

除填充題外，每道題必須整齊列出有效之計算、推導式子於給定空白處方予計分。不依指示作答 該題 0 分。

1. (11) differential (微分) 的中文意譯是 微變化量，derivative (導數) 的中文意譯是 微變化率

$$(\ln x)' = \frac{1}{x}, (e^x)' = e^x, (\csc x)' = -\csc x \cot x, (\sec x)' = \sec x \tan x, (\cot x)' = -\csc^2 x \quad \S 2.4$$

$$(fg)' = f'g + fg', (f/g)' = \frac{f'g - fg'}{g^2}, (fg)'' = f''g + 2f'g' + fg'', (fgh)' = f'gh + fg'h + fgh'$$

2. (7) 求  $\frac{d}{dx} [x + (x + \sin^2 x)^3]^4$ 。由外到內 限用  $f, g, h$ 、每層都要令好。有先令過，最後才可以不回代。 §2.5#42

$$\begin{aligned} (\text{sol.}) \text{ 令 } f &:= g^4, g := x + h^3, h := x + \sin^2 x \quad (+2), \text{ 則原式 } \frac{df}{dx} = \frac{df}{dg} \frac{dg}{dx} = 4g^3 \left[ 1 + \frac{d}{dx} [h^3] \right] \quad (+1) = 4g^3 \left[ 1 + 3h^2 \frac{dh}{dx} \right] \quad (+1) = \\ &4g^3 \left[ 1 + 3h^2 (1 + \frac{d}{dx} [\sin^2 x]) \right] \quad (+1) = 4g^3 \left[ 1 + 3h^2 (1 + 2 \sin x \cos x) \right] \quad (+2) \end{aligned}$$

3. (5)  $\tan(x - y) = \frac{y}{1+x^2}$  是  $x, y$  的關係式，以 differential 求  $\frac{dx}{dy}$ 。(用 derivative 做 不給分) §2.6#16

$$\begin{aligned} (\text{sol.}) \xrightarrow{d} \sec^2(x - y) \cdot d(x - y) &= \frac{dy \cdot (1 + x^2) - y \cdot d(1 + x^2)}{(1 + x^2)^2} \quad (+1), \\ (1 + x^2)^2 \sec^2(x - y) \cdot (dx - dy) &= dy \cdot (1 + x^2) - 2xy \cdot dx \quad (+1) \\ ((1 + x^2)^2 \sec^2(x - y) + 2xy) \cdot dx &= ((1 + x^2)^2 \sec^2(x - y) + (1 + x^2)) \cdot dy \quad (+1) \\ \frac{dx}{dy} &= \frac{(1+x^2)^2 \sec^2(x-y) + (1+x^2)}{(1+x^2)^2 \sec^2(x-y) + 2xy} \quad (+1) \end{aligned}$$

4. (5) 以「某函數的 linear approximation」去估計  $(8.06)^{2/3}$ 。 §2.8# 13

$$\begin{aligned} (\text{sol.}) \text{ 令 } f(x) &= x^{2/3} \quad (+1), \text{ 則 } f'(x) = \frac{2}{3}x^{-1/3} \quad (+1), f(x) \approx f(8) + f'(8)(x - 8) \text{ for } x \approx 8, \\ \therefore f(8.06) &\approx 4 + (1/3) \cdot (0.06) \quad (+2) = 4.02 \quad (+1). \end{aligned}$$

任何東西 寫於此線以下或背面 (草稿區) 皆不看 不受理 不記分