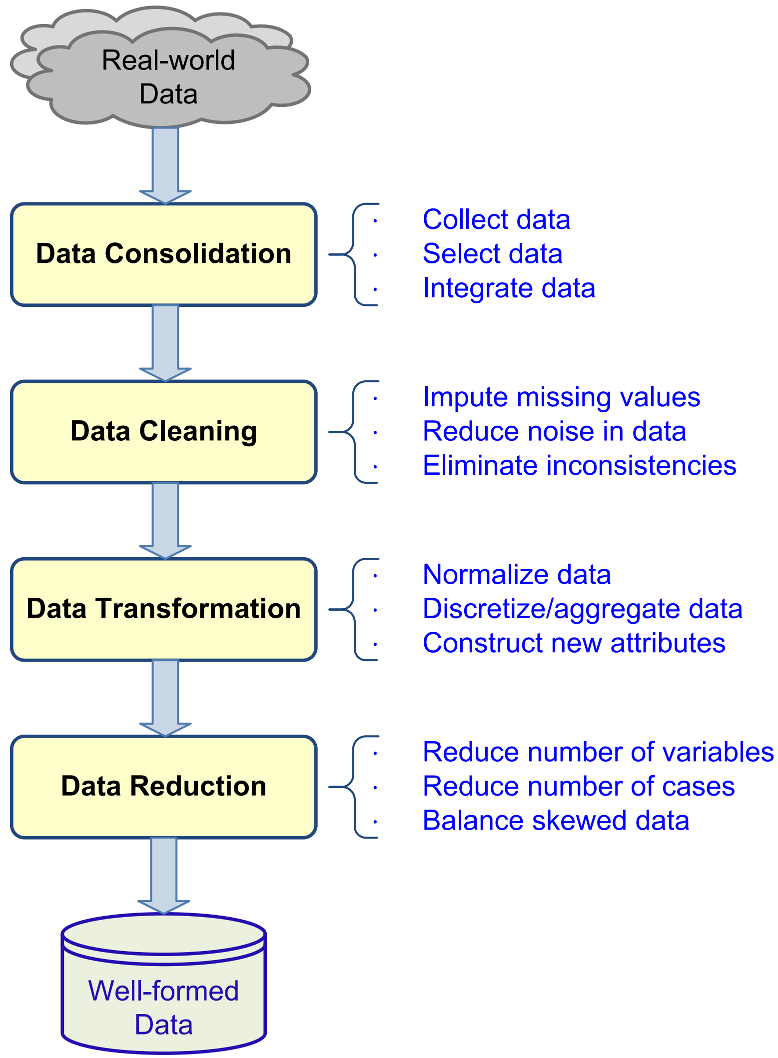
Step 5: Testing and Evaluation

Step 6: Deployment

- The process is highly repetitive and experimental (DM: art versus science?)

