Web Mining
(網路探勘)

Association Rules and Sequential Patterns
(關聯規則和序列模式)

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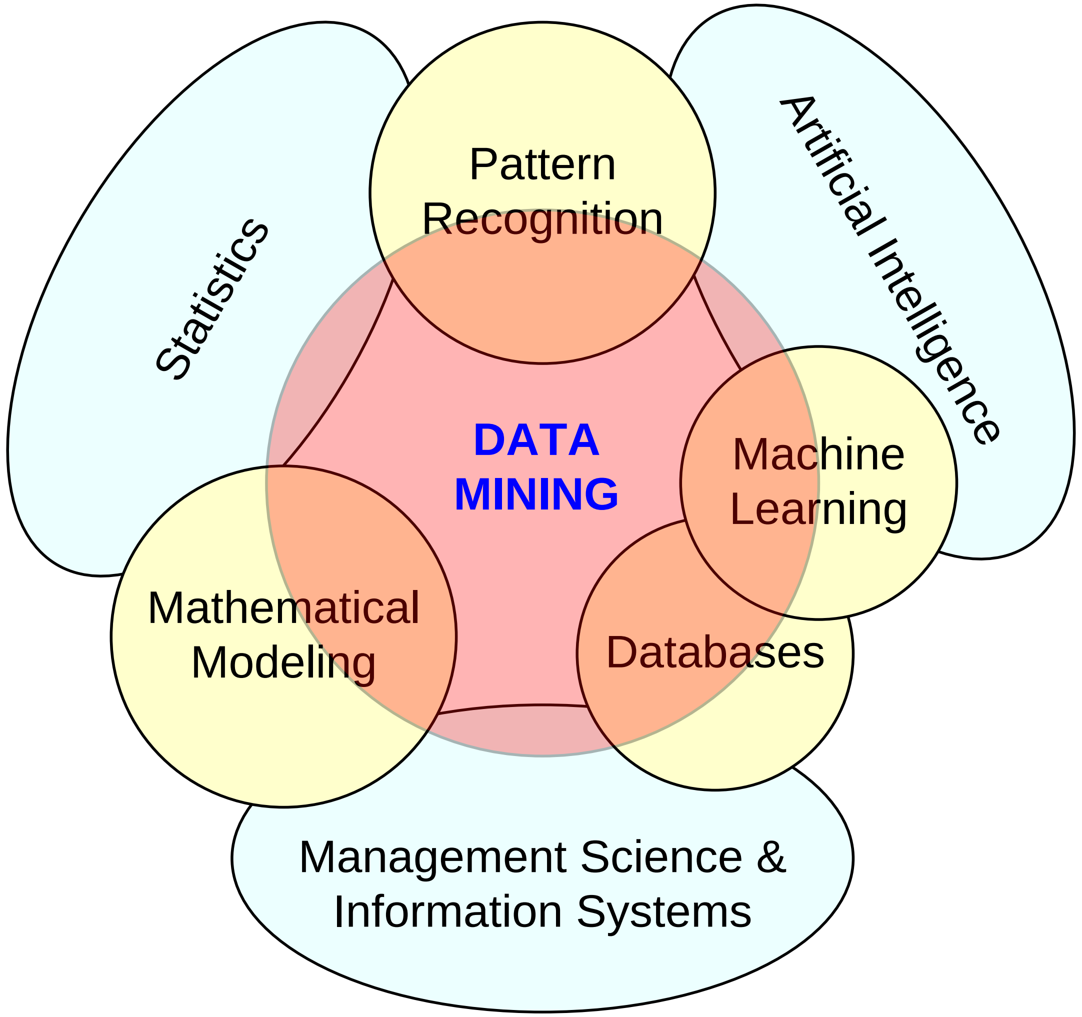
課程大綱 (Syllabus)

週次	日期	內容（Subject/Topics）
1	101/09/12	Introduction to Web Mining (網路探勘導論)
2	101/09/19	Association Rules and Sequential Patterns (關聯規則和序列模式)
3	101/09/26	Supervised Learning (監督式學習)
4	101/10/03	Unsupervised Learning (非監督式學習)
5	101/10/10	國慶紀念日(放假一天)
6	101/10/17	Paper Reading and Discussion (論文研讀與討論)
7	101/10/24	Partially Supervised Learning (部分監督式學習)
8	101/10/31	Information Retrieval and Web Search (資訊檢索與網路搜尋)
9	101/11/07	Social Network Analysis (社會網路分析)

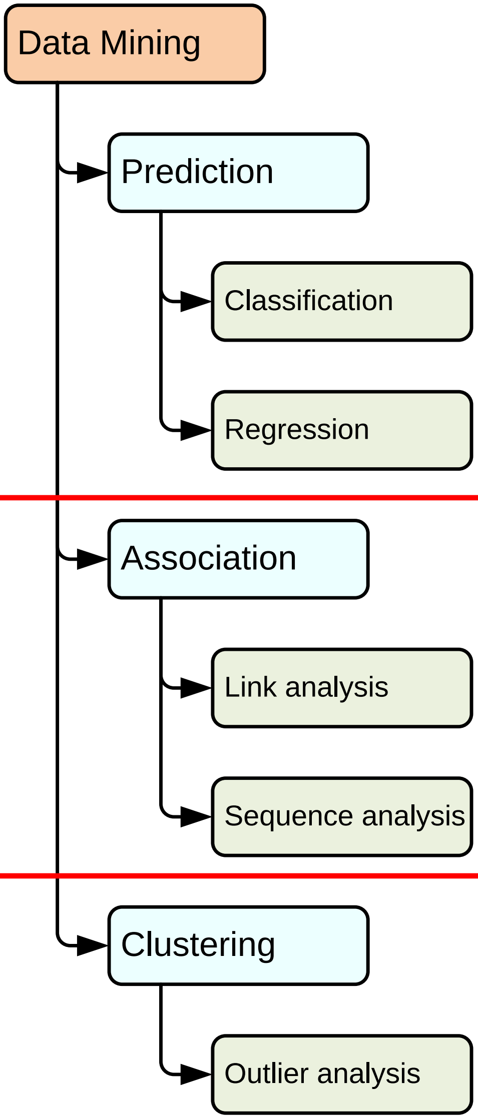
課程大綱 (Syllabus)

週次	日期	內容（Subject/Topics）
10	101/11/14	Midterm Presentation (期中報告)
11	101/11/21	Web Crawling (網路爬行)
12	101/11/28	Structured Data Extraction (結構化資料擷取)
13	101/12/05	Information Integration (資訊整合)
14	101/12/12	Opinion Mining and Sentiment Analysis (意見探勘與情感分析)
15	101/12/19	Paper Reading and Discussion (論文研讀與討論)
16	101/12/26	Web Usage Mining (網路使用挖掘)
17	102/01/02	Project Presentation 1 (期末報告1)
18	102/01/09	Project Presentation 2 (期末報告2)

Data Mining at the Intersection of Many Disciplines



A Taxonomy for Data Mining Tasks

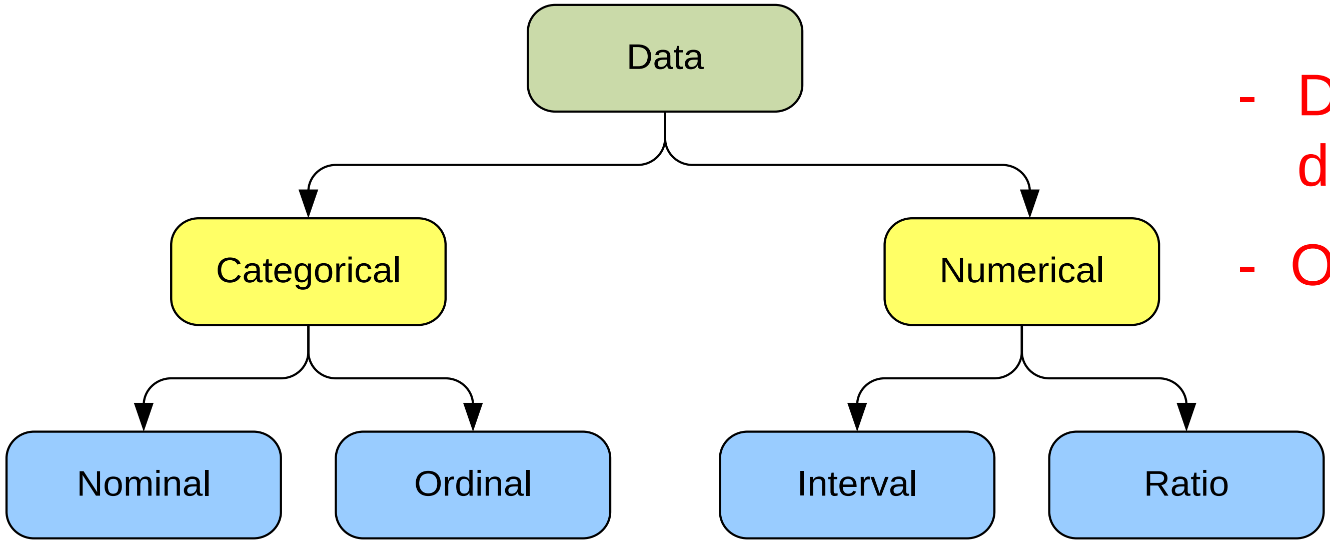


Data Mining	Learning Method	Popular Algorithms
Prediction	Supervised	Classification and Regression Trees, ANN, SVM, Genetic Algorithms
Classification	Supervised	Decision trees, ANN/MLP, SVM, Rough sets, Genetic Algorithms
Regression	Supervised	Linear/Nonlinear Regression, Regression trees, ANN/MLP, SVM
Association	Unsupervised	Apriory, OneR, ZeroR, Eclat
Link analysis	Unsupervised	Expectation Maximization, Apriory Algorithm, Graph-based Matching
Sequence analysis	Unsupervised	Apriory Algorithm, FP-Growth technique
Clustering	Unsupervised	K-means, ANN/SOM
Outlier analysis	Unsupervised	K-means, Expectation Maximization (EM)

Source: Turban et al. (2011), Decision Support and Business Intelligence Systems

Data in Data Mining

- Data: a collection of facts usually obtained as the result of experiences, observations, or experiments
- Data may consist of numbers, words, images, …
- Data: lowest level of abstraction (from which information and knowledge are derived)



- DM with different data types?
- Other data types?

What Does DM Do?

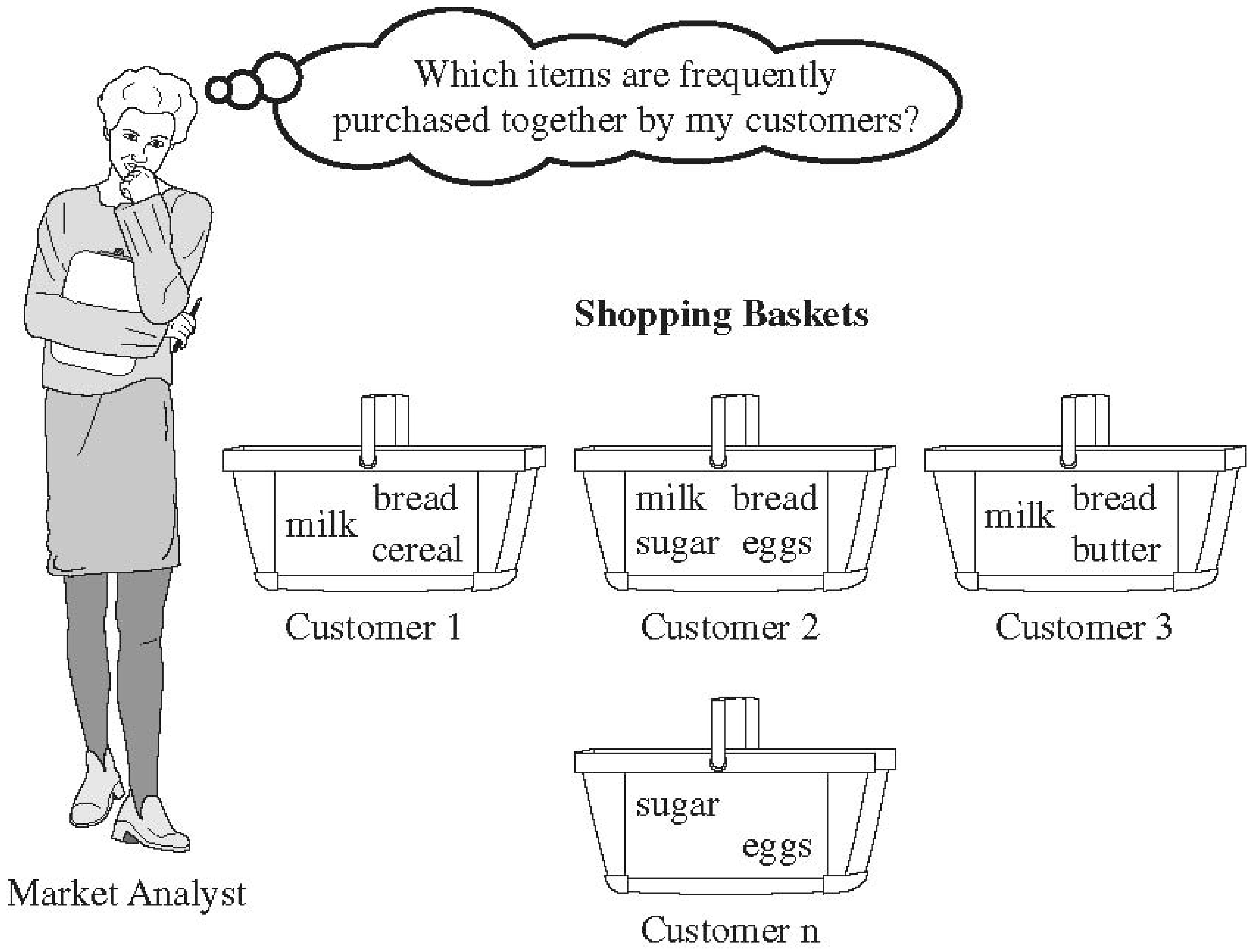
- DM extract patterns from data
  - Pattern?
    A mathematical (numeric and/or symbolic) relationship among data items

