BLOWUP RATE ESTIMATE FOR A SYSTEM OF SEMILINEAR PARABOLIC EQUATIONS

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Abstract. In this paper, we study the blowup rate estimate for a system of semilinear parabolic equations. The blowup rate depends on whether the two components of the solution of this system blow up simultaneously or not.

Keywords: parabolic system, blowup rate, simultaneous blowup, non-simultaneous blowup

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1. Introduction

We consider the Cauchy problem for the following system of semilinear parabolic equations:

$$(1.1)\qquad \begin{cases} u_t = \Delta u + u^{p_1} v^{q_1} & \text{in } \mathbf{R}^n \times (0,T), \\ v_t = \Delta v + u^{p_2} v^{q_2} & \text{in } \mathbf{R}^n \times (0,T) \end{cases}$$

with the initial condition

$$(1.2)\qquad \begin{cases} u(x,0) = u_0(x) & \text{in } \mathbf{R}^n, \\ v(x,0) = v_0(x) & \text{in } \mathbf{R}^n, \end{cases}$$

where we assume that $T > 0$, all powers p_i, q_i, $i = 1, 2$, are positive, and initial data u_0 and v_0 are positive bounded smooth functions.

In this paper, we always assume that the solution (u, v) of (1.1) blows up at the finite time T in the sense that

$$\limsup_{t\to T}\{\sup_{x\in\mathbf{R}^n} u(x,t) + \sup_{x\in\mathbf{R}^n} v(x,t)\} = \infty.$$

It is well known that there exist solutions (u, v) of (1.1) that blow up at finite time T under certain conditions on the exponents p_1, p_2, q_1, q_2. We refer to [4] for more details.

For a blowup solution (u, v) of (1.1), the sup norm of one of the components must tend to infinity as t tends to the blowup time T. In this paper we always assume that the sup norm of the component u tends to infinity as t tends to T. The case when u blows up and v remains bounded is called *non-simultaneous blowup*. We shall call the case when both components u and v blow up at the same time as *simultaneous blowup*.

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The purpose of this paper is to obtain the blowup rate estimates for both simultaneous and non-simultaneous blowup cases.

First, we shall assume the initial data satisfy the following conditions:

$$u_0 = u_0(r), v_0 = v_0(r), u_0'(r), v_0'(r) \le 0 \text{ for } r \ge 0, \quad r := |x|, \tag{1.3}$$

$$\Delta u_0 + u_0^{p_1} v_0^{q_1} \ge 0, \quad \Delta v_0 + u_0^{p_2} v_0^{q_2} \ge 0 \text{ in } \mathbf{R}^n. \tag{1.4}$$

Under these assumptions, we obtain the following upper bound estimate.

Theorem 1.1. *Let (u, v) be a blowup solution of (1.1)-(1.2) such that (1.3) and (1.4) hold. Suppose that $p_1 > 1$ and $(n-2)p_1 < n+2$. Then there exists a positive constant C depending only on n, v_0, p_1, q_1 and T such that*

$$u(x,t) \le C(T-t)^{-1/(p_1-1)} \quad \forall x \in \mathbf{R}^n, t \in (0,T). \tag{1.5}$$

We remark that the estimate (1.5) for the system (1.1) is the same as that for the scalar equation $u_t = \Delta u + u^{p_1}$, under the same assumption on p_1. In the non-simultaneous blowup case, since u blows up in finite time T and v remains bounded, we also have the lower bound estimate

$$u(0,t) \ge c(T-t)^{-1/(p_1-1)} \text{ for } t \in (0,T)$$

for some positive constant c. Indeed, this lower bound estimate follows from

$$u_t(0,t) \le \Delta u(0,t) + L^{q_1} u(0,t)^{p_1} \le L^{q_1} u(0,t)^{p_1}$$

and an integration, where L is an upper bound for v. This gives the blowup rate estimate with exponent $1/(p_1 - 1)$ for the blowup component u.

Indeed the upper bound estimate (1.5) is given in [11] without a detailed proof. We are not sure whether the classical method of Giga-Kohn [6] is applicable to this estimate. Here we provide a detailed proof by using a different approach. In [11], they shown that $p_1 > p_2 + 1$ (which implies $p_1 > 1$, since $p_2 \ge 0$) is a sufficient condition for the non-simultaneous blowup for (1.1) for some initial data. The proof of this result depends on the estimate (1.5). It is also shown in [11] that $p_1 > p_2 + 1$ is a necessary condition for the non-simultaneous blowup for (1.1).