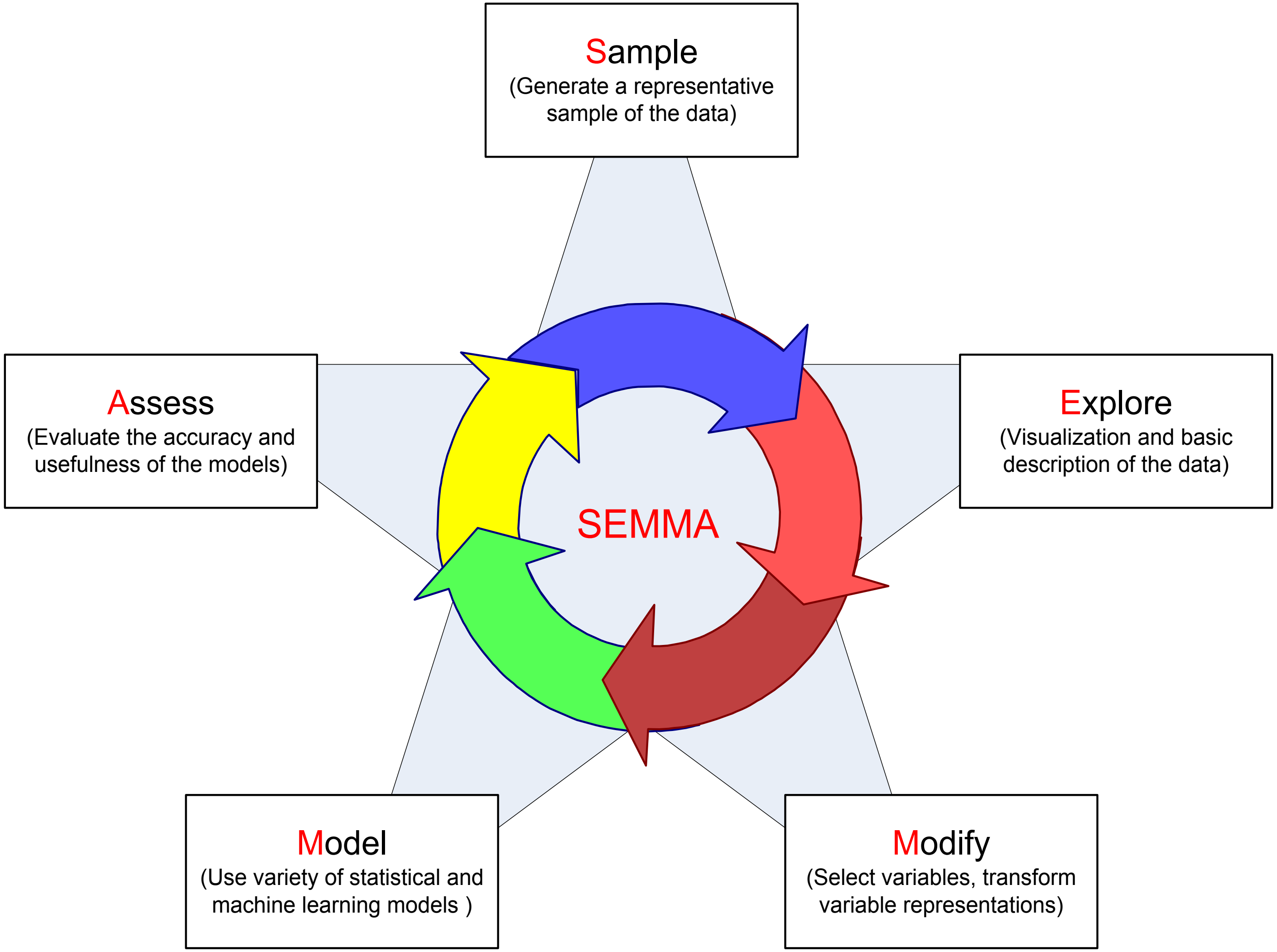
Source: Turban et al. (2011), Decision Support and Business Intelligence Systems

Data Preparation – A Critical DM Task



Source: Turban et al. (2011), Decision Support and Business Intelligence Systems

Data Mining Process: SEMMA



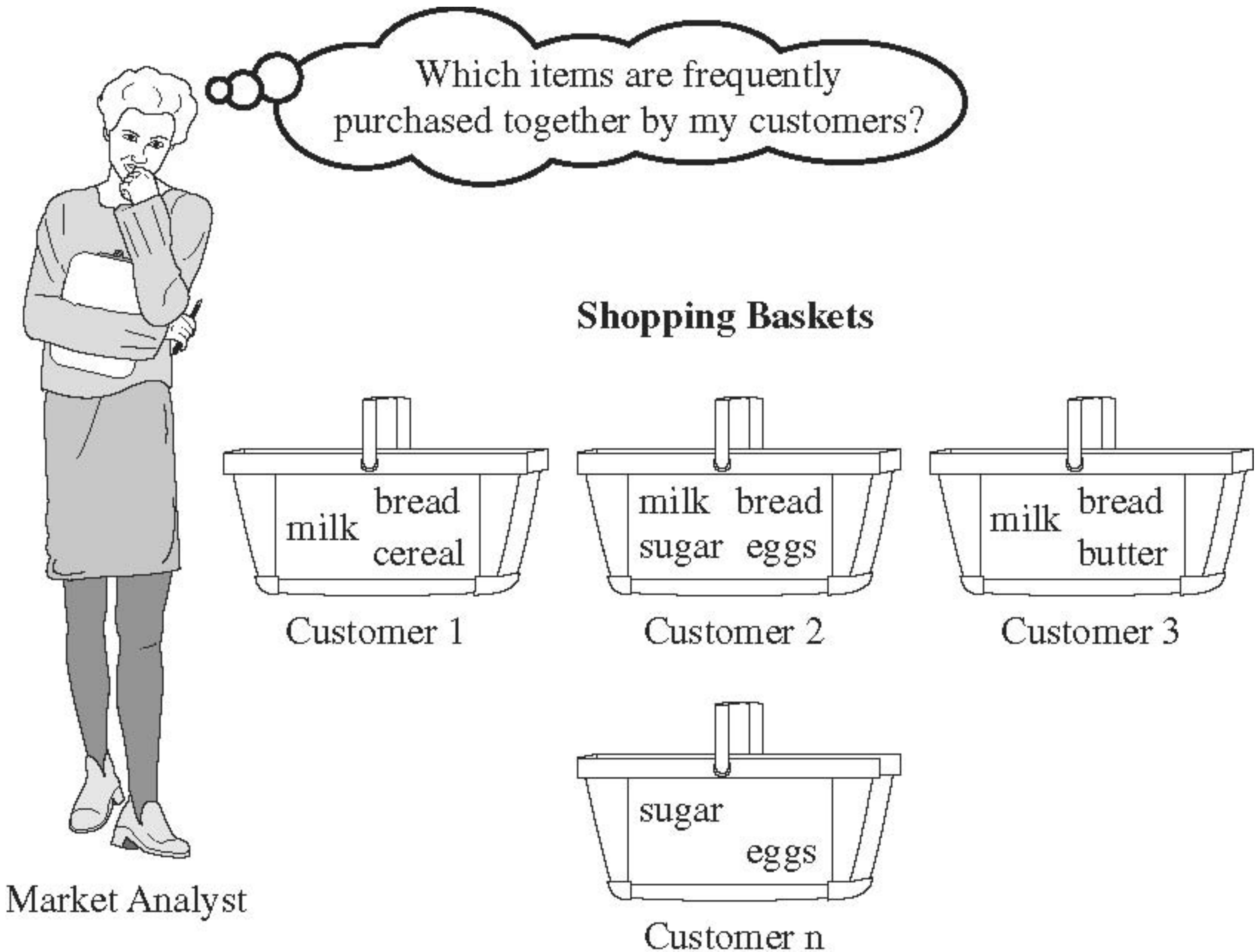
Source: Turban et al. (2011), Decision Support and Business Intelligence Systems