- Types of patterns
  - Association
  - Prediction
  - Cluster (segmentation)
  - Sequential (or time series) relationships

Road map

- Basic concepts of Association Rules
- Apriori algorithm
- Different data formats for mining
- Mining with multiple minimum supports
- Mining class association rules
- Sequential pattern mining
- Summary

Market Basket Analysis



Source: Han & Kamber (2006)

Association Rule Mining

- Apriori Algorithm

Raw Transaction Data

Transaction No	SKUs (Item No)
1	1, 2, 3, 4
1	2, 3, 4
1	2, 3
1	1, 2, 4
1	1, 2, 3, 4
1	2, 4

One-item Itemsets

Itemset (SKUs)	Support
1	3
2	6
3	4
4	5

Two-item Itemsets

Itemset (SKUs)	Support
1, 2	3
1, 3	2
1, 4	3
2, 3	4
2, 4	5
3, 4	3

Three-item Itemsets

Itemset (SKUs)	Support
1, 2, 4	3
2, 3, 4	3

Association Rule Mining

- A very popular DM method in business
- Finds interesting relationships (affinities) between variables (items or events)
- Part of machine learning family
- Employs unsupervised learning
- There is no output variable
- Also known as market basket analysis
- Often used as an example to describe DM to ordinary people, such as the famous "relationship between diapers and beers!"

Association Rule Mining

- Input: the simple point-of-sale transaction data
- Output: Most frequent affinities among items
- Example: according to the transaction data…

  "Customer who bought a laptop computer and a virus protection software, also bought extended service plan 70 percent of the time."
- How do you use such a pattern/knowledge?
  - Put the items next to each other for ease of finding
  - Promote the items as a package (do not put one on sale if the other(s) are on sale)
  - Place items far apart from each other so that the customer has to walk the aisles to search for it, and by doing so potentially seeing and buying other items

Association Rule Mining

- A representative applications of association rule mining include
  - In business: cross-marketing, cross-selling, store design, catalog design, e-commerce site design, optimization of online advertising, product pricing, and sales/promotion configuration
  - In medicine: relationships between symptoms and illnesses; diagnosis and patient characteristics and treatments (to be used in medical DSS); and genes and their functions (to be used in genomics projects)…

Source: Turban et al. (2011), Decision Support and Business Intelligence Systems

Association Rule Mining

- Are all association rules interesting and useful?

A Generic Rule: **X $\Rightarrow$ Y [S%, C%]**

X, Y: products and/or services

X: Left-hand-side (LHS)

Y: Right-hand-side (RHS)

S: Support: how often **X** and **Y** go together

C: Confidence: how often **Y** go together with the **X**

Example: {Laptop Computer, Antivirus Software} $\Rightarrow$ {Extended Service Plan} [30%, 70%]

Association Rule Mining

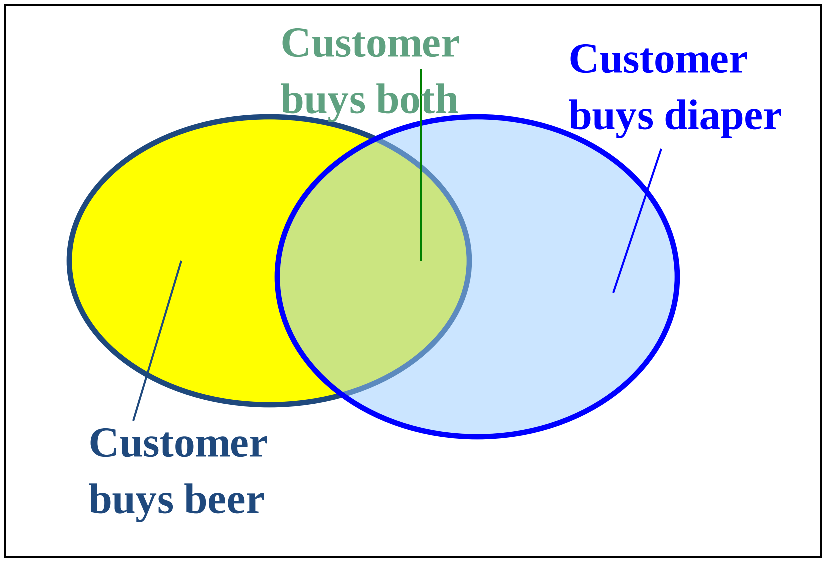
- Algorithms are available for generating association rules
  - Apriori
  - Eclat
  - FP-Growth
  - + Derivatives and hybrids of the three
- The algorithms help identify the frequent item sets, which are, then converted to association rules

Association Rule Mining

- Apriori Algorithm
  - Finds subsets that are common to at least a minimum number of the itemsets
  - uses a bottom-up approach
    - frequent subsets are extended one item at a time (the size of frequent subsets increases from one-item subsets to two-item subsets, then three-item subsets, and so on), and
    - groups of candidates at each level are tested against the data for minimum

Basic Concepts: Frequent Patterns and Association Rules

Transaction-id	Items bought
10	A, B, D
20	A, C, D
30	A, D, E
40	B, E, F
50	B, C, D, E, F



- Itemset X = {x_1, …, x_k}
- Find all the rules $X \rightarrow Y$ with minimum support and confidence
  - support, *s*, probability that a transaction contains $X \cup Y$
  - confidence, *c*, conditional probability that a transaction having X also contains *Y*

Let sup_{min} = 50%, $conf_{min}$ = 50%
Freq. Pat.: {*A:3, B:3, D:4, E:3, AD:3*}
Association rules:

$A \rightarrow D$ (60%, 100%)
$D \rightarrow A$ (60%, 75%)

$A \rightarrow D$ (support = 3/5 = 60%, confidence = 3/3 =100%)
$D \rightarrow A$ (support = 3/5 = 60%, confidence = 3/4 = 75%)

Source: Han & Kamber (2006)

Market basket analysis

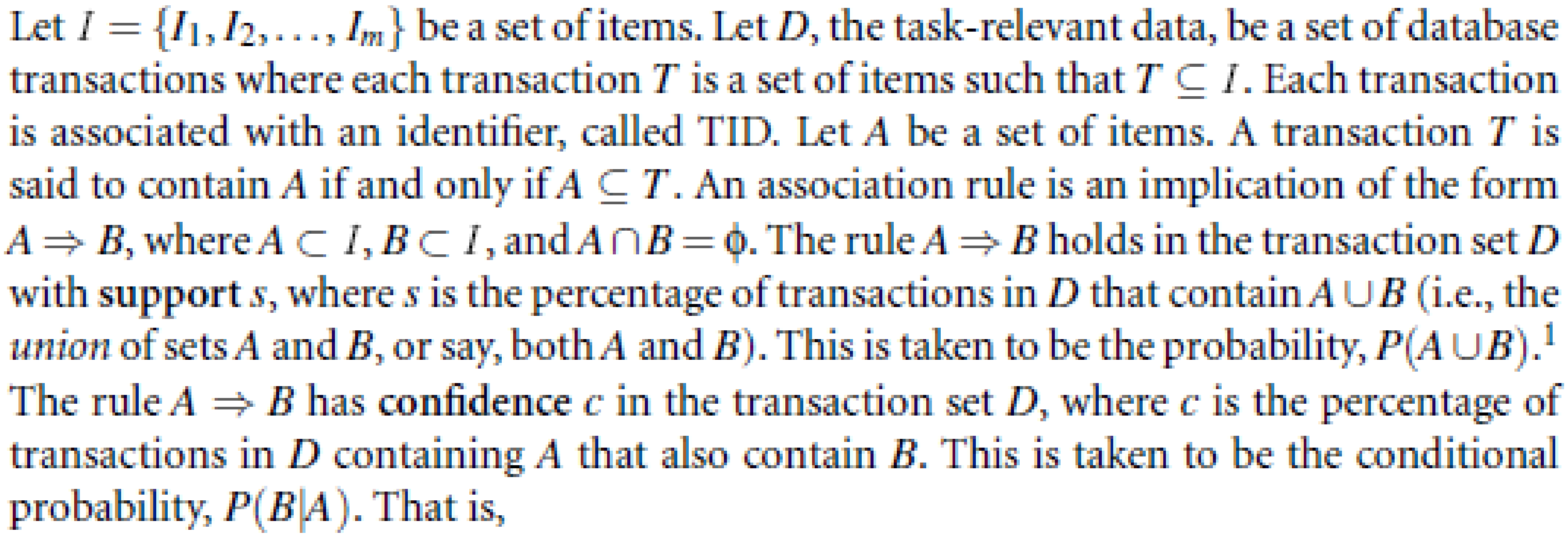
- Example
  - Which groups or sets of items are customers likely to purchase on a given trip to the store?
- Association Rule
  - *Computer → antivirus_software [support = 2%; confidence = 60%]*
    - A support of 2% means that 2% of all the transactions under analysis show that computer and antivirus software are purchased together.
    - A confidence of 60% means that 60% of the customers who purchased a computer also bought the software.

Source: Han & Kamber (2006)

Association rules

- Association rules are considered interesting if they satisfy both
  - a minimum support threshold and
  - a minimum confidence threshold.

Frequent Itemsets, Closed Itemsets, and Association Rules



Let $I = \{I_1, I_2, \ldots, I_m\}$ be a set of items. Let D, the task-relevant data, be a set of database transactions where each transaction T is a set of items such that $T \subseteq I$. Each transaction is associated with an identifier, called TID. Let A be a set of items. A transaction T is said to contain A if and only if $A \subseteq T$. An association rule is an implication of the form $A \Rightarrow B$, where $A \subset I$, $B \subset I$, and $A \cap B = \phi$. The rule $A \Rightarrow B$ holds in the transaction set D with **support** s, where s is the percentage of transactions in D that contain $A \cup B$ (i.e., the *union* of sets A and B, or say, both A and B). This is taken to be the probability, $P(A \cup B)$.[1] The rule $A \Rightarrow B$ has **confidence** c in the transaction set D, where c is the percentage of transactions in D containing A that also contain B. This is taken to be the conditional probability, $P(B|A)$. That is,

Support $(A \rightarrow B) = P(A \cup B)$

Confidence $(A \rightarrow B) = P(B|A)$

Source: Han & Kamber (2006)

Support ($A \rightarrow B$) = P($A \cup B$)
Confidence ($A \rightarrow B$) = P($B|A$)

- The notation *P($A \cup B$) indicates the probability that a transaction contains the union of set A and set B*
  - *(i.e., it contains every item in A and in B).*
- *This should not be confused with P(A or B),* which indicates the probability that a transaction contains either *A or B.*

- Rules that satisfy both a minimum support threshold (*min_sup*) and a minimum confidence threshold (*min_conf*) are called **strong**.
- By convention, we write support and confidence values so as to occur between 0% and 100%, rather than 0 to 1.0.

- itemset
  - A set of items is referred to as an itemset.
- K-itemset
  - An itemset that contains *k items is a k*-itemset*.*
- Example:
  - The set {*computer, antivirus software*} is a 2-itemset.

Absolute Support and Relative Support

- Absolute Support
  - The occurrence frequency of an itemset is the number of transactions that contain the itemset
    - frequency, support count, or count of the itemset
  - Ex: 3
- Relative support
  - Ex: 60%

- If the relative support of an itemset *I satisfies a prespecified minimum support threshold, then I is a* frequent itemset.
  - *i.e., the absolute* support of *I satisfies the corresponding minimum support count threshold*
- The set of frequent *k-itemsets is commonly denoted by* L_K

$$confidence(A \Rightarrow B) = P(B|A) = \frac{support(A \cup B)}{support(A)} = \frac{support\_count(A \cup B)}{support\_count(A)}$$

- the confidence of rule *A* → *B can be easily derived from the* support counts of *A and A ∪ B.*
- once the support counts of *A, B, and A ∪ B are* found, it is straightforward to derive the corresponding association rules *A* → *B and B* → *A* and check whether they are strong.
- Thus the problem of mining association rules can be reduced to that of mining frequent itemsets.