Next, we consider the simultaneous blowup case. For this case, we assume that

$$(p_1 - 1)(q_2 - 1) - p_2 q_1 \neq 0 \tag{1.6}$$

and define the components

$$\alpha := \frac{q_2 - q_1 - 1}{(p_1 - 1)(q_2 - 1) - p_2 q_1}, \quad \beta := \frac{p_1 - p_2 - 1}{(p_1 - 1)(q_2 - 1) - p_2 q_1}.$$

Moreover, in addition to the assumption (1.3), we assume there exists a positive constant ε such that the initial data satisfy

$$\Delta u_0 + (1-\varepsilon) u_0^{p_1} v_0^{q_1} \ge 0, \quad \Delta v_0 + (1-\varepsilon) u_0^{p_2} v_0^{q_2} \ge 0 \text{ in } \mathbf{R}^n. \tag{1.7}$$

Then we have the following theorem on the blowup rate estimate.

Theorem 1.2. *Let (u, v) be a blowup solution of (1.1)-(1.2) such that (1.3), (1.6) and (1.7) are in force. If $p_i, q_i > 1$ $(i = 1, 2)$, $p_2 - p_1 + 1 > 0$ and $q_1 - q_2 + 1 > 0$, then u and v blow up simultaneously. Moreover, there exist positive constants c and C depending only on p_1, p_2, q_1, q_2, u_0 and v_0 such that*

$$c(T-t)^{-\alpha} \le u(0,t) \le C(T-t)^{-\alpha}, \quad c(T-t)^{-\beta} \le v(0,t) \le C(T-t)^{-\beta}$$

for all $t \in [0, T)$.

Note that Theorem 1.2 gives a sufficient condition for simultaneous blowup. We also remark that a similar blowup rate estimate was obtained by Wang [13] for the initial boundary value problem for the same system (1.1). For some of the recent studies of blowup for parabolic systems, we refer the reader to, e.g., [1, 2, 3, 9, 10, 14, 15, 16, 17, 18].

We organize this paper as follows. In Section 2, we study the blowup rate estimate for the non-simultaneous blowup case. We shall derive the upper bound estimate for the blowup rate and give a proof of Theorem 1.1. Then, in Section 3, we give a proof of Theorem 1.2 for the blowup rate estimate in the simultaneous blowup case. Finally, in Section 4, we discuss the criteria for simultaneous and non-simultaneous blowup for the corresponding ODE system.

2. Non-simultaneous Blowup Rate

In this section, we shall prove Theorem 1.1 for the blowup rate estimate in the case of non-simultaneous blowup.

Since the solution is positive, it is easy to derive the following linear estimate (cf. [8]).

Lemma 2.1. (Linear estimate) *Let (u, v) be a solution of the system (1.1) with positive initial data u_0 and v_0. Then there exists a positive constant C depending only on n, u_0 and v_0 such that*

$$\begin{aligned} u(x,t) &\ge C(1+t)^{-n/2}\exp\left(-\frac{|x|^2}{2t}\right), \\ v(x,t) &\ge C(1+t)^{-n/2}\exp\left(-\frac{|x|^2}{2t}\right) \end{aligned}$$

for all $t > 0$.

By this linear estimate, Lemma 2.1, we obtain the following inequality:

$$u_t \ge \Delta u + C^{q_1}(1+T)^{-nq_1/2}\exp\left(-\frac{q_1|x|^2}{2t}\right)u^{p_1}$$

in $(x,t) \in \mathbf{R}^n \times (0,T)$. Using assumptions (1.3) and (1.4), it follows from the maximum principle that

$$u_r, v_r \le 0 \quad \text{and} \quad u_t, v_t > 0$$

in $\mathbf{R} \times (0,T)$. Therefore, $(0,t)$ is the maximum point of $u(x,t)$ and $v(x,t)$.

We set a positive constant $K := C(1+T)^{-nq_1/2}\exp\{-q_1R^2/(2t_1)\}$, where R and $t_1 < T$ are some positive constants. Then we have the following uniform lower bound for v:

$$v(x,t) \geq K^{1/q_1} \text{ in } (x,t) \in B_R \times (t_1,T),$$

where B_R is the open ball of radius R centered at the origin in $\mathbf{R}^n$. Hence u satisfies

$$u_t \geq \Delta u + Ku^{p_1}$$

in $B_R \times (t_1,T)$.