Association Analysis: Mining Frequent Patterns, Association and Correlations

- Association Analysis
- Mining Frequent Patterns
- Association and Correlations
- Apriori Algorithm

Source: Han & Kamber (2006)

Market Basket Analysis



Source: Han & Kamber (2006)

Association Rule Mining

- Apriori Algorithm

Raw Transaction Data

Transaction No	SKUs (Item No)
1	1, 2, 3, 4
1	2, 3, 4
1	2, 3
1	1, 2, 4
1	1, 2, 3, 4
1	2, 4

One-item Itemsets

Itemset (SKUs)	Support
1	3
2	6
3	4
4	5

Two-item Itemsets

Itemset (SKUs)	Support
1, 2	3
1, 3	2
1, 4	3
2, 3	4
2, 4	5
3, 4	3

Three-item Itemsets

Itemset (SKUs)	Support
1, 2, 4	3
2, 3, 4	3

Source: Turban et al. (2011), Decision Support and Business Intelligence Systems

Association Rule Mining

- A very popular DM method in business
- Finds interesting relationships (affinities) between variables (items or events)
- Part of machine learning family
- Employs unsupervised learning
- There is no output variable
- Also known as market basket analysis
- Often used as an example to describe DM to ordinary people, such as the famous "relationship between diapers and beers!"

Source: Turban et al. (2011), Decision Support and Business Intelligence Systems

Association Rule Mining

- Input: the simple point-of-sale transaction data
- Output: Most frequent affinities among items
- Example: according to the transaction data…

  “Customer who bought a laptop computer and a virus protection software, also bought extended service plan 70 percent of the time."
- How do you use such a pattern/knowledge?
  - Put the items next to each other for ease of finding
  - Promote the items as a package (do not put one on sale if the other(s) are on sale)
  - Place items far apart from each other so that the customer has to walk the aisles to search for it, and by doing so potentially seeing and buying other items

Source: Turban et al. (2011), Decision Support and Business Intelligence Systems

Association Rule Mining

- A representative applications of association rule mining include
  - In business: cross-marketing, cross-selling, store design, catalog design, e-commerce site design, optimization of online advertising, product pricing, and sales/promotion configuration
  - In medicine: relationships between symptoms and illnesses; diagnosis and patient characteristics and treatments (to be used in medical DSS); and genes and their functions (to be used in genomics projects)…

Source: Turban et al. (2011), Decision Support and Business Intelligence Systems

Association Rule Mining

- Are all association rules interesting and useful?

A Generic Rule: **X ⇒ Y [S%, C%]**

X, Y: products and/or services

X: Left-hand-side (LHS)

Y: Right-hand-side (RHS)

S: Support: how often **X** and **Y** go together

C: Confidence: how often **Y** go together with the **X**

Example: {Laptop Computer, Antivirus Software} ⇒ {Extended Service Plan} [30%, 70%]

Source: Turban et al. (2011), Decision Support and Business Intelligence Systems

Association Rule Mining

- Algorithms are available for generating association rules
  - Apriori
  - Eclat
  - FP-Growth
  - + Derivatives and hybrids of the three
- The algorithms help identify the frequent item sets, which are, then converted to association rules

