THE SIGN OF THE WAVE SPEED FOR THE LOTKA-VOLTERRA COMPETITION-DIFFUSION SYSTEM

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Abstract. In this paper, we study the traveling front solutions of the Lotka-Volterra competition-diffusion system with bistable nonlinearity. It is well-known that the wave speed of traveling front is unique. Although little is known for the sign of the wave speed. In this paper, we first study the standing wave which gives some criteria when the speed is zero. Then, by the monotone dependence on parameters, we obtain some criteria about the sign of the wave speed under some parameter restrictions.

1. Introduction

In this paper, we study the following Lotka-Volterra competition-diffusion system

$$\begin{cases} u_t = u_{xx} + u(1-u-kv), \\ v_t = dv_{xx} + av(1-v-hu), \end{cases} \tag{1.1}$$

where $u = u(x,t)$ and $v = v(x,t)$ represent population densities of two competing species, and a, h, k, d are positive constants with certain ecological meanings. Indeed, a is the intrinsic growth rate and d is the diffusion coefficient of the species v, h is the inter-specific competition coefficient of the species u and k is the inter-specific competition coefficient of the species v.

The relations of parameters h and k influence the asymptotic behaviors of (u,v). In fact, for (1.1) with initial data $(u,v)(x,0) \geq 0$, the asymptotic behaviors of (u,v) can be divided into the following four cases:

(1) If $k < 1 < h$, then $\lim_{t\to\infty}(u,v)(x,t) = (1,0)$,
(2) If $h < 1 < k$, then $\lim_{t\to\infty}(u,v)(x,t) = (0,1)$,
(3) If $\min\{h,k\} > 1$, then $(1,0)$ and $(0,1)$ are locally stable and almost every solution converges to one of them as $t \to \infty$.
(4) If $\max\{h,k\} < 1$, then

$$\lim_{t\to\infty}(u,v)(x,t) = (\frac{1-k}{1-hk}, \frac{1-h}{1-hk}),$$

The system (1.1) has been studied very extensively with monostable or bistable nonlinearity. For instance see [1, 2, 3, 4, 5, 6, 8, 9, 11, 12] and references therein. Throughout this paper, we only focus on the bistable nonlinearity. In other words, the parameters h and k satisfy the bistability condition $\min\{h,k\} > 1$.

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We say that $(u,v)(x,t)$ is a traveling front solution of (1.1) with speed s if $(u,v)(x,t) = (U,V)(\xi)$, where $\xi = x - st$ for some functions U, V (called *wave profiles*), such that $(U,V)(\pm\infty) \in \{(1,0),(0,1)\}$ and $(U,V)(\infty) \neq (U,V)(-\infty)$. Therefore, the traveling front problem $(\mathbf{P})$ can be written as the system:

$$U'' + sU' + U(1 - U - kV) = 0 < U', \tag{1.2}$$

$$dV'' + sV' + aV(1 - V - hU) = 0 > V', \tag{1.3}$$

with the boundary condition

$$(U,V)(-\infty) = (0,1),\ (U,V)(+\infty) = (1,0), \tag{1.4}$$

where $(a,h,k,d) \in \mathcal{P} := \{(a,h,k,d) \mid a > 0,\ h > 1,\ k > 1,\ d > 0\}$ and $s = s(a,h,k,d)$. For the study of traveling front solution of (1.1), we refer to, e.g., [1, 2, 5, 6, 10].

By the change of the variables $(\tilde{U},\tilde{V}) = (U, aV)$, problem $(\mathbf{P})$ is reduced to the following problem $(\tilde{\mathbf{P}})$:

$$\tilde{U}'' + \tilde{s}\tilde{U}' + \tilde{U}(1 - \tilde{U} - c\tilde{V}) \ = \ 0 < \tilde{U}', \tag{1.5}$$

$$d\tilde{V}'' + \tilde{s}\tilde{V}' + \tilde{V}(a - b\tilde{U} - \tilde{V}) \ = \ 0 > \tilde{V}', \tag{1.6}$$

with

$$(\tilde{U},\tilde{V})(-\infty) = (0,a),\ (\tilde{U},\tilde{V})(+\infty) = (1,0), \tag{1.7}$$

where $(a,b,c,d) \in \tilde{\mathcal{P}} := \{(a,b,c,d) \mid 0 < 1/c < a < b,\ d > 0\}$, $\tilde{s} = \tilde{s}(a,b,c,d) = s$. Here, for given a and d, we have the following relations between parameters (h,k) and (b,c):