Now, we define a function

$$w(y,t) := \frac{u(K^{-1/2}\alpha^{-1}(t)y,t)}{u(0,t)} \quad \text{for } (y,t) \in B_{K^{1/2}\alpha(t)R} \times (t_1,T),$$

where $\alpha(t) = u(0,t)^{(p_1-1)/2}$. Then

$$\Delta w + w^{p_1} = K^{-1}\frac{\Delta u(K^{-1/2}\alpha^{-1}(t)y,t)}{u(0,t)^{p_1}} + \frac{u(K^{-1/2}\alpha^{-1}(t)y,t)^{p_1}}{u(0,t)^{p_1}}.$$

Note that w is radial and decreasing with respect to $r := |y|$, since u is radial and decreasing with respect to r. Therefore, we can easily get the following inequality

$$w_{rr} + \frac{n-1}{r}w_r + w^{p_1} \leq \frac{u_t(K^{-1/2}\alpha^{-1}(t)r,t)}{u(0,t)^{p_1}}K^{-1}. \tag{2.1}$$

Next, we define a function

$$z(r,t) := \frac{1}{\bar{t}-t}\int_t^{\bar{t}} w(r,s)ds,$$

where $\bar{t} = (t+T)/2$, for $t_1 < t < T$ and $0 \leq r < K^{1/2}\alpha(t)R$. By Hölder's inequality, we obtain

$$\begin{aligned} z(r,t) &\leq \frac{1}{\bar{t}-t}\left(\int_t^{\bar{t}} 1ds\right)^{(p_1-1)/p_1}\left(\int_t^{\bar{t}} w(r,s)^{p_1}ds\right)^{1/p_1} \\ &= \frac{1}{(\bar{t}-t)^{1/p_1}}\left(\int_t^{\bar{t}} w(r,s)^{p_1}ds\right)^{1/p_1}. \end{aligned}$$

Integrating (2.1) over $(t,\bar{t})$, we obtain

$$z_{rr} + \frac{n-1}{r}z_r + z^{p_1} \leq \frac{2K^{-1}}{T-t}\int_t^T \frac{u_t(K^{-1/2}\alpha^{-1}(s)r,s)}{u(0,s)^{p_1}}ds. \tag{2.2}$$

We can estimate the right-hand side of (2.2) as follows.

Lemma 2.2. *For all $r \in [0, K^{1/2}\alpha(t)R)$ and all $t \in (0,T)$, we have*

$$\int_t^T \frac{u_t(K^{-1/2}\alpha^{-1}(s)r,s)}{u(0,s)^{p_1}}ds \leq \frac{p_1}{p_1-1}u^{1-p_1}(0,t).$$

Proof. First we set a function

$$\beta(t) := u(K^{-1/2}\alpha^{-1}(t)r, t).$$

Since $u_t \geq 0$ and $u_r \leq 0$, we obtain

$$\begin{aligned}\beta'(t) &= u_t(K^{-1/2}\alpha^{-1}(t)r, t) - \frac{K^{-1/2}\alpha'(t)r}{\alpha(t)^2} u_r(K^{-1/2}\alpha^{-1}(t)r, t) \\ &\geq u_t(K^{-1/2}\alpha^{-1}(t)r, t).\end{aligned}$$

Since $u_r \leq 0$, $\beta(t) \leq u(0,t)$. For $\tau \in (t, T)$, integrating by parts gives

$$\begin{aligned}\int_t^\tau \frac{u_t(K^{-1/2}\alpha^{-1}(s)r, s)}{u^{p_1}(0,s)} ds &\leq \int_t^\tau \frac{\beta'(s)}{u^{p_1}(0,s)} ds \\ &= \left[\frac{\beta(s)}{u^{p_1}(0,s)}\right]_t^\tau + p_1 \int_t^\tau \frac{\beta(s)u_t(0,s)}{u^{p_1+1}(0,s)} ds \\ &\leq \frac{\beta(\tau)}{u^{p_1}(0,\tau)} + p_1 \int_t^\tau \frac{u_t(0,s)}{u^{p_1}(0,s)} ds \\ &\leq u^{1-p_1}(0,\tau) - \frac{p_1}{p_1 - 1}\left[u^{1-p_1}(0,s)\right]_t^\tau \\ &\leq \frac{p_1}{p_1-1} u^{1-p_1}(0,t).\end{aligned}$$

By letting $\tau \to T$, the lemma follows. ∎

From Lemma 2.2 and (2.2), we obtain that z satisfies

$$z_{rr} + \frac{n-1}{r} z_r + z^{p_1} \leq \frac{2p_1 K^{-1}}{p_1 - 1} \frac{1}{T-t} u^{1-p_1}(0,t). \tag{2.3}$$

We now prove a nonexistence result as follows. The case $n = 1$ is already given in [12] by a different proof.