Source: Turban et al. (2011), Decision Support and Business Intelligence Systems

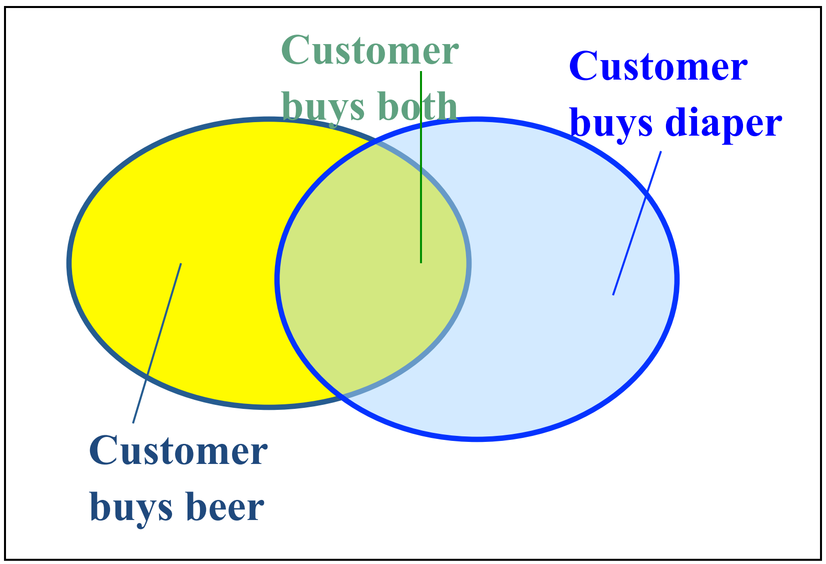
Association Rule Mining

- Apriori Algorithm
  - Finds subsets that are common to at least a minimum number of the itemsets
  - uses a bottom-up approach
    - frequent subsets are extended one item at a time (the size of frequent subsets increases from one-item subsets to two-item subsets, then three-item subsets, and so on), and
    - groups of candidates at each level are tested against the data for minimum

Source: Turban et al. (2011), Decision Support and Business Intelligence Systems

Basic Concepts: Frequent Patterns and Association Rules

Transaction-id	Items bought
10	A, B, D
20	A, C, D
30	A, D, E
40	B, E, F
50	B, C, D, E, F



- Itemset X = $\{x_1, \ldots, x_k\}$
- Find all the rules $X \rightarrow Y$ with minimum support and confidence
  - support, *s*, probability that a transaction contains $X \cup Y$
  - confidence, *c*, conditional probability that a transaction having X also contains *Y*

Let sup_{min} *= 50%,* $conf_{min}$ *= 50%*

Freq. Pat.: {A:3, B:3, D:4, E:3, AD:3}

Association rules:

A → *D* (60%, 100%)

D → *A* (60%, 75%)

A → *D* (support = 3/5 = 60%, confidence = 3/3 =100%)

D → *A* (support = 3/5 = 60%, confidence = 3/4 = 75%)

Source: Han & Kamber (2006)

Market basket analysis

- Example
  - Which groups or sets of items are customers likely to purchase on a given trip to the store?
- Association Rule
  - *Computer → antivirus_software [support = 2%; confidence = 60%]*
    - A support of 2% means that 2% of all the transactions under analysis show that computer and antivirus software are purchased together.
    - A confidence of 60% means that 60% of the customers who purchased a computer also bought the software.

Source: Han & Kamber (2006)

Association rules

- Association rules are considered interesting if they satisfy both
  - a minimum support threshold and
  - a minimum confidence threshold.

Frequent Itemsets, Closed Itemsets, and Association Rules

Let $I = \{I_1, I_2, \ldots, I_m\}$ be a set of items. Let D, the task-relevant data, be a set of database transactions where each transaction T is a set of items such that $T \subseteq I$. Each transaction is associated with an identifier, called TID. Let A be a set of items. A transaction T is said to contain A if and only if $A \subseteq T$. An association rule is an implication of the form $A \Rightarrow B$, where $A \subset I$, $B \subset I$, and $A \cap B = \phi$. The rule $A \Rightarrow B$ holds in the transaction set D with **support** s, where s is the percentage of transactions in D that contain $A \cup B$ (i.e., the *union* of sets A and B, or say, both A and B). This is taken to be the probability, $P(A \cup B)$.[1] The rule $A \Rightarrow B$ has **confidence** c in the transaction set D, where c is the percentage of transactions in D containing A that also contain B. This is taken to be the conditional probability, $P(B|A)$. That is,

Support (A→ B) = P(A ∪ B)

Confidence (A→ B) = P(B|A)

Source: Han & Kamber (2006)

Support $(A \rightarrow B) = P(A \cup B)$
Confidence $(A \rightarrow B) = P(B|A)$

- The notation *$P(A \cup B)$ indicates the probability that a transaction contains the union of set A and set B*
  - *(i.e., it contains every item in A and in B).*
- *This should not be confused with P(A or B),* which indicates the probability that a transaction contains either *A or B.*

Source: Han & Kamber (2006)

Does diaper purchase predict beer purchase?