Association rule mining: Two-step process

1. Find all frequent itemsets
   - By definition, each of these itemsets will occur at least as frequently as a predetermined minimum support count, *min_sup.*
2. Generate strong association rules from the frequent itemsets
   - By definition, these rules must satisfy minimum support and minimum confidence.

Efficient and Scalable Frequent Itemset Mining Methods

- The Apriori Algorithm
  - Finding Frequent Itemsets Using Candidate Generation

Association rule mining

- Proposed by Agrawal et al in 1993.
- It is an important data mining model studied extensively by the database and data mining community.
- Assume all data are categorical.
- No good algorithm for numeric data.
- Initially used for Market Basket Analysis to find how items purchased by customers are related.

Bread $\rightarrow$ Milk [sup = 5%, conf = 100%]

The model: data

- $I = \{i_1, i_2, \ldots, i_m\}$: a set of *items*.
- Transaction t :
  - t a set of items, and $t \subseteq I$.
- Transaction Database T: a set of transactions $T = \{t_1, t_2, \ldots, t_n\}$.

Transaction data: supermarket data

- Market basket transactions:

  t1: {bread, cheese, milk}

  t2: {apple, eggs, salt, yogurt}

  …   …

  tn: {biscuit, eggs, milk}

- Concepts:
  - An *item*: an item/article in a basket
  - *I*: the set of all items sold in the store
  - A *transaction*: items purchased in a basket; it may have TID (transaction ID)
  - A *transactional dataset*: A set of transactions

Transaction data: a set of documents

- **A text document data set. Each document is treated as a "bag" of keywords**

doc1:	Student, Teach, School
doc2:	Student, School
doc3:	Teach, School, City, Game
doc4:	Baseball, Basketball
doc5:	Basketball, Player, Spectator
doc6:	Baseball, Coach, Game, Team
doc7:	Basketball, Team, City, Game

The model: rules

- A transaction t contains X, a set of items (itemset) in I, if $X \subseteq t$.
- An association rule is an implication of the form:

$$X \rightarrow Y, \text{ where } X, Y \subset I, \textit{ and } X \cap Y = \emptyset$$

- An itemset is a set of items.
  - E.g., X = {milk, bread, cereal} is an itemset.
- A *k*-itemset is an itemset with *k* items.
  - E.g., {milk, bread, cereal} is a 3-itemset

Rule strength measures

- Support: The rule holds with support *sup* in *T* (the transaction data set) if sup% of transactions contain $X \cup Y$.
  - $sup = \Pr(X \cup Y)$.
- Confidence: The rule holds in *T* with confidence *conf* if *conf*% of tranactions that contain *X* also contain *Y*.
  - $conf = \Pr(Y \mid X)$
- An association rule is a pattern that states when *X* occurs, *Y* occurs with certain probability.

Support and Confidence

- Support count: The support count of an itemset *X*, denoted by *X.count*, in a data set *T* is the number of transactions in *T* that contain *X*. Assume *T* has *n* transactions.
- Then,

$$support = \frac{(X \cup Y).count}{n}$$

$$confidence = \frac{(X \cup Y).count}{X.count}$$

Goal and key features

- **Goal:** Find all rules that satisfy the user-specified *minimum support* (minsup) and *minimum confidence* (minconf).

- **Key Features**
  - Completeness: find all rules.
  - No target item(s) on the right-hand-side
  - Mining with data on hard disk (not in memory)

An example

- Transaction data
- Assume:

  minsup = 30%

  minconf = 80%

t1:	Beef, Chicken, Milk
t2:	Beef, Cheese
t3:	Cheese, Boots
t4:	Beef, Chicken, Cheese
t5:	Beef, Chicken, Clothes, Cheese, Milk
t6:	Chicken, Clothes, Milk
t7:	Chicken, Milk, Clothes

- An example frequent *itemset*:

{Chicken, Clothes, Milk} [sup = 3/7]

- Association rules from the itemset:

Clothes $\rightarrow$ Milk, Chicken	[sup = 3/7, conf = 3/3]
…	…
Clothes, Chicken $\rightarrow$ Milk,	[sup = 3/7, conf = 3/3]

Transaction data representation

- A simplistic view of shopping baskets,
- Some important information not considered. E.g,
  - the quantity of each item purchased and
  - the price paid.

Many mining algorithms

- There are a large number of them!!
- They use different strategies and data structures.
- Their resulting sets of rules are all the same.
  - Given a transaction data set *T*, and a minimum support and a minimum confident, the set of association rules existing in *T* is uniquely determined.
- Any algorithm should find the same set of rules although their computational efficiencies and memory requirements may be different.
- We study only one: the Apriori Algorithm

Road map

- Basic concepts of Association Rules
- Apriori algorithm
- Different data formats for mining
- Mining with multiple minimum supports
- Mining class association rules
- Sequential pattern mining
- Summary

Apriori Algorithm

- **Apriori** is a seminal algorithm proposed by R. Agrawal and R. Srikant in 1994 for mining frequent itemsets for Boolean association rules.
- The name of the algorithm is based on the fact that the algorithm uses *prior knowledge of frequent itemset properties, as we shall* see following.

Apriori Algorithm

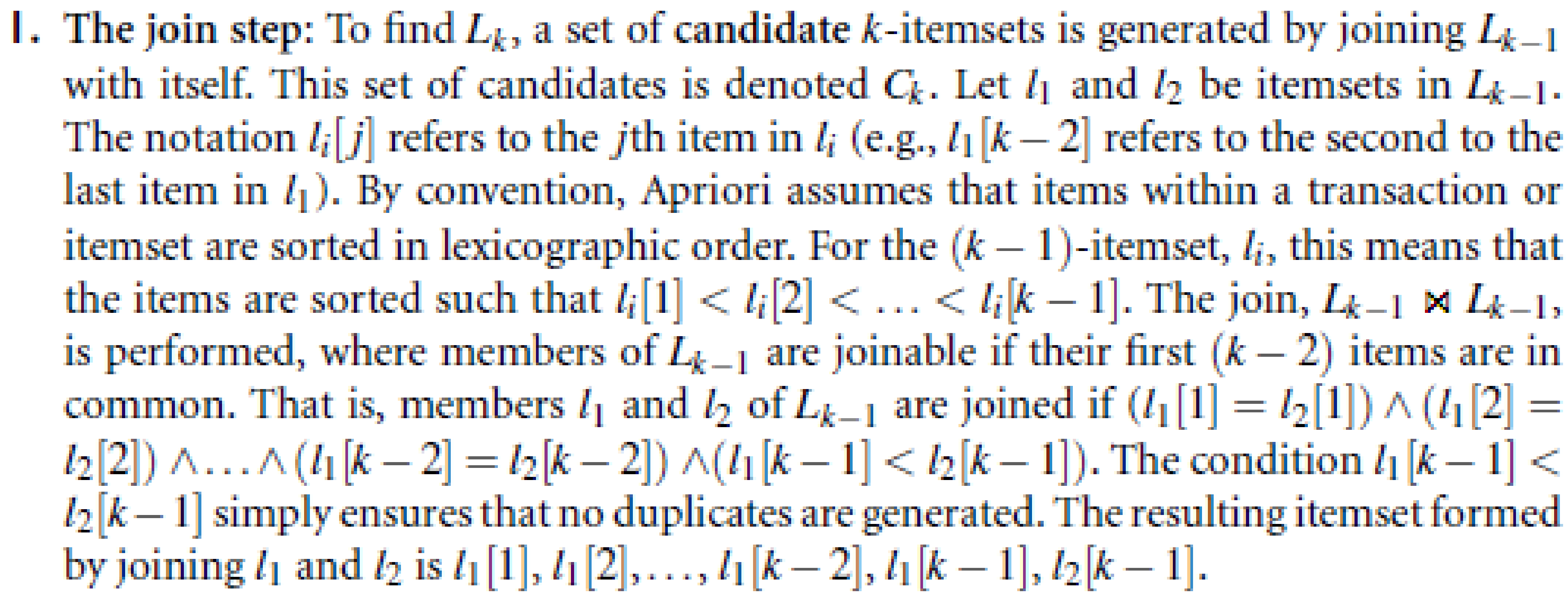
- Apriori employs an iterative approach known as a *level-wise search, where k-itemsets are used to explore (k+1)-itemsets.*
- *First, the set of frequent 1-itemsets is found* by scanning the database to accumulate the count for each item, and collecting those items that satisfy minimum support. The resulting set is denoted L_1.
- *Next,* L_1 *is used to find* L_2, the set of frequent 2-itemsets, which is used to find L_3, *and so on, until no more frequent k-itemsets can be found.*
- *The finding of each* L_k *requires one full scan of the database.*

Apriori Algorithm

- To improve the efficiency of the level-wise generation of frequent itemsets, an important property called the Apriori property.
- Apriori property
  - *All nonempty subsets of a frequent itemset must also be frequent.*

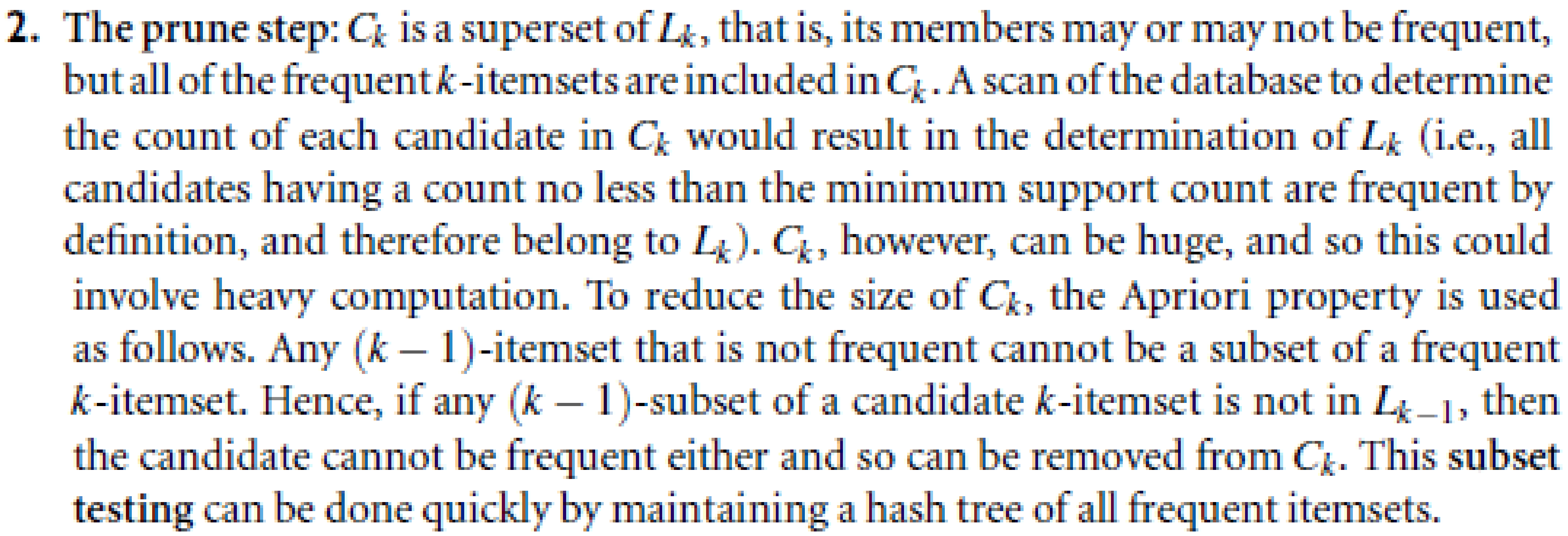
- *How is the Apriori property used in the algorithm?*
  - How L_{k-1} *is used to find* L_k *for k >= 2.*
  - A two-step process is followed, consisting of join and prune actions.

Apriori property used in algorithm 1. The join step



1. **The join step:** To find L_k, a set of **candidate** k-itemsets is generated by joining L_{k-1} with itself. This set of candidates is denoted C_k. Let l_1 and l_2 be itemsets in L_{k-1}. The notation $l_i[j]$ refers to the jth item in l_i (e.g., $l_1[k-2]$ refers to the second to the last item in l_1). By convention, Apriori assumes that items within a transaction or itemset are sorted in lexicographic order. For the $(k-1)$-itemset, l_i, this means that the items are sorted such that $l_i[1] < l_i[2] < \ldots < l_i[k-1]$. The join, $L_{k-1} \bowtie L_{k-1}$, is performed, where members of L_{k-1} are joinable if their first $(k-2)$ items are in common. That is, members l_1 and l_2 of L_{k-1} are joined if $(l_1[1] = l_2[1]) \wedge (l_1[2] = l_2[2]) \wedge \ldots \wedge (l_1[k-2] = l_2[k-2]) \wedge (l_1[k-1] < l_2[k-1])$. The condition $l_1[k-1] < l_2[k-1]$ simply ensures that no duplicates are generated. The resulting itemset formed by joining l_1 and l_2 is $l_1[1], l_1[2], \ldots, l_1[k-2], l_1[k-1], l_2[k-1]$.