Lemma 2.3. *Let $p > 1, \varepsilon > 0, R > 0$. Assume that $(n-2)p < n+2$. Then there exist two positive constants R_0 and ε_0 depending only on p such that there is no solution $z \in C^2([0, R])$ to the problem*

$$\begin{cases} z_{rr} + \frac{n-1}{r} z_r + z^p \leq \varepsilon, \quad 0 < r < R, \\ z \geq 0, \quad z_r \leq 0, \quad 0 < r < R, \\ z(0) = 1, \quad z_r(0) = 0, \end{cases}$$

if $R \geq R_0$ and $\varepsilon \leq \varepsilon_0$.

Proof. Recall from [7] that the solution to the initial value problem

$$w'' + \frac{n-1}{r} w' + w^p = h, \ 0 < r < R, \tag{2.4}$$

$$w'(0) = 0, \ w(0) = 1. \tag{2.5}$$

is decreasing in r and becomes negative at R_1 for some large R_1 (depending on p), if $h = 0$ and $(n-2)p < n+2$. Here it is realized that $w^p := |w|^{p-1}w$ for any $w \in \mathbf{R}$. It follows from the theory of continuous dependence on parameter for the initial value problem that there exists a positive constant ε_0 such that the same property holds for the solution of (2.4)-(2.5) whenever $|h| \le \varepsilon_0$.

On the other hand, multiplying (2.4) by r^{n-1} and integrating it from 0 to s, we obtain

$$s^{n-1}w'(s) = -\int_0^s r^{n-1}w^p(r)dr + hs^n/n \le hs^n/n, \quad s>0,$$

and so $w'(s) \le hs/n$ for all $s>0$. It follows that

$$w(R) \le 1 + hR^2/(2n) < 0, \quad \text{if } h \le -\varepsilon_0 \text{ and } R > \sqrt{2n/\varepsilon_0}.$$

Therefore, the lemma follows by taking $R_0 = \max\{R_1, \sqrt{2n/\varepsilon_0}+1\}$. □

Note that $z(0,t)=1, z_r(0,t)=0$ and $z_r \le 0 \le z$. Applying Lemma 2.3 with $R=R_0$, we obtain from (2.3) that

$$\frac{2p_1K^{-1}}{p_1-1}\frac{1}{T-t}u^{1-p_1}(0,t) > \varepsilon_0,$$

where ε_0, R_0 are the constants defined in Lemma 2.3. Therefore, we obtain the estimate

$$u(0,t) \le \left(\frac{2p_1}{\varepsilon_0(p_1-1)}\right)^{1/(p_1-1)} K^{-1/(p_1-1)}(T-t)^{-1/(p_1-1)}.$$

This completes the proof of Theorem 1.1.

3. Proof of Theorem 1.2

In this section, we shall give a proof of Theorem 1.2.

Proof of Theorem 1.2. Following [5] (see also [13]), we consider the functions

$$G := u_t - \varepsilon u^{p_1}v^{q_1} \text{ and } J := v_t - \varepsilon u^{p_2}v^{q_2}.$$

Then these functions satisfy following identities

$$\begin{cases} G_t - \Delta G = p_1u^{p_1-1}v^{q_1}G + q_1u^{p_1}v^{q_1-1}J + \varepsilon p_1(p_1-1)u^{p_1-2}|\nabla u|^2v^{q_1} \\ \qquad\qquad +\varepsilon q_1(q_1-1)u^{p_1}|\nabla v|^2v^{q_1-2} + 2\varepsilon p_1q_1u^{p_1-1}v^{q_1-1}\nabla u\cdot\nabla v \\ J_t - \Delta J = p_2u^{p_2-1}v^{q_2}G + q_2u^{p_2}v^{q_2-1}J + \varepsilon p_2(p_2-1)u^{p_2-2}|\nabla u|^2v^{q_2} \\ \qquad\qquad +\varepsilon q_2(q_2-1)u^{p_2}|\nabla v|^2v^{q_2-2} + 2\varepsilon p_2q_2u^{p_2-1}v^{q_2-1}\nabla u\cdot\nabla v. \end{cases}$$