- Contingency tables

	Beer: Yes	Beer: No	
No diapers	6	94	100
diapers	40	60	100

DEPENDENT (yes)

	Beer: Yes	Beer: No
No diapers	23	77
diapers	23	77

INDEPENDENT (no predictability)

Source: Dickey (2012) http://www4.stat.ncsu.edu/~dickey/SAScode/Encore_2012.ppt

Support $(A \rightarrow B) = P(A \cup B)$

Confidence $(A \rightarrow B) = P(B|A)$

Conf $(A \rightarrow B) = $ Supp $(A \cup B)/$ Supp (A)

Lift $(A \rightarrow B) = $ Supp $(A \cup B)$ / (Supp (A) x Supp (B))

Lift (Correlation)

Lift $(A \rightarrow B) = $ Confidence $(A \rightarrow B)$ / Support(B)

Source: Dickey (2012) http://www4.stat.ncsu.edu/~dickey/SAScode/Encore_2012.ppt

Lift

Lift = Confidence / Expected Confidence if Independent

Checking Saving	No (1500)	Yes (8500)	 (10000)
No	500	3500	4000
Yes	1000	5000	6000

SVG=>CHKG Expect 8500/10000 = 85% if independent
Observed Confidence is 5000/6000 = 83%
Lift = 83/85 < 1.
Savings account holders actually LESS likely than others to have checking account !!!

Source: Dickey (2012) http://www4.stat.ncsu.edu/~dickey/SAScode/Encore_2012.ppt

- Rules that satisfy both a minimum support threshold (*min_sup*) and a minimum confidence threshold (*min_conf*) are called **strong**.
- By convention, we write support and confidence values so as to occur between 0% and 100%, rather than 0 to 1.0.

- itemset
  - A set of items is referred to as an itemset.
- K-itemset
  - An itemset that contains *k items is a k*-itemset*.*
- Example:
  - The set {*computer, antivirus software*} is a 2-itemset.

Absolute Support and Relative Support

- Absolute Support
  - The occurrence frequency of an itemset is the number of transactions that contain the itemset
    - frequency, support count, or count of the itemset
  - Ex: 3
- Relative support
  - Ex: 60%

Source: Han & Kamber (2006)

- If the relative support of an itemset *I satisfies a prespecified minimum support threshold, then I is a* frequent itemset.
  - *i.e., the absolute* support of *I satisfies the corresponding minimum support count threshold*
- The set of frequent *k-itemsets is commonly denoted by* L_K

$$confidence(A \Rightarrow B) = P(B|A) = \frac{support(A \cup B)}{support(A)} = \frac{support\_count(A \cup B)}{support\_count(A)}$$

- the confidence of rule *A→ B can be easily derived from the* support counts of *A and A ∪ B.*
- once the support counts of *A, B, and A ∪ B are* found, it is straightforward to derive the corresponding association rules *A→B and B→A* and check whether they are strong.
- Thus the problem of mining association rules can be reduced to that of mining frequent itemsets.

Source: Han & Kamber (2006)

Association rule mining: Two-step process

1. Find all frequent itemsets
   – By definition, each of these itemsets will occur at least as frequently as a predetermined minimum support count, *min_sup.*
2. Generate strong association rules from the frequent itemsets
   – By definition, these rules must satisfy minimum support and minimum confidence.

Source: Han & Kamber (2006)

Efficient and Scalable Frequent Itemset Mining Methods

- The Apriori Algorithm
  - Finding Frequent Itemsets Using Candidate Generation

Apriori Algorithm

- **Apriori** is a seminal algorithm proposed by R. Agrawal and R. Srikant in 1994 for mining frequent itemsets for Boolean association rules.
- The name of the algorithm is based on the fact that the algorithm uses *prior knowledge of frequent itemset properties, as we shall* see following.

Source: Han & Kamber (2006)

Apriori Algorithm

- Apriori employs an iterative approach known as a *level-wise search, where k-itemsets are used to explore (k+1)-itemsets.*
- *First, the set of frequent 1-itemsets is found* by scanning the database to accumulate the count for each item, and collecting those items that satisfy minimum support. The resulting set is denoted L_1.
- *Next,* L_1 *is used to find* L_2, the set of frequent 2-itemsets, which is used to find L_3, *and so on, until no more frequent k-itemsets can be found.*
- *The finding of each* L_k *requires one full scan of the database.*

Source: Han & Kamber (2006)

Apriori Algorithm

- To improve the efficiency of the level-wise generation of frequent itemsets, an important property called the Apriori property.
- Apriori property
  - *All nonempty subsets of a frequent itemset must also be frequent.*

Source: Han & Kamber (2006)

- *How is the Apriori property used in the algorithm?*
  - How L_{k-1} *is used to find* L_k *for* $k >= 2$.
  - A two-step process is followed, consisting of join and prune actions.