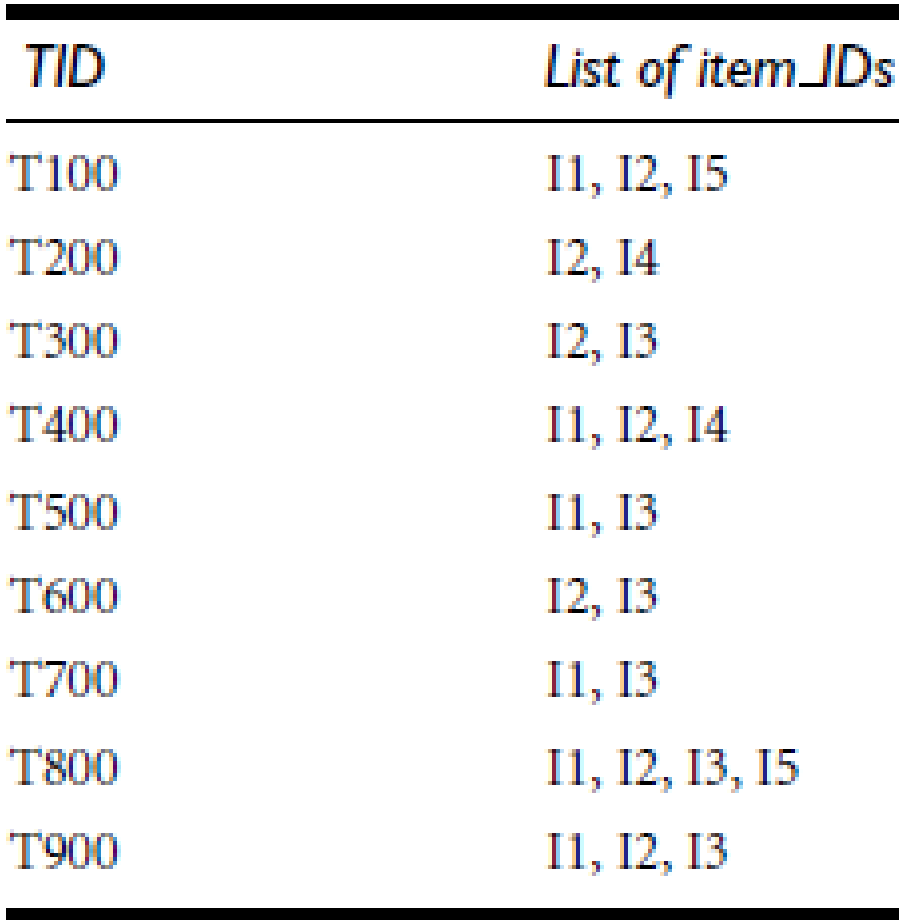
Apriori property used in algorithm 2. The prune step



2. **The prune step:** C_k is a superset of L_k, that is, its members may or may not be frequent, but all of the frequent k-itemsets are included in C_k. A scan of the database to determine the count of each candidate in C_k would result in the determination of L_k (i.e., all candidates having a count no less than the minimum support count are frequent by definition, and therefore belong to L_k). C_k, however, can be huge, and so this could involve heavy computation. To reduce the size of C_k, the Apriori property is used as follows. Any $(k-1)$-itemset that is not frequent cannot be a subset of a frequent k-itemset. Hence, if any $(k-1)$-subset of a candidate k-itemset is not in L_{k-1}, then the candidate cannot be frequent either and so can be removed from C_k. This **subset testing** can be done quickly by maintaining a hash tree of all frequent itemsets.

Source: Han & Kamber (2006)

Transactional data for an *AllElectronics* branch

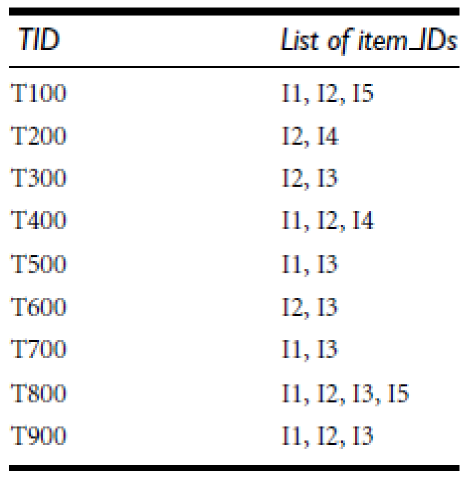


TID	*List of item_IDs*
T100	I1, I2, I5
T200	I2, I4
T300	I2, I3
T400	I1, I2, I4
T500	I1, I3
T600	I2, I3
T700	I1, I3
T800	I1, I2, I3, I5
T900	I1, I2, I3

Source: Han & Kamber (2006)

Example: Apriori

- Let’s look at a concrete example, based on the *AllElectronics transaction database, D.*
- *There are nine transactions in this database, that is, |D| = 9.*
- Apriori algorithm for finding frequent itemsets in *D*



TID	*List of item_IDs*
T100	I1, I2, I5
T200	I2, I4
T300	I2, I3
T400	I1, I2, I4
T500	I1, I3
T600	I2, I3
T700	I1, I3
T800	I1, I2, I3, I5
T900	I1, I2, I3

Source: Han & Kamber (2006)

Example: Apriori Algorithm

Generation of candidate itemsets and frequent itemsets, where the minimum support count is 2.

TID	*List of item_IDs*
T100	I1, I2, I5
T200	I2, I4
T300	I2, I3
T400	I1, I2, I4
T500	I1, I3
T600	I2, I3
T700	I1, I3
T800	I1, I2, I3, I5
T900	I1, I2, I3

Scan *D* for count of each candidate →

C_1

Itemset	Sup. count
{I1}	6
{I2}	7
{I3}	6
{I4}	2
{I5}	2

Compare candidate support count with minimum support count →

L_1

Itemset	Sup. count
{I1}	6
{I2}	7
{I3}	6
{I4}	2
{I5}	2

Generate C_2 candidates from L_1 →

C_2

Itemset
{I1, I2}
{I1, I3}
{I1, I4}
{I1, I5}
{I2, I3}
{I2, I4}
{I2, I5}
{I3, I4}
{I3, I5}
{I4, I5}

Scan *D* for count of each candidate →

C_2

Itemset	Sup. count
{I1, I2}	4
{I1, I3}	4
{I1, I4}	1
{I1, I5}	2
{I2, I3}	4
{I2, I4}	2
{I2, I5}	2
{I3, I4}	0
{I3, I5}	1
{I4, I5}	0

Compare candidate support count with minimum support count →

L_2

Itemset	Sup. count
{I1, I2}	4
{I1, I3}	4
{I1, I5}	2
{I2, I3}	4
{I2, I4}	2
{I2, I5}	2

Generate C_3 candidates from L_2 →

C_3

Itemset
{I1, I2, I3}
{I1, I2, I5}

Scan *D* for count of each candidate →

C_3

Itemset	Sup. count
{I1, I2, I3}	2
{I1, I2, I5}	2

Compare candidate support count with minimum support count →

L_3

Itemset	Sup. count
{I1, I2, I3}	2
{I1, I2, I5}	2

Source: Han & Kamber (2006)

Example: Apriori Algorithm
$C_1 \rightarrow L_1$

TID	*List of item_IDs*
T100	I1, I2, I5
T200	I2, I4
T300	I2, I3
T400	I1, I2, I4
T500	I1, I3
T600	I2, I3
T700	I1, I3
T800	I1, I2, I3, I5
T900	I1, I2, I3

Scan *D* for count of each candidate $\longrightarrow$

C_1

Itemset	Sup. count
{I1}	6
{I2}	7
{I3}	6
{I4}	2
{I5}	2

Compare candidate support count with minimum support count $\longrightarrow$

L_1

Itemset	Sup. count
{I1}	6
{I2}	7
{I3}	6
{I4}	2
{I5}	2

Source: Han & Kamber (2006)

Example: Apriori Algorithm $C_2 \rightarrow L_2$

TID	*List of item_IDs*
T100	I1, I2, I5
T200	I2, I4
T300	I2, I3
T400	I1, I2, I4
T500	I1, I3
T600	I2, I3
T700	I1, I3
T800	I1, I2, I3, I5
T900	I1, I2, I3

Scan *D* for count of each candidate $\rightarrow$

C_1

Itemset	Sup. count
{I1}	6
{I2}	7
{I3}	6
{I4}	2
{I5}	2

Compare candidate support count with minimum support count $\rightarrow$

L_1

Itemset	Sup. count
{I1}	6
{I2}	7
{I3}	6
{I4}	2
{I5}	2

Generate C_2 candidates from L_1 $\rightarrow$

C_2

Itemset
{I1, I2}
{I1, I3}
{I1, I4}
{I1, I5}
{I2, I3}
{I2, I4}
{I2, I5}
{I3, I4}
{I3, I5}
{I4, I5}

Scan *D* for count of each candidate $\rightarrow$

C_2

Itemset	Sup. count
{I1, I2}	4
{I1, I3}	4
{I1, I4}	1
{I1, I5}	2
{I2, I3}	4
{I2, I4}	2
{I2, I5}	2
{I3, I4}	0
{I3, I5}	1
{I4, I5}	0

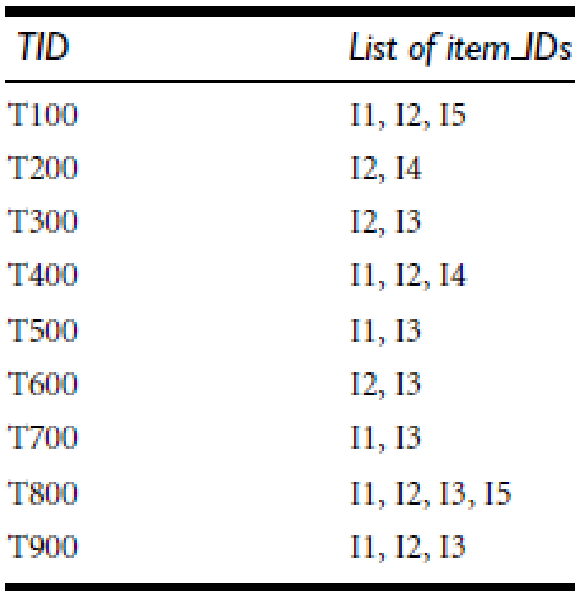
Compare candidate support count with minimum support count $\rightarrow$

L_2

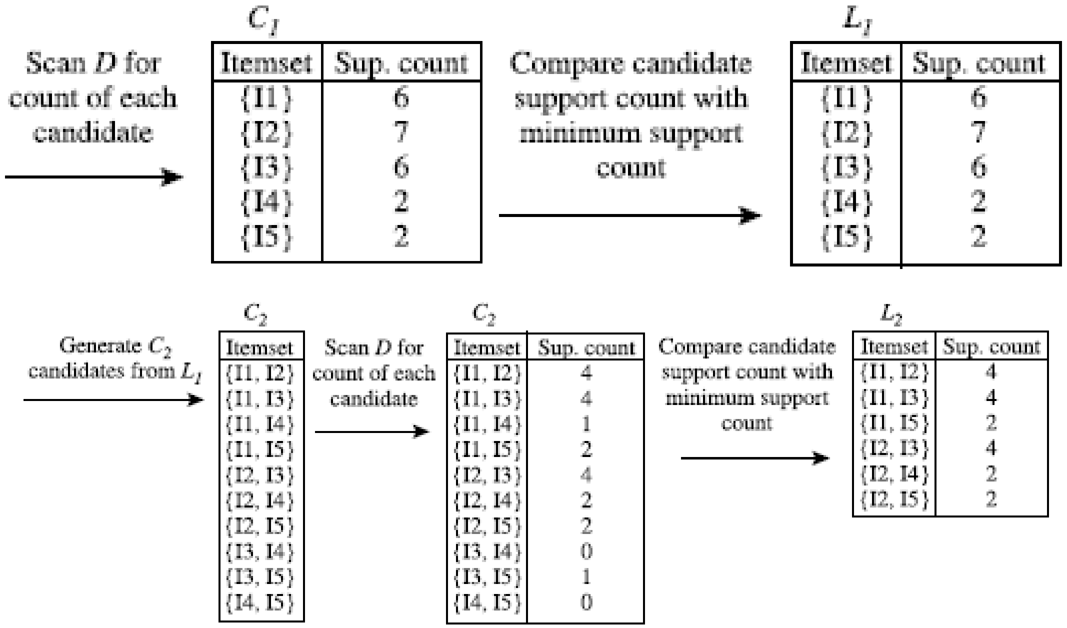
Itemset	Sup. count
{I1, I2}	4
{I1, I3}	4
{I1, I5}	2
{I2, I3}	4
{I2, I4}	2
{I2, I5}	2