Since u and v are radial and decreasing in $|x|$, we have $\nabla u\cdot\nabla v = u_rv_r \ge 0$. Using this positivity and $p_i, q_i > 1$, we obtain

$$\begin{cases} G_t - \Delta G \ge p_1u^{p_1-1}v^{q_1}G + q_1u^{p_1}v^{q_1-1}J, \\ J_t - \Delta J \ge p_2u^{p_2-1}v^{q_2}G + q_1u^{p_2}v^{q_2-1}J. \end{cases} \tag{3.1}$$

Also, $G(x,0) = \Delta u_0 + u_0^{p_1}v_0^{q_1} - \varepsilon u_0^{p_1}v_0^{q_1} \ge 0$ and $J(x,0) = \Delta v_0 + u_0^{p_2}v_0^{q_2} - \varepsilon u_0^{p_2}v_0^{q_2} \ge 0$, by the assumption (1.7). By the maximum principle, we obtain that G and J are non-negative. Therefore, u and v satisfy

$$u_t \ge \varepsilon u^{p_1}v^{q_1}, \quad v_t \ge \varepsilon u^{p_2}v^{q_2} \quad \text{in } \mathbf{R}^n \times (0,T). \tag{3.2}$$

By (1.3), one can show that 0 is the unique maximum point for both u and v. Hence $\Delta u(0,t)$ and $\Delta v(0,t)$ are non-positive. Therefore, we obtain that

$$u_t(0,t) \le u^{p_1}(0,t)v^{q_1}(0,t) \text{ and } v_t(0,t) \le u^{p_2}(0,t)v^{q_2}(0,t) \tag{3.3}$$

By (3.2) and (3.3), we deduce that

$$\varepsilon v^{q_1-q_2}(0,t)v_t(0,t) \le u^{p_2-p_1}(0,t)u_t(0,t) \le \frac{1}{\varepsilon}v^{q_1-q_2}(0,t)v_t(0,t). \tag{3.4}$$

From the second inequality in (3.4), we have

$$\frac{u(0,t)^{p_2-p_1+1}}{p_2-p_1+1} \le \frac{v(0,t)^{q_1-q_2+1} - v_0(0)^{q_1-q_2+1}}{\varepsilon(q_1-q_2+1)} + \frac{u_0(0)^{p_2-p_1+1}}{p_2-p_1+1}.$$

Since we always assume that u blows up at T, we have $u(0,t) \to \infty$ as $t \to T$. Then v must tend to ∞ as $t \to T$, because $p_2 - p_1 + 1 > 0$ and $q_1 - q_2 + 1 > 0$. So, u and v blow up simultaneously and there exists a positive constant C_1 such that

$$u^{p_2-p_1+1}(0,t) \le C_1 v^{q_1-q_2+1}(0,t), \quad 0 < T-t \ll 1. \tag{3.5}$$

Similarly, we can show that there exists a positive constant C_2 such that

$$v^{q_1-q_2+1}(0,t) \le C_2 u^{p_2-p_1+1}(0,t), \quad 0 < T-t \ll 1. \tag{3.6}$$

By (3.2), (3.5) and (3.6), we obtain

$$u_t(0,t) \ge C_3 u^{(\alpha+1)/\alpha}(0,t), \quad v_t(0,t) \ge C_4 v^{(\beta+1)/\beta}(0,t).$$

By integrating the above inequalities, we obtain the following upper bound estimates

$$u(0,t) \le C(T-t)^{-\alpha},\ v(0,t) \le C(T-t)^{-\beta},\ 0 < T-t \ll 1,$$

where C is a positive constant depending only on $\varepsilon, p_1, p_2, q_1, q_2, u_0$ and v_0. Similarly, by (3.3), (3.5) and (3.6), we can obtain the lower bound estimates

$$u(0,t) \ge c(T-t)^{-\alpha},\ v(0,t) \ge c(T-t)^{-\beta},\ 0 < T-t \ll 1,$$

where c is a positive constant depending only on $\varepsilon, p_1, p_2, q_1, q_2, u_0$ and v_0. This completes the proof of Theorem 1.2. ∎

4. Criteria for ODE System

In this section, we consider the initial value problem with positive initial data for the corresponding ODE system to (1.1):

$$u_t = u^{p_1}v^{q_1}, \tag{4.1}$$

$$v_t = u^{p_2}v^{q_2}, \tag{4.2}$$

where $p_1, p_2, q_1, q_2 \ge 0$. Then we have the following theorem on the criteria of simultaneous blowup.