Apriori property used in algorithm 1. The join step

1. **The join step**: To find L_k, a set of **candidate** k-itemsets is generated by joining L_{k-1} with itself. This set of candidates is denoted C_k. Let l_1 and l_2 be itemsets in L_{k-1}. The notation $l_i[j]$ refers to the jth item in l_i (e.g., $l_1[k-2]$ refers to the second to the last item in l_1). By convention, Apriori assumes that items within a transaction or itemset are sorted in lexicographic order. For the $(k-1)$-itemset, l_i, this means that the items are sorted such that $l_i[1] < l_i[2] < \ldots < l_i[k-1]$. The join, $L_{k-1} \bowtie L_{k-1}$, is performed, where members of L_{k-1} are joinable if their first $(k-2)$ items are in common. That is, members l_1 and l_2 of L_{k-1} are joined if $(l_1[1] = l_2[1]) \wedge (l_1[2] = l_2[2]) \wedge \ldots \wedge (l_1[k-2] = l_2[k-2]) \wedge (l_1[k-1] < l_2[k-1])$. The condition $l_1[k-1] < l_2[k-1]$ simply ensures that no duplicates are generated. The resulting itemset formed by joining l_1 and l_2 is $l_1[1], l_1[2], \ldots, l_1[k-2], l_1[k-1], l_2[k-1]$.

Source: Han & Kamber (2006)

Apriori property used in algorithm 2. The prune step

2. The prune step: C_k is a superset of L_k, that is, its members may or may not be frequent, but all of the frequent k-itemsets are included in C_k. A scan of the database to determine the count of each candidate in C_k would result in the determination of L_k (i.e., all candidates having a count no less than the minimum support count are frequent by definition, and therefore belong to L_k). C_k, however, can be huge, and so this could involve heavy computation. To reduce the size of C_k, the Apriori property is used as follows. Any $(k-1)$-itemset that is not frequent cannot be a subset of a frequent k-itemset. Hence, if any $(k-1)$-subset of a candidate k-itemset is not in L_{k-1}, then the candidate cannot be frequent either and so can be removed from C_k. This **subset testing** can be done quickly by maintaining a hash tree of all frequent itemsets.

Source: Han & Kamber (2006)

Transactional data for an *AllElectronics* branch

TID	*List of item_IDs*
T100	I1, I2, I5
T200	I2, I4
T300	I2, I3
T400	I1, I2, I4
T500	I1, I3
T600	I2, I3
T700	I1, I3
T800	I1, I2, I3, I5
T900	I1, I2, I3

Source: Han & Kamber (2006)

Example: Apriori

- Let's look at a concrete example, based on the *AllElectronics transaction database, D.*
- *There are nine transactions in this database, that is, |D| = 9.*
- Apriori algorithm for finding frequent itemsets in *D*

TID	*List of item_IDs*
T100	I1, I2, I5
T200	I2, I4
T300	I2, I3
T400	I1, I2, I4
T500	I1, I3
T600	I2, I3
T700	I1, I3
T800	I1, I2, I3, I5
T900	I1, I2, I3

Source: Han & Kamber (2006)

Example: Apriori Algorithm

Generation of candidate itemsets and frequent itemsets, where the minimum support count is 2.

TID	*List of item_IDs*
T100	I1, I2, I5
T200	I2, I4
T300	I2, I3
T400	I1, I2, I4
T500	I1, I3
T600	I2, I3
T700	I1, I3
T800	I1, I2, I3, I5
T900	I1, I2, I3

Scan D for count of each candidate →

C_1

Itemset	Sup. count
{I1}	6
{I2}	7
{I3}	6
{I4}	2
{I5}	2

Compare candidate support count with minimum support count →

L_1

Itemset	Sup. count
{I1}	6
{I2}	7
{I3}	6
{I4}	2
{I5}	2

Generate C_2 candidates from L_1 →

C_2

Itemset
{I1, I2}
{I1, I3}
{I1, I4}
{I1, I5}
{I2, I3}
{I2, I4}
{I2, I5}
{I3, I4}
{I3, I5}
{I4, I5}

Scan D for count of each candidate →

C_2

Itemset	Sup. count
{I1, I2}	4
{I1, I3}	4
{I1, I4}	1
{I1, I5}	2
{I2, I3}	4
{I2, I4}	2
{I2, I5}	2
{I3, I4}	0
{I3, I5}	1
{I4, I5}	0

Compare candidate support count with minimum support count →

L_2

Itemset	Sup. count
{I1, I2}	4
{I1, I3}	4
{I1, I5}	2
{I2, I3}	4
{I2, I4}	2
{I2, I5}	2

Generate C_3 candidates from L_2 →

C_3

Itemset
{I1, I2, I3}
{I1, I2, I5}

Scan D for count of each candidate →

C_3

Itemset	Sup. count
{I1, I2, I3}	2
{I1, I2, I5}	2

Compare candidate support count with minimum support count →

L_3

Itemset	Sup. count
{I1, I2, I3}	2
{I1, I2, I5}	2

Source: Han & Kamber (2006)