Source: Han & Kamber (2006)

Example: Apriori Algorithm
$C_3 \rightarrow L_3$



TID	List of item_IDs
T100	I1, I2, I5
T200	I2, I4
T300	I2, I3
T400	I1, I2, I4
T500	I1, I3
T600	I2, I3
T700	I1, I3
T800	I1, I2, I3, I5
T900	I1, I2, I3



Scan D for count of each candidate
Compare candidate support count with minimum support count
Generate C_2 candidates from L_1
Scan D for count of each candidate
Compare candidate support count with minimum support count

C_1

Itemset	Sup. count
{I1}	6
{I2}	7
{I3}	6
{I4}	2
{I5}	2

L_1

Itemset	Sup. count
{I1}	6
{I2}	7
{I3}	6
{I4}	2
{I5}	2

C_2

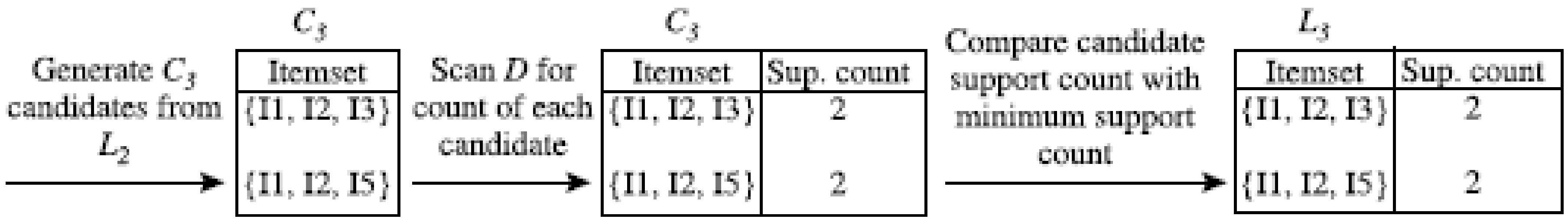
Itemset
{I1, I2}
{I1, I3}
{I1, I4}
{I1, I5}
{I2, I3}
{I2, I4}
{I2, I5}
{I3, I4}
{I3, I5}
{I4, I5}

C_2

Itemset	Sup. count
{I1, I2}	4
{I1, I3}	4
{I1, I4}	1
{I1, I5}	2
{I2, I3}	4
{I2, I4}	2
{I2, I5}	2
{I3, I4}	0
{I3, I5}	1
{I4, I5}	0

L_2

Itemset	Sup. count
{I1, I2}	4
{I1, I3}	4
{I1, I5}	2
{I2, I3}	4
{I2, I4}	2
{I2, I5}	2



Generate C_3 candidates from L_2
Scan D for count of each candidate
Compare candidate support count with minimum support count

C_3

Itemset
{I1, I2, I3}
{I1, I2, I5}

C_3

Itemset	Sup. count
{I1, I2, I3}	2
{I1, I2, I5}	2

L_3

Itemset	Sup. count
{I1, I2, I3}	2
{I1, I2, I5}	2

Source: Han & Kamber (2006)

The Apriori algorithm for discovering frequent itemsets for mining Boolean association rules.

Algorithm: Apriori. Find frequent itemsets using an iterative level-wise approach based on candidate generation.

Input:

- D, a database of transactions;
- *min_sup*, the minimum support count threshold.

Output: L, frequent itemsets in D.

Method:

```
(1)     $L_1$ = find_frequent_1-itemsets(D);
(2)     for ($k = 2; L_{k-1} \neq \phi; k{+}{+}$) {
(3)        $C_k$ = apriori_gen($L_{k-1}$);
(4)        for each transaction $t \in D$ { // scan $D$ for counts
(5)           $C_t$ = subset($C_k, t$); // get the subsets of $t$ that are candidates
(6)           for each candidate $c \in C_t$
(7)              c.count++;
(8)        }
(9)        $L_k = \{c \in C_k | c.count \geq min\_sup\}$
(10)    }
(11)    return $L = \cup_k L_k$;

procedure apriori_gen($L_{k-1}$:frequent $(k-1)$-itemsets)
(1)     for each itemset $l_1 \in L_{k-1}$
(2)        for each itemset $l_2 \in L_{k-1}$
(3)           if $(l_1[1] = l_2[1]) \wedge (l_1[2] = l_2[2]) \wedge ... \wedge (l_1[k-2] = l_2[k-2]) \wedge (l_1[k-1] < l_2[k-1])$ then {
(4)              $c = l_1 \bowtie l_2$; // join step: generate candidates
(5)              if has_infrequent_subset($c, L_{k-1}$) then
(6)                 delete $c$; // prune step: remove unfruitful candidate
(7)              else add $c$ to $C_k$;
(8)           }
(9)     return $C_k$;

procedure has_infrequent_subset($c$: candidate $k$-itemset;
             $L_{k-1}$: frequent $(k-1)$-itemsets); // use prior knowledge
(1)     for each $(k-1)$-subset $s$ of $c$
(2)        if $s \notin L_{k-1}$ then
(3)           return TRUE;
(4)     return FALSE;
```

Source: Han & Kamber (2006)

The Apriori Algorithm—An Example

$Sup_{min} = 2$

Database TDB

Tid	Items
10	A, C, D
20	B, C, E
30	A, B, C, E
40	B, E

1st scan →

C_1

Itemset	sup
{A}	2
{B}	3
{C}	3
{D}	1
{E}	3

→ L_1

Itemset	sup
{A}	2
{B}	3
{C}	3
{E}	3

C_2

Itemset
{A, B}
{A, C}
{A, E}
{B, C}
{B, E}
{C, E}

← 2nd scan

C_2

Itemset	sup
{A, B}	1
{A, C}	2
{A, E}	1
{B, C}	2
{B, E}	3
{C, E}	2

← L_2

Itemset	sup
{A, C}	2
{B, C}	2
{B, E}	3
{C, E}	2

C_3

Itemset
{B, C, E}

3rd scan → L_3

Itemset	sup
{B, C, E}	2

Source: Han & Kamber (2006)

The Apriori Algorithm

- Pseudo-code:

```
$C_k$: Candidate itemset of size k
$L_k$ : frequent itemset of size k

$L_1$ = {frequent items};
for ($k$ = 1; $L_k$ !=∅; $k$++) do begin
    $C_{k+1}$ = candidates generated from $L_k$;
    for each transaction $t$ in database do
            increment the count of all candidates in $C_{k+1}$
        that are contained in $t$
    $L_{k+1}$ = candidates in $C_{k+1}$ with min_support
    end
return $\cup_k L_k$;
```

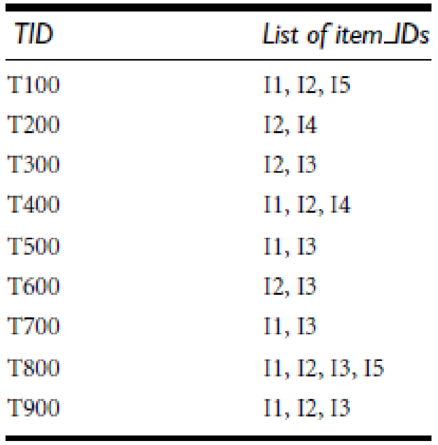
Generating Association Rules from Frequent Itemsets

$$confidence(A \Rightarrow B) = P(B|A) = \frac{support\_count(A \cup B)}{support\_count(A)}$$

- For each frequent itemset l, generate all nonempty subsets of l.
- For every nonempty subset s of l, output the rule "$s \Rightarrow (l - s)$" if $\frac{support\_count(l)}{support\_count(s)} \geq min\_conf$, where min_conf is the minimum confidence threshold.

Source: Han & Kamber (2006)

Example: Generating association rules



TID	List of item_IDs
T100	I1, I2, I5
T200	I2, I4
T300	I2, I3
T400	I1, I2, I4
T500	I1, I3
T600	I2, I3
T700	I1, I3
T800	I1, I2, I3, I5
T900	I1, I2, I3

- frequent itemset *l = {I1, I2, I5}*

$$
\begin{array}{ll}
I1 \wedge I2 \Rightarrow I5, & confidence = 2/4 = 50\% \\
I1 \wedge I5 \Rightarrow I2, & confidence = 2/2 = 100\% \\
I2 \wedge I5 \Rightarrow I1, & confidence = 2/2 = 100\% \\
I1 \Rightarrow I2 \wedge I5, & confidence = 2/6 = 33\% \\
I2 \Rightarrow I1 \wedge I5, & confidence = 2/7 = 29\% \\
I5 \Rightarrow I1 \wedge I2, & confidence = 2/2 = 100\%
\end{array}
$$

- If the minimum confidence threshold is, say, 70%, then only the second, third, and last rules above are output, because these are the only ones generated that are strong.

Source: Han & Kamber (2006)

The Apriori algorithm

- **The best known algorithm**
- **Two steps**:
  - Find all itemsets that have minimum support (*frequent itemsets*, also called large itemsets).
  - Use frequent itemsets to generate rules.

- E.g., a frequent itemset

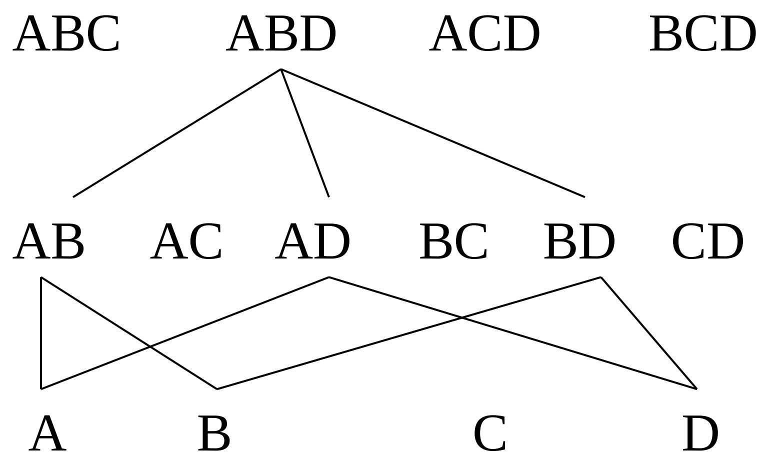
  {Chicken, Clothes, Milk}   [sup = 3/7]

  and one rule from the frequent itemset

  Clothes $\rightarrow$ Milk, Chicken   [sup = 3/7, conf = 3/3]

Step 1: Mining all frequent itemsets

- A frequent *itemset* is an itemset whose support is ≥ minsup.
- Key idea: The apriori property (downward closure property): any subsets of a frequent itemset are also frequent itemsets



The Algorithm

- Iterative algo. (also called level-wise search): Find all 1-item frequent itemsets; then all 2-item frequent itemsets, and so on.
  - In each iteration k, only consider itemsets that contain some k-1 frequent itemset.

- Find frequent itemsets of size 1: F_1
- From $k = 2$
  - C_k = candidates of size k: those itemsets of size k that could be frequent, given F_{k-1}
  - F_k = those itemsets that are actually frequent, $F_k \subseteq C_k$ (need to scan the database once).

Example – Finding frequent itemsets

Dataset T

minsup=0.5

TID	Items
T100	1, 3, 4
T200	2, 3, 5
T300	1, 2, 3, 5
T400	2, 5

itemset:count

1. scan T ➔ C_1: {1}:2, {2}:3, {3}:3, {4}:1, {5}:3

➔ F_1: {1}:2, {2}:3, {3}:3, {5}:3

➔ C_2: {1,2}, {1,3}, {1,5}, {2,3}, {2,5}, {3,5}

2. scan T ➔ C_2: {1,2}:1, {1,3}:2, {1,5}:1, {2,3}:2, {2,5}:3, {3,5}:2

➔ F_2: **{1,3}**:2, **{2,3}**:2, **{2,5}**:3, **{3,5}**:2

➔ C_3: {2, 3,5}

3. scan T ➔ C_3: **{2, 3, 5}**:2 ➔ F_3: **{2, 3, 5}**

Details: ordering of items

- The items in *I* are sorted in lexicographic order (which is a total order).
- The order is used throughout the algorithm in each itemset.
- {w[1], w[2], …, w[k]} represents a k-itemset w consisting of items w[1], w[2], …, w[k], where $w[1] < w[2] < \ldots < w[k]$ according to the total order.