$$(h,k) = (b/a, ac), \quad (b,c) = (ah, k/a). \tag{1.8}$$

We now recall some known results as follows. In [1] or [2], they proved the existence of traveling front solutions. In [10] (or [6]), they studied the existence of traveling front solutions with $s = 0$ (i.e. the standing wave). In [5], Kan-on derived the monotone dependence of the wave speed on the parameters a, b, c. In [11], some exact solutions of (1.5)-(1.7) are given and the wave speed can also be represented explicitly under some parameter restrictions. But, little is known about the sign of the wave speed. In fact, the speed sign is an important matter because it decides which species becomes dominant and eventually occupies the whole domain. From the biological point of view, when the speed s is positive, the species v is dominant and the species u goes extinct eventually for those initial distributions which are close to traveling waves in an appropriate function space (cf. [2, 7]). On the other hand, the species u wins the competition in the above sense when the speed s is negative.

Now, we list the main theorems of this paper as follows.

Theorem 1. *Suppose that $a = d$. Then we have*

$$s(a,h,k,d) = \begin{cases} > 0, & \text{if} \quad k > h > 1; \\ = 0, & \text{if} \quad h = k > 1; \\ < 0, & \text{if} \quad h > k > 1. \end{cases}$$

Theorem 2. (i). *Suppose that $a > d$. Then $s(a,h,k,d) > 0$, if $h > 1$ and $k \geq (a/d)^2 h$.*
(ii). *Suppose that $a < d$. Then $s(a,h,k,d) < 0$, if $k > 1$ and $h \geq (d/a)^2 k$.*

Theorem 3. *For any $l > 0$, $s(a,h,k,d)$ and $s(la,h,k,ld)$ have the same sign.*

Theorem 4. *Suppose that $a > d$. If $1 < h \leq 1 + d/a$ and $k \geq 2$, then $s(a,h,k,d) > 0$.*

Note that the condition in Theorem 4 is not *totally* included in the condition in Theorem 2 for $a > d$, since $(1 + d/a)(a/d)^2 > 2$. More precisely, we have

$$\{(h,k) \mid 1 < h \leq 1 + d/a, k \geq 2\} \setminus \{(h,k) \mid h > 1, k \geq (a/d)^2 h\} \neq \emptyset.$$

Theorem 5. *Suppose that $a > d$. If $h > 1$, $k \geq 5a/d$ and $(3ah - d)h \leq (4a - d)k$, then $s(a,h,k,d) > 0$.*

Rewriting the set $\{(h,k) \mid h > 1, k \geq 5a/d, (3ah - d)h \leq (4a - d)k\}$ by

$$\left\{ (h,k) \middle| \, h > 1, \; k \geq \frac{5a}{d}, \; k \geq \frac{3a}{4a-d} \left(h - \frac{d}{6a} \right)^2 - \frac{d^2}{12a(4a-d)} \right\},$$

we also see that the condition in Theorem 5 is not *totally* included in the condition in Theorem 2, if $a > d$ and $(4a - d)a^2 > 14d^3$.

Theorem 6. *If $a = d/4$, then we have*

$$s(a,h,k,d) = \begin{cases} > 0, & \text{if } 1 < h \leq 4/3 \text{ and } k \geq 5/4, \text{ except } (h,k) = (4/3, 5/4); \\ = 0, & \text{if} \quad h = 4/3, \; k = 5/4; \\ < 0, & \text{if } h \geq 4/3 \text{ and } 1 < k \leq 5/4, \text{ except } (h,k) = (4/3, 5/4). \end{cases}$$

Theorem 6 shows that, when $a = d/4$, we have $s(a,h,k,d) < 0$ in the region

$$\{(h,k) \mid h \in [4/3, 16], k \in (1, 5/4]\} \cup \{(h,k) \mid 16 < h < 16k, k \in (1, 5/4]\} \setminus \{(4/3, 5/4)\}$$

which is not contained in the set obtained in Theorem 2 for $a < d$.

We organize this paper as follows. In Section 2, we give some preliminaries and some results about the standing wave in terms of parameters. Next, in Section 3, we offer the proofs of the main Theorems. Our strategy is to derive some useful information about the standing waves in terms of parameters. Then, by the monotone dependence on parameters, we can determine the sign of the speed in those special situations. There are still many cases left open.

2. Preliminaries

In this section, we recall some