Example: Apriori Algorithm
$C_1 \rightarrow L_1$

TID	*List of item_IDs*
T100	I1, I2, I5
T200	I2, I4
T300	I2, I3
T400	I1, I2, I4
T500	I1, I3
T600	I2, I3
T700	I1, I3
T800	I1, I2, I3, I5
T900	I1, I2, I3

Scan *D* for count of each candidate

C_1

Itemset	Sup. count
{I1}	6
{I2}	7
{I3}	6
{I4}	2
{I5}	2

Compare candidate support count with minimum support count

L_1

Itemset	Sup. count
{I1}	6
{I2}	7
{I3}	6
{I4}	2
{I5}	2

Source: Han & Kamber (2006)

TID	List of item_IDs
T100	I1, I2, I5
T200	I2, I4
T300	I2, I3
T400	I1, I2, I4
T500	I1, I3
T600	I2, I3
T700	I1, I3
T800	I1, I2, I3, I5
T900	I1, I2, I3

Scan D for count of each candidate →

C_1

Itemset	Sup. count
{I1}	6
{I2}	7
{I3}	6
{I4}	2
{I5}	2

Compare candidate support count with minimum support count →

L_1

Itemset	Sup. count
{I1}	6
{I2}	7
{I3}	6
{I4}	2
{I5}	2

Example: Apriori Algorithm $C_2 \rightarrow L_2$

Generate C_2 candidates from L_1 →

C_2

Itemset
{I1, I2}
{I1, I3}
{I1, I4}
{I1, I5}
{I2, I3}
{I2, I4}
{I2, I5}
{I3, I4}
{I3, I5}
{I4, I5}

Scan D for count of each candidate →

C_2

Itemset	Sup. count
{I1, I2}	4
{I1, I3}	4
{I1, I4}	1
{I1, I5}	2
{I2, I3}	4
{I2, I4}	2
{I2, I5}	2
{I3, I4}	0
{I3, I5}	1
{I4, I5}	0

Compare candidate support count with minimum support count →

L_2

Itemset	Sup. count
{I1, I2}	4
{I1, I3}	4
{I1, I5}	2
{I2, I3}	4
{I2, I4}	2
{I2, I5}	2

Source: Han & Kamber (2006)

Example: Apriori Algorithm $C_3 \rightarrow L_3$

TID	*List of item_IDs*
T100	I1, I2, I5
T200	I2, I4
T300	I2, I3
T400	I1, I2, I4
T500	I1, I3
T600	I2, I3
T700	I1, I3
T800	I1, I2, I3, I5
T900	I1, I2, I3

Scan D for count of each candidate →

C_1

Itemset	Sup. count
{I1}	6
{I2}	7
{I3}	6
{I4}	2
{I5}	2

Compare candidate support count with minimum support count →

L_1

Itemset	Sup. count
{I1}	6
{I2}	7
{I3}	6
{I4}	2
{I5}	2

Generate C_2 candidates from L_1 →

C_2

Itemset
{I1, I2}
{I1, I3}
{I1, I4}
{I1, I5}
{I2, I3}
{I2, I4}
{I2, I5}
{I3, I4}
{I3, I5}
{I4, I5}

Scan D for count of each candidate →

C_2

Itemset	Sup. count
{I1, I2}	4
{I1, I3}	4
{I1, I4}	1
{I1, I5}	2
{I2, I3}	4
{I2, I4}	2
{I2, I5}	2
{I3, I4}	0
{I3, I5}	1
{I4, I5}	0

Compare candidate support count with minimum support count →

L_2

Itemset	Sup. count
{I1, I2}	4
{I1, I3}	4
{I1, I5}	2
{I2, I3}	4
{I2, I4}	2
{I2, I5}	2

Generate C_3 candidates from L_2 →

C_3

Itemset
{I1, I2, I3}
{I1, I2, I5}

Scan D for count of each candidate →

C_3

Itemset	Sup. count
{I1, I2, I3}	2
{I1, I2, I5}	2

Compare candidate support count with minimum support count →

L_3

Itemset	Sup. count
{I1, I2, I3}	2
{I1, I2, I5}	2

Source: Han & Kamber (2006)

The Apriori algorithm for discovering frequent itemsets for mining Boolean association rules.

Algorithm: Apriori. Find frequent itemsets using an iterative level-wise approach based on candidate generation.

Input:

- D, a database of transactions;
- *min_sup*, the minimum support count threshold.

Output: L, frequent itemsets in D.

Method:

L_1 = find_frequent_1-itemsets(D);
for ($k = 2; L_{k-1} \neq \phi; k{+}{+}$) {
    C_k = apriori_gen(L_{k-1});
    for each transaction $t \in D$ { // scan D for counts
        C_t = subset(C_k, t); // get the subsets of t that are candidates
        for each candidate $c \in C_t$
            c.count++;
    }
    $L_k = \{c \in C_k | c.count \geq min\_sup\}$
}
return $L = \cup_k L_k$;

procedure apriori_gen(L_{k-1}:frequent $(k-1)$-itemsets)
for each itemset $l_1 \in L_{k-1}$
    for each itemset $l_2 \in L_{k-1}$
        if $(l_1[1] = l_2[1]) \wedge (l_1[2] = l_2[2]) \wedge ... \wedge (l_1[k-2] = l_2[k-2]) \wedge (l_1[k-1] < l_2[k-1])$ **then** {
            $c = l_1 \bowtie l_2$; // join step: generate candidates
            if has_infrequent_subset(c, L_{k-1}) **then**
                delete c; // prune step: remove unfruitful candidate
            else add c **to** C_k;
        }
return C_k;

procedure has_infrequent_subset(c: candidate k-itemset;
    L_{k-1}: frequent $(k-1)$-itemsets); // use prior knowledge
for each $(k-1)$-subset s **of** c
    if $s \notin L_{k-1}$ **then**
        return TRUE;
return FALSE;

Source: Han & Kamber (2006)

The Apriori Algorithm—An Example

$Sup_{min} = 2$

Database TDB

Tid	Items
10	A, C, D
20	B, C, E
30	A, B, C, E
40	B, E

1[st] scan

C_1

Itemset	sup
{A}	2
{B}	3
{C}	3
{D}	1
{E}	3

L_1

Itemset	sup
{A}	2
{B}	3
{C}	3
{E}	3

C_2

Itemset
{A, B}
{A, C}
{A, E}
{B, C}
{B, E}
{C, E}

2[nd] scan

C_2

Itemset	sup
{A, B}	1
{A, C}	2
{A, E}	1
{B, C}	2
{B, E}	3
{C, E}	2

L_2

Itemset	sup
{A, C}	2
{B, C}	2
{B, E}	3
{C, E}	2

C_3

Itemset
{B, C, E}

3[rd] scan

L_3

Itemset	sup
{B, C, E}	2

Source: Han & Kamber (2006)

The Apriori Algorithm

- Pseudo-code:

  C_k: Candidate itemset of size k
  L_k : frequent itemset of size k

  L_1 = {frequent items};
  for (k = 1; L_k !=∅; k++) **do begin**
  C_{k+1} = candidates generated from L_k;
  for each transaction t in database do
  increment the count of all candidates in C_{k+1}
  that are contained in t
  L_{k+1} = candidates in C_{k+1} with min_support
  end
  return $\cup_k L_k$;

Generating Association Rules from Frequent Itemsets

$$confidence(A \Rightarrow B) = P(B|A) = \frac{support\_count(A \cup B)}{support\_count(A)}$$

- For each frequent itemset l, generate all nonempty subsets of l.
- For every nonempty subset s of l, output the rule "$s \Rightarrow (l - s)$" if $\frac{support\_count(l)}{support\_count(s)} \geq min\_conf$, where min_conf is the minimum confidence threshold.

Source: Han & Kamber (2006)

Example: Generating association rules

- frequent itemset *l = {I1, I2, I5}*

TID	List of item_IDs
T100	I1, I2, I5
T200	I2, I4
T300	I2, I3
T400	I1, I2, I4
T500	I1, I3
T600	I2, I3
T700	I1, I3
T800	I1, I2, I3, I5
T900	I1, I2, I3

$$
\begin{array}{ll}
I1 \wedge I2 \Rightarrow I5, & confidence = 2/4 = 50\% \\
I1 \wedge I5 \Rightarrow I2, & confidence = 2/2 = 100\% \\
I2 \wedge I5 \Rightarrow I1, & confidence = 2/2 = 100\% \\
I1 \Rightarrow I2 \wedge I5, & confidence = 2/6 = 33\% \\
I2 \Rightarrow I1 \wedge I5, & confidence = 2/7 = 29\% \\
I5 \Rightarrow I1 \wedge I2, & confidence = 2/2 = 100\%
\end{array}
$$

- If the minimum confidence threshold is, say, 70%, then only the second, third, and last rules above are output, because these are the only ones generated that are strong.

Source: Han & Kamber (2006)

Summary

- Association Analysis
- Mining Frequent Patterns
- Association and Correlations
- Apriori Algorithm

References

- Jiawei Han and Micheline Kamber, Data Mining: Concepts and Techniques, Second Edition, 2006, Elsevier
- Efraim Turban, Ramesh Sharda, Dursun Delen, Decision Support and Business Intelligence Systems, Ninth Edition, 2011, Pearson.