Details: the algorithm

```
Algorithm Apriori($T$)
    $C_1 \leftarrow$ init-pass($T$);
    $F_1 \leftarrow \{f \mid f \in C_1, f.\text{count}/n \geq minsup\}$;    // n: no. of transactions in T
    for ($k$ = 2; $F_{k-1} \neq \emptyset$; $k$++) do
        $C_k \leftarrow$ candidate-gen($F_{k-1}$);
        for each transaction $t \in T$ do
            for each candidate $c \in C_k$ do
                if $c$ is contained in $t$ then
                    $c.count$++;
            end
        end
        $F_k \leftarrow \{c \in C_k \mid c.count/n \geq minsup\}$
    end
return $F \leftarrow \bigcup_k F_k$;
```

Apriori candidate generation

- The candidate-gen function takes F_{k-1} and returns a superset (called the candidates) of the set of all frequent *k*-itemsets. It has two steps
  - *join* step: Generate all possible candidate itemsets C_k of length k
  - *prune* step: Remove those candidates in C_k that cannot be frequent.

Candidate-gen function

```
Function candidate-gen($F_{k-1}$)
   $C_k \leftarrow \varnothing$;
   forall $f_1, f_2 \in F_{k-1}$
         with $f_1 = \{i_1, \ldots, i_{k-2}, i_{k-1}\}$
         and $f_2 = \{i_1, \ldots, i_{k-2}, i'_{k-1}\}$
         and $i_{k-1} < i'_{k-1}$ do
      $c \leftarrow \{i_1, \ldots, i_{k-1}, i'_{k-1}\}$;          // join $f_1$ and $f_2$
      $C_k \leftarrow C_k \cup \{c\}$;
      for each ($k$-1)-subset $s$ of $c$ do
         if ($s \notin F_{k-1}$) then
            delete $c$ from $C_k$;                  // prune
      end
   end
   return $C_k$;
```

An example

- F_3 = {{1, 2, 3}, {1, 2, 4}, {1, 3, 4}, {1, 3, 5}, {2, 3, 4}}

- After join
  - C_4 = {{1, 2, 3, 4}, {1, 3, 4, 5}}
- After pruning:
  - C_4 = {{1, 2, 3, 4}}
    because {1, 4, 5} is not in F_3 ({1, 3, 4, 5} is removed)

Step 2: Generating rules from frequent itemsets

- Frequent itemsets $\neq$ association rules
- One more step is needed to generate association rules
- For each frequent itemset *X*,

  For each proper nonempty subset *A* of *X*,
  - Let *B* = X - *A*
  - A $\rightarrow$ B is an association rule if
    - Confidence(A $\rightarrow$ B) $\geq$ minconf,

      support(A $\rightarrow$ B) = support(A$\cup$B) = support(X)

      confidence(A $\rightarrow$ B) = support(A $\cup$ B) / support(A)

Generating rules: an example

- Suppose {2,3,4} is frequent, with sup=50%
  - Proper nonempty subsets: {2,3}, {2,4}, {3,4}, {2}, {3}, {4}, with sup=50%, 50%, 75%, 75%, 75%, 75% respectively
  - These generate these association rules:
    - $2,3 \rightarrow 4$, confidence=100%
    - $2,4 \rightarrow 3$, confidence=100%
    - $3,4 \rightarrow 2$, confidence=67%
    - $2 \rightarrow 3,4$, confidence=67%
    - $3 \rightarrow 2,4$, confidence=67%
    - $4 \rightarrow 2,3$, confidence=67%
    - All rules have support = 50%

Generating rules: summary

- To recap, in order to obtain A $\rightarrow$ B, we need to have support(A $\cup$ B) and support(A)
- All the required information for confidence computation has already been recorded in itemset generation. No need to see the data *T* any more.
- This step is not as time-consuming as frequent itemsets generation.

On Apriori Algorithm

Seems to be very expensive

- Level-wise search
- K = the size of the largest itemset
- It makes at most K passes over data
- In practice, K is bounded (10).
- The algorithm is very fast. Under some conditions, all rules can be found in linear time.
- Scale up to large data sets

More on association rule mining

- Clearly the space of all association rules is exponential, $O(2^m)$, where m is the number of items in *I*.
- The mining exploits sparseness of data, and high minimum support and high minimum confidence values.
- Still, it always produces a huge number of rules, thousands, tens of thousands, millions, …

Road map

- Basic concepts of Association Rules
- Apriori algorithm
- Different data formats for mining
- Mining with multiple minimum supports
- Mining class association rules
- Sequential pattern mining
- Summary

Different data formats for mining

- The data can be in transaction form or table form

Transaction form:

a, b
a, c, d, e
a, d, f

Table form:

Attr1	Attr2	Attr3
a,	b,	d
b,	c,	e

- Table data need to be converted to transaction form for association mining

From a table to a set of transactions

Table form:

Attr1	Attr2	Attr3
a,	b,	d
b,	c,	e

⇒Transaction form:

(Attr1, a), (Attr2, b), (Attr3, d)

(Attr1, b), (Attr2, c), (Attr3, e)

candidate-gen can be slightly improved. Why?

Road map

- Basic concepts of Association Rules
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Problems with the association mining

- Single minsup: It assumes that all items in the data are of the same nature and/or have similar frequencies.
- Not true: In many applications, some items appear very frequently in the data, while others rarely appear.

  E.g., in a supermarket, people buy *food processor* and *cooking pan* much less frequently than they buy *bread* and *milk*.

Rare Item Problem

- If the frequencies of items vary a great deal, we will encounter two problems
  - If minsup is set too high, those rules that involve rare items will not be found.
  - To find rules that involve both frequent and rare items, minsup has to be set very low. This may cause combinatorial explosion because those frequent items will be associated with one another in all possible ways.

Multiple minsups model

- The minimum support of a rule is expressed in terms of *minimum item supports* (MIS) of the items that appear in the rule.
- Each item can have a minimum item support.
- By providing different MIS values for different items, the user effectively expresses different support requirements for different rules.
- To prevent very frequent items and very rare items from appearing in the same itemsets, we introduce a **support difference constraint**.

  $max_{i \in s}\{sup\{i\} - min_{i \in s}\{sup(i)\} \leq \varphi,$

Minsup of a rule

- Let MIS(i) be the MIS value of item i. The *minsup* of a rule R is the lowest MIS value of the items in the rule.
- I.e., a rule R: $a_1, a_2, \ldots, a_k \rightarrow a_{k+1}, \ldots, a_r$ satisfies its minimum support if its actual support is $\geq$

  $\min(\text{MIS}(a_1), \text{MIS}(a_2), \ldots, \text{MIS}(a_r))$.

An Example

- Consider the following items:

  bread, shoes, clothes

  The user-specified MIS values are as follows:

  MIS(*bread*) = 2%   MIS(*shoes*) = 0.1%

  MIS(*clothes*) = 0.2%

  The following rule doesn't satisfy its minsup:

  clothes $\rightarrow$ *bread* [sup=0.15%,conf =70%]

  The following rule satisfies its minsup:

  clothes $\rightarrow$ *shoes* [sup=0.15%,conf =70%]

Downward closure property

- In the new model, the property no longer holds (?)

E.g., Consider four items 1, 2, 3 and 4 in a database. Their minimum item supports are

MIS(1) = 10%   MIS(2) = 20%

MIS(3) = 5%   MIS(4) = 6%

{1, 2} with support 9% is infrequent, but {1, 2, 3} and {1, 2, 4} could be frequent.

To deal with the problem

- We sort all items in *I* according to their MIS values (make it a total order).
- The order is used throughout the algorithm in each itemset.
- Each itemset *w* is of the following form:

  $\{w[1], w[2], \ldots, w[k]\}$, consisting of items,

  $w[1], w[2], \ldots, w[k]$,

  where $\text{MIS}(w[1]) \leq \text{MIS}(w[2]) \leq \ldots \leq \text{MIS}(w[k])$.

The MSapriori algorithm

```
Algorithm MSapriori($T, MS, \varphi$) // $\varphi$ is for support difference constraint
    $M \leftarrow$ sort($I, MS$);
    $L \leftarrow$ init-pass($M, T$);
    $F_1 \leftarrow \{\{i\} \mid i \in L, i.count/n \geq \text{MIS}(i)\}$;
    for ($k = 2$; $F_{k-1} \neq \emptyset$; $k$++) do
            if $k$=2 then
              $C_k \leftarrow$ level2-candidate-gen($L, \varphi$)
            else $C_k \leftarrow$ MScandidate-gen($F_{k-1}, \varphi$);
            end;
            for each transaction $t \in T$ do
              for each candidate $c \in C_k$ do
                  if $c$ is contained in $t$ then
                        $c.count$++;
                  if $c - \{c[1]\}$ is contained in $t$ then
                        $c.tailCount$++
              end
            end
        $F_k \leftarrow \{c \in C_k \mid c.count/n \geq MIS(c[1])\}$
    end
    return $F \leftarrow \bigcup_k F_k$;
```

Candidate itemset generation

- Special treatments needed:
  - Sorting the items according to their MIS values
  - First pass over data (the first three lines)
    - Let us look at this in detail.
  - Candidate generation at level-2
    - Read it in the handout.
  - Pruning step in level-*k* (*k* > 2) candidate generation.
    - Read it in the handout.

First pass over data

- It makes a pass over the data to record the support count of each item.
- It then follows the sorted order to find the first item *i* in *M* that meets MIS(*i*).
  - *i* is inserted into *L*.
  - For each subsequent item *j* in *M* after *i*, if *j.count*/*n* $\geq$ MIS(*i*) then *j* is also inserted into *L*, where *j.count* is the support count of *j* and *n* is the total number of transactions in *T*. Why?
- *L* is used by function level2-candidate-gen

First pass over data: an example

- Consider the four items 1, 2, 3 and 4 in a data set. Their minimum item supports are:

  MIS(1) = 10% MIS(2) = 20%

  MIS(3) = 5% MIS(4) = 6%

- Assume our data set has 100 transactions. The first pass gives us the following support counts:

  {3}.*count* = 6, {4}.*count* = 3,

  {1}.*count* = 9, {2}.*count* = 25.

- **Then** L = {3, 1, 2}, and F_1 = {{3}, {2}}
- Item 4 is not in L because 4.*count*/*n* < MIS(3) (= 5%),
- {1} is not in F_1 because 1.*count*/*n* < MIS(1) (= 10%).

Rule generation

- The following two lines in MSapriori algorithm are important for rule generation, which are not needed for the Apriori algorithm

```
**if** $c$ – {$c$[1]} is contained in $t$ **then**
    $c.tailCount$++
```

- Many rules cannot be generated without them.
- Why?

On multiple minsup rule mining

- Multiple minsup model subsumes the single support model.
- It is a more realistic model for practical applications.
- The model enables us to found rare item rules yet without producing a huge number of meaningless rules with frequent items.
- By setting MIS values of some items to 100% (or more), we effectively instruct the algorithms not to generate rules only involving these items.

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Mining class association rules (CAR)

- Normal association rule mining does not have any target.
- It finds all possible rules that exist in data, i.e., any item can appear as a consequent or a condition of a rule.
- However, in some applications, the user is interested in some targets.
  - E.g, the user has a set of text documents from some known topics. He/she wants to find out what words are associated or correlated with each topic.

Problem definition

- Let T be a transaction data set consisting of n transactions.
- Each transaction is also labeled with a class y.
- Let I be the set of all items in T, Y be the set of all class labels and $I \cap Y = \emptyset$.
- A **class association rule** (**CAR**) is an implication of the form

  $X \rightarrow y$, where $X \subseteq I$, and $y \in Y$.
- The definitions of **support** and **confidence** are the same as those for normal association rules.

An example

- **A text document data set**

doc 1:	Student, Teach, School	: Education
doc 2:	Student, School	: Education
doc 3:	Teach, School, City, Game	: Education
doc 4:	Baseball, Basketball	: Sport
doc 5:	Basketball, Player, Spectator	: Sport
doc 6:	Baseball, Coach, Game, Team	: Sport
doc 7:	Basketball, Team, City, Game	: Sport

- Let *minsup* = 20% and *minconf* = 60%. The following are two examples of class association rules:

  Student, School $\rightarrow$ Education [sup= 2/7, conf = 2/2]

  game $\rightarrow$ Sport [sup= 2/7, conf = 2/3]

Mining algorithm

- Unlike normal association rules, CARs can be mined directly in one step.
- The key operation is to find all **ruleitems** that have support above *minsup*. A **ruleitem** is of the form:

  $(condset, y)$

  where **condset** is a set of items from I (*i.e.,* $condset \subseteq I$), and $y \in Y$ is a class label.
- Each ruleitem basically represents a rule:

  $condset \rightarrow y$,
- The Apriori algorithm can be modified to generate CARs

Multiple minimum class supports

- The multiple minimum support idea can also be applied here.
- The user can specify different minimum supports to different classes, which effectively assign a different minimum support to rules of each class.
- For example, we have a data set with two classes, Yes and No. We may want
  - rules of class Yes to have the minimum support of 5% and
  - rules of class No to have the minimum support of 10%.
- By setting minimum class supports to 100% (or more for some classes), we tell the algorithm not to generate rules of those classes.
  - This is a very useful trick in applications.

Road map

- Basic concepts of Association Rules
- Apriori algorithm
- Different data formats for mining
- Mining with multiple minimum supports
- Mining class association rules
- Sequential pattern mining
- Summary

Sequential pattern mining

- Association rule mining does not consider the order of transactions.
- In many applications such orderings are significant. E.g.,
  - in market basket analysis, it is interesting to know whether people buy some items in sequence,
    - e.g., buying bed first and then bed sheets some time later.
  - In Web usage mining, it is useful to find navigational patterns of users in a Web site from sequences of page visits of users

Basic concepts

- Let $I = \{i_1, i_2, \ldots, i_m\}$ be a set of items.
- **Sequence**: An ordered list of itemsets.
- **Itemset/element**: A non-empty set of items $X \subseteq I$. We denote a sequence s by $\langle a_1 a_2 \ldots a_r \rangle$, where a_i is an itemset, which is also called an **element** of s.
- An element (or an itemset) of a sequence is denoted by $\{x_1, x_2, \ldots, x_k\}$, where $x_j \in I$ is an item.
- We assume without loss of generality that items in an element of a sequence are in **lexicographic order**.

Basic concepts (contd)

- **Size**: The **size** of a sequence is the number of elements (or itemsets) in the sequence.
- **Length**: The **length** of a sequence is the number of items in the sequence.
  - A sequence of length *k* is called ***k*-sequence**.
- A sequence $s_1 = \langle a_1 a_2 \ldots a_r \rangle$ is a **subsequence** of another sequence $s_2 = \langle b_1 b_2 \ldots b_v \rangle$, or s_2 is a **supersequence** of s_1, if there exist integers $1 \leq j_1 < j_2 < \ldots < j_{r-1} < j_r \leq v$ such that $a_1 \subseteq b_{j1}$, $a_2 \subseteq b_{j2}$, …, $a_r \subseteq b_{jr}$. We also say that s_2 **contains** s_1.

An example

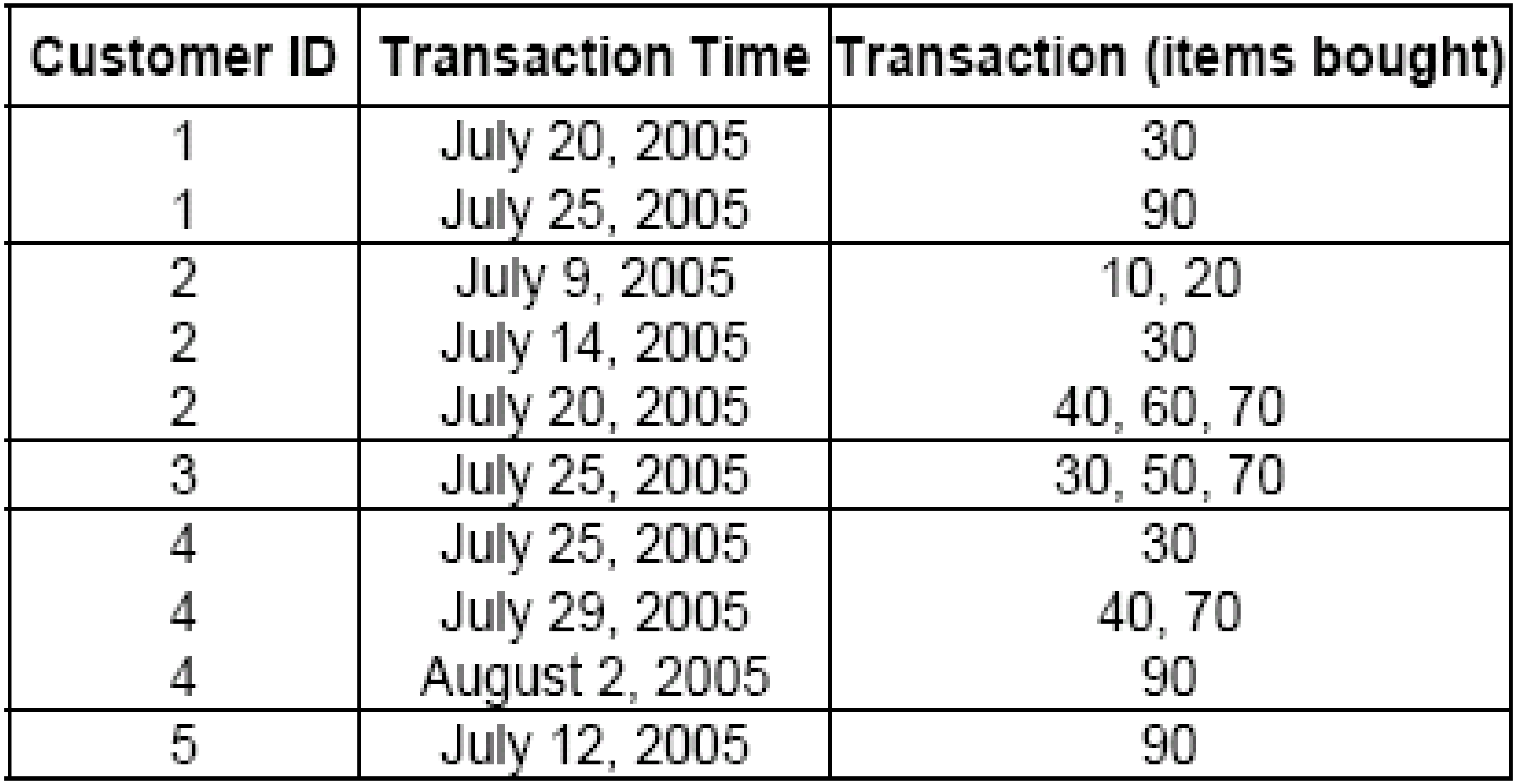
- Let *I* = {1, 2, 3, 4, 5, 6, 7, 8, 9}.
- Sequence ⟨{3}{4, 5}{8}⟩ is **contained** in (or is a **subsequence** of) ⟨{6} {3, 7}{9}{4, 5, 8}{3, 8}⟩
  - because {3} $\subseteq$ {3, 7}, {4, 5} $\subseteq$ {4, 5, 8}, and {8} $\subseteq$ {3, 8}.
  - However, ⟨{3}{8}⟩ is not contained in ⟨{3, 8}⟩ or vice versa.
  - The size of the sequence ⟨{3}{4, 5}{8}⟩ is 3, and the length of the sequence is 4.

Objective

- Given a set *S* of input data sequences (or sequence database), the problem of mining sequential patterns is to find all the sequences that have a user-specified minimum support.
- Each such sequence is called a **frequent sequence**, or a **sequential pattern**.
- The **support** for a sequence is the fraction of total data sequences in *S* that contains this sequence.

Example

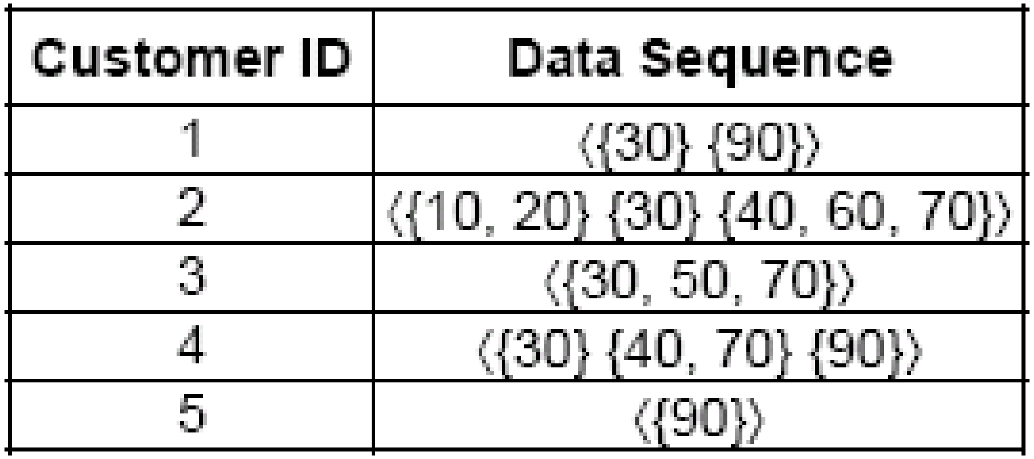
Table 1. A set of transactions sorted by customer ID and transaction time



Customer ID	Transaction Time	Transaction (items bought)
1	July 20, 2005	30
1	July 25, 2005	90
2	July 9, 2005	10, 20
2	July 14, 2005	30
2	July 20, 2005	40, 60, 70
3	July 25, 2005	30, 50, 70
4	July 25, 2005	30
4	July 29, 2005	40, 70
4	August 2, 2005	90
5	July 12, 2005	90

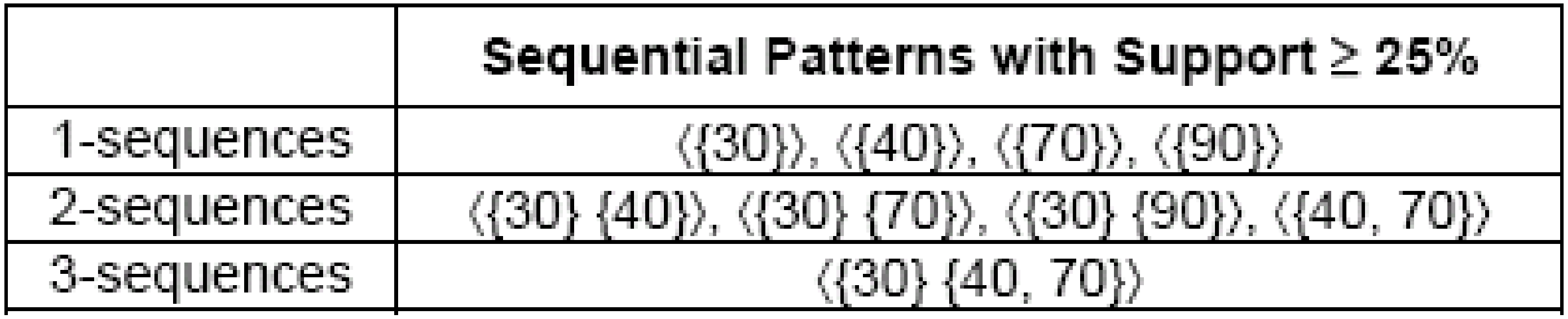
Example (cond)

Table 2. Data sequences produced from the transaction database in Table 1.



Customer ID	Data Sequence
1	⟨{30} {90}⟩
2	⟨{10, 20} {30} {40, 60, 70}⟩
3	⟨{30, 50, 70}⟩
4	⟨{30} {40, 70} {90}⟩
5	⟨{90}⟩

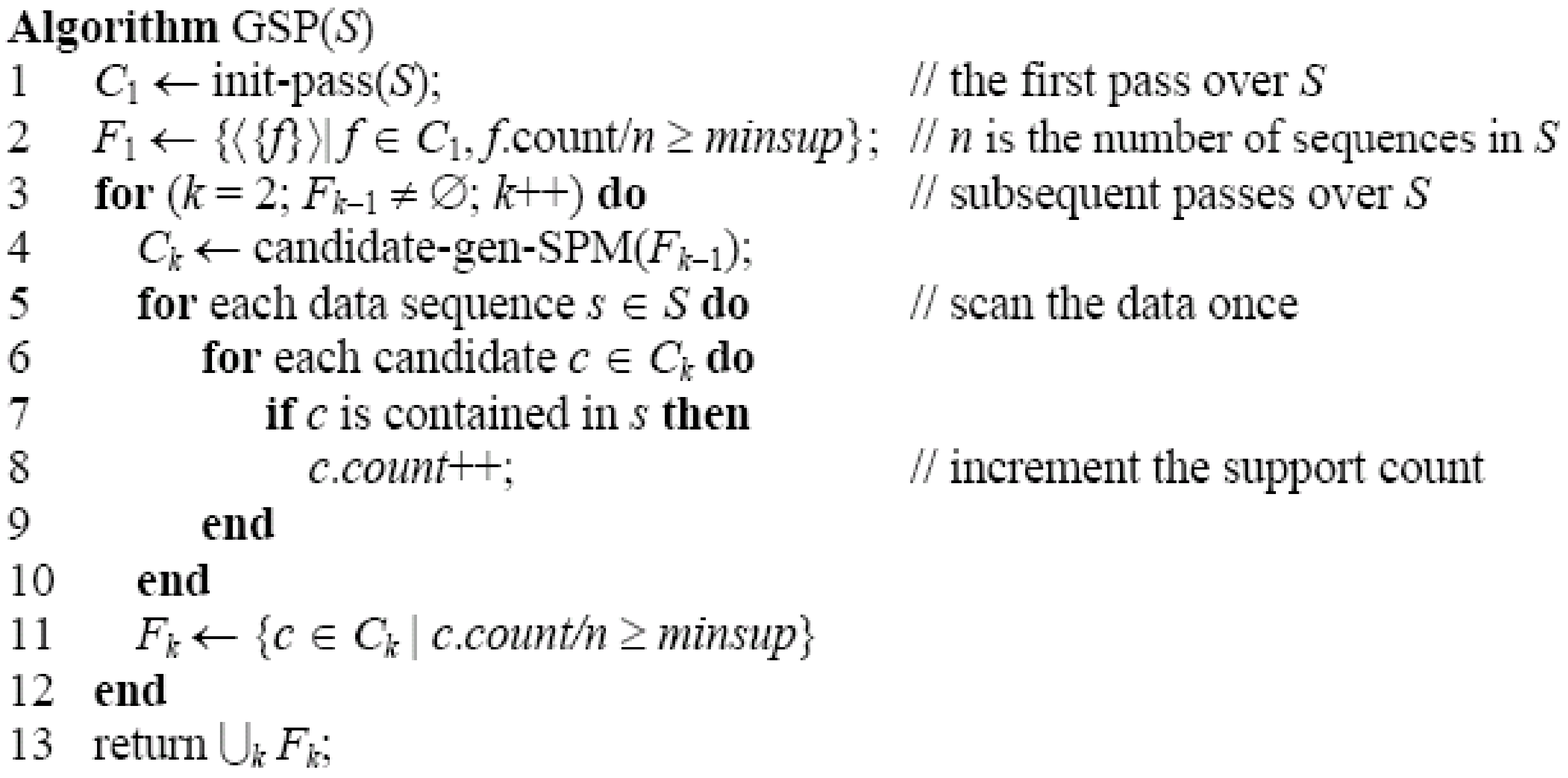
Table 3. The final output sequential patterns



	Sequential Patterns with Support ≥ 25%
1-sequences	⟨{30}⟩, ⟨{40}⟩, ⟨{70}⟩, ⟨{90}⟩
2-sequences	⟨{30} {40}⟩, ⟨{30} {70}⟩, ⟨{30} {90}⟩, ⟨{40, 70}⟩
3-sequences	⟨{30} {40, 70}⟩

GSP mining algorithm

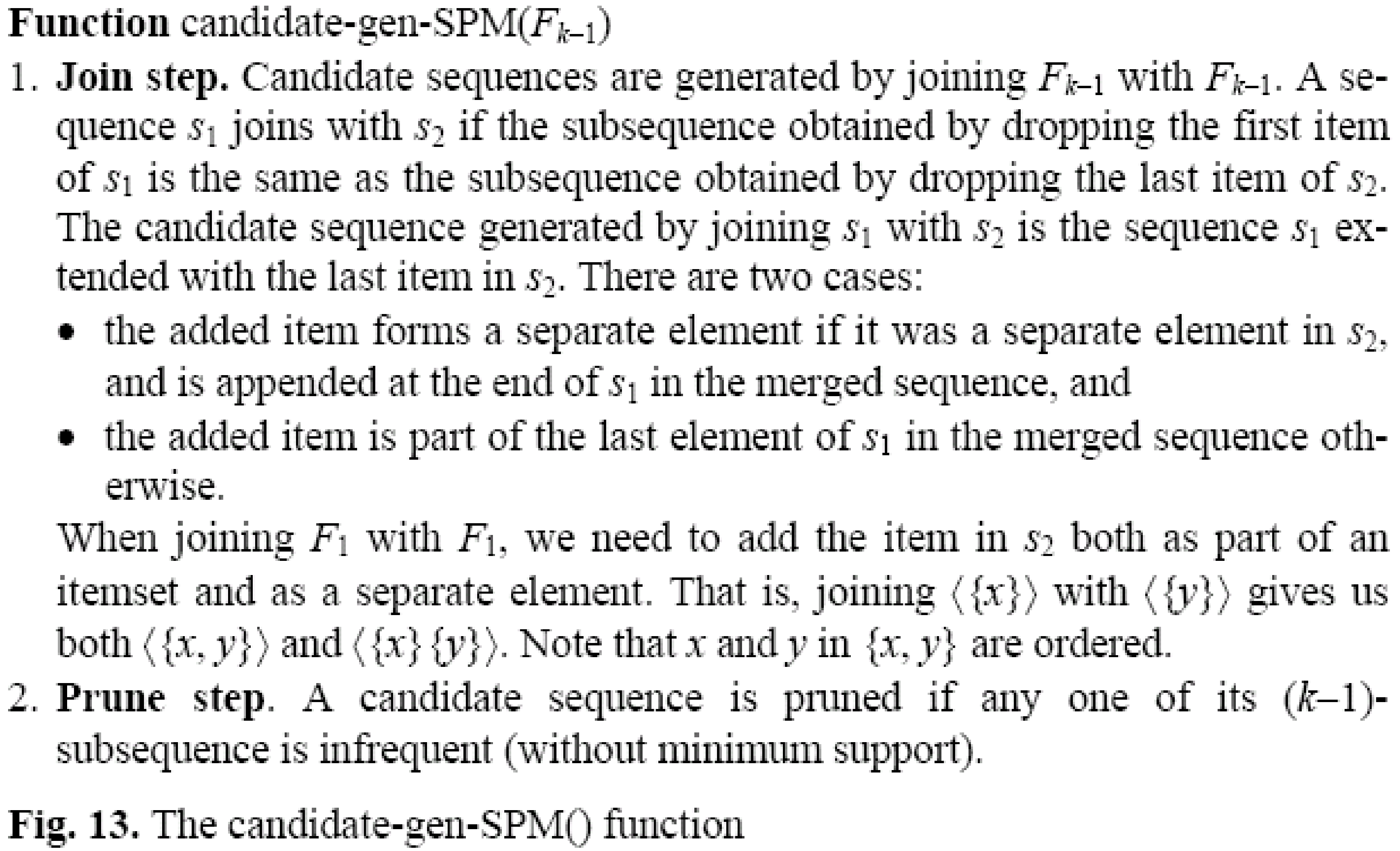
- Very similar to the Apriori algorithm



```
**Algorithm** GSP($S$)
C_1 ← init-pass($S$);                                  // the first pass over $S$
$F_1 \leftarrow \{\langle\{f\}\rangle \mid f \in C_1, f.\text{count}/n \geq minsup\}$;   // $n$ is the number of sequences in $S$
**for** ($k = 2$; $F_{k-1} \neq \emptyset$; $k$++) **do**    // subsequent passes over $S$
    $C_k$ ← candidate-gen-SPM($F_{k-1}$);
    **for** each data sequence $s \in S$ **do**          // scan the data once
        **for** each candidate $c \in C_k$ **do**
            **if** $c$ is contained in $s$ **then**
                $c.count$++;                            // increment the support count
        **end**
    **end**
    $F_k \leftarrow \{c \in C_k \mid c.count/n \geq minsup\}$
**end**
return $\bigcup_k F_k$;
```

Fig. 12. The GSP Algorithm for generating sequential patterns

Candidate generation



Function candidate-gen-SPM(F_{k-1})

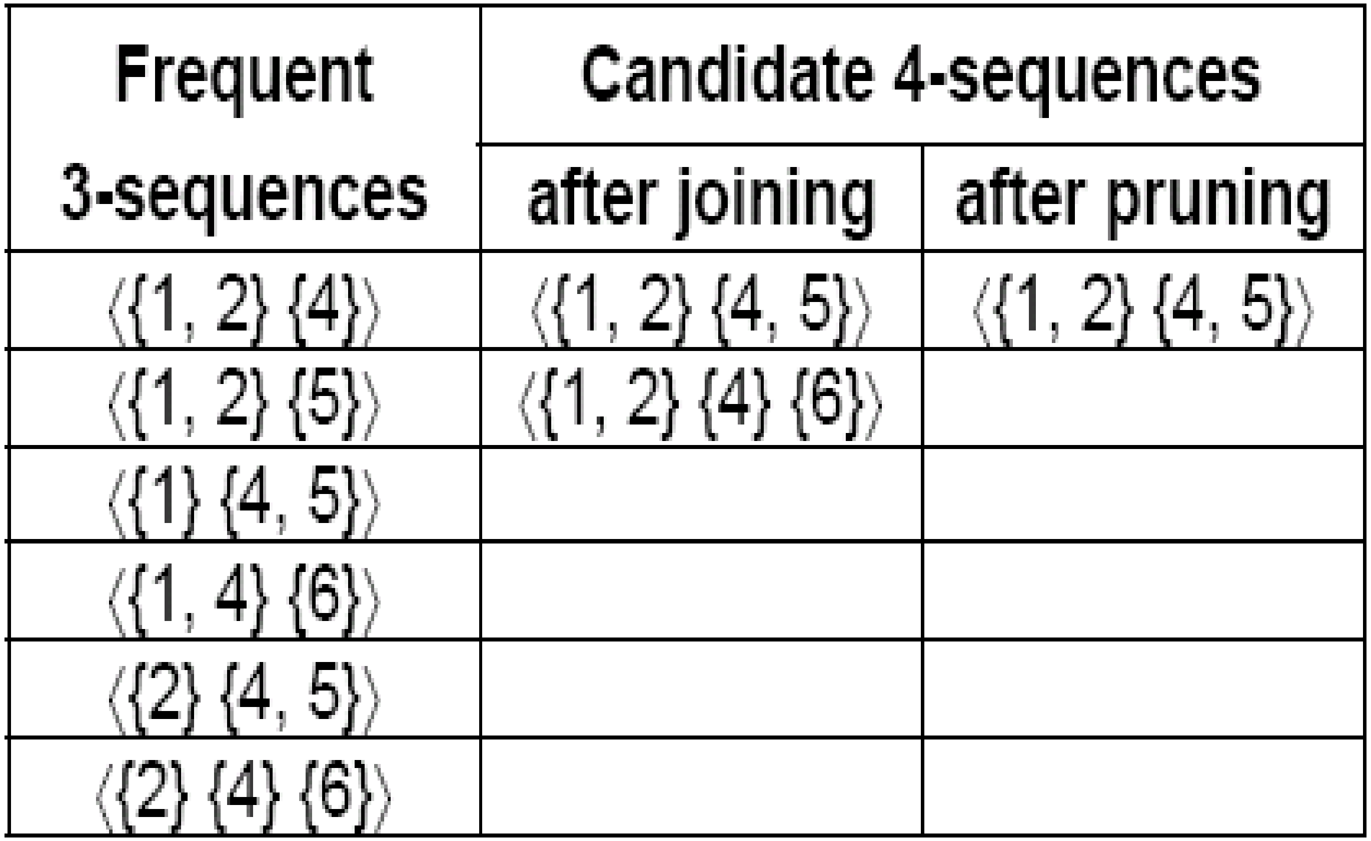
1. **Join step.** Candidate sequences are generated by joining F_{k-1} with F_{k-1}. A sequence s_1 joins with s_2 if the subsequence obtained by dropping the first item of s_1 is the same as the subsequence obtained by dropping the last item of s_2. The candidate sequence generated by joining s_1 with s_2 is the sequence s_1 extended with the last item in s_2. There are two cases:
   - the added item forms a separate element if it was a separate element in s_2, and is appended at the end of s_1 in the merged sequence, and
   - the added item is part of the last element of s_1 in the merged sequence otherwise.

   When joining F_1 with F_1, we need to add the item in s_2 both as part of an itemset and as a separate element. That is, joining $\langle\{x\}\rangle$ with $\langle\{y\}\rangle$ gives us both $\langle\{x, y\}\rangle$ and $\langle\{x\}\{y\}\rangle$. Note that x and y in $\{x, y\}$ are ordered.
2. **Prune step**. A candidate sequence is pruned if any one of its (k–1)-subsequence is infrequent (without minimum support).

Fig. 13. The candidate-gen-SPM() function

An example

Table 4. Candidate generation: an example



Frequent 3-sequences	Candidate 4-sequences	
	after joining	after pruning
⟨{1, 2} {4}⟩	⟨{1, 2} {4, 5}⟩	⟨{1, 2} {4, 5}⟩
⟨{1, 2} {5}⟩	⟨{1, 2} {4} {6}⟩	
⟨{1} {4, 5}⟩		
⟨{1, 4} {6}⟩		
⟨{2} {4, 5}⟩		
⟨{2} {4} {6}⟩		

Road map

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Summary

- Association rule mining has been extensively studied in the data mining community.
- So is sequential pattern mining
- There are many efficient algorithms and model variations.
- Other related work includes
  - Multi-level or generalized rule mining
  - Constrained rule mining
  - Incremental rule mining
  - Maximal frequent itemset mining
  - Closed itemset mining
  - Rule interestingness and visualization
  - Parallel algorithms
  - …

References

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- Efraim Turban, Ramesh Sharda, Dursun Delen (2011), “Decision Support and Business Intelligence Systems,” 9th Edition, Pearson.
- Jiawei Han and Micheline Kamber (2006), “Data Mining: Concepts and Techniques”, 2nd Edition, Elsevier.