Theorem 4.1. *Suppose that the solution (u, v) of (4.1)-(4.2) with positive initial data (u_0, v_0) blows up in finite time. Then simultaneous blowup occurs if and only if either*

$$p_1 \le p_2 + 1 \quad \text{and} \quad q_2 \le q_1 + 1, \tag{4.3}$$

or

$$p_1 > p_2 + 1, \quad q_2 > q_1 + 1 \quad \text{and} \quad \frac{u_0^{p_2-p_1+1}}{p_2 - p_1 + 1} = \frac{v_0^{q_1-q_2+1}}{q_1 - q_2 + 1}. \tag{4.4}$$

Proof. By assumption, the solution (u, v) of (4.1)-(4.2) must be positive for all $t \ge 0$ and is increasing in t. Then, by integrating the identity

$$u^{p_2-p_1} u_t = v^{q_1-q_2} v_t$$

from 0 to t for some $t > 0$, we obtain

$$[u(t)^{p_2-p_1+1} - u_0^{p_2-p_1+1}]/(p_2 - p_1 + 1) = [v(t)^{q_1-q_2+1} - v_0^{q_1-q_2+1}]/(q_1 - q_2 + 1), \tag{4.5}$$
$$\text{if } p_2 - p_1 + 1 \ne 0 \text{ and } q_1 - q_2 + 1 \ne 0;$$

$$[u(t)^{p_2-p_1+1} - u_0^{p_2-p_1+1}]/(p_2 - p_1 + 1) = \ln v(t) - \ln v_0, \tag{4.6}$$
$$\text{if } p_2 - p_1 + 1 \ne 0 \text{ and } q_1 - q_2 + 1 = 0;$$

$$\ln u(t) - \ln u_0 = [v(t)^{q_1-q_2+1} - v_0^{q_1-q_2+1}]/(q_1 - q_2 + 1), \tag{4.7}$$
$$\text{if } p_2 - p_1 + 1 = 0 \text{ and } q_1 - q_2 + 1 \ne 0;$$

$$\ln u(t) - \ln u_0 = \ln v(t) - \ln v_0, \quad \text{if } p_2 - p_1 + 1 = 0 \text{ and } q_1 - q_2 + 1 = 0. \tag{4.8}$$

First, suppose that $p_2 - p_1 + 1 \ge 0$ and $q_1 - q_2 + 1 \ge 0$. If u blows up at T, then v must blow up at the same time T, by using (4.5)-(4.8). Similarly, if v blows up at T, then u must blows up at the same time T.

Next, suppose that $p_2 - p_1 + 1 < 0$ and $q_1 - q_2 + 1 < 0$. Then, using (4.5), it is easy to show that u and v blows up simultaneously if and only if the condition

$$\frac{u_0^{p_2-p_1+1}}{p_2 - p_1 + 1} = \frac{v_0^{q_1-q_2+1}}{q_1 - q_2 + 1}$$

holds.

Conversely, it follows from (4.5)-(4.8) that simultaneous blowup occurs only if either (4.3) or (4.4) holds. This completes the proof of the theorem. ∎

The following criteria for non-simultaneous blowup is a direct consequence of Theorem 4.1.

Corollary 4.1. *Suppose that the solution (u, v) of (4.1)-(4.2) with positive initial data (u_0, v_0) blows up in finite time. Then non-simultaneous blowup occurs if and only if one of the following statements holds:*

(1) $p_1 \le p_2 + 1$ *and* $q_2 > q_1 + 1$;
(2) $p_1 > p_2 + 1$ *and* $q_2 \le q_1 + 1$;
(3) $p_1 > p_2 + 1$, $q_2 > q_1 + 1$ *and* $(u_0^{p_2-p_1+1})/(p_2 - p_1 + 1) \ne (v_0^{q_1-q_2+1})/(q_1 - q_2 + 1)$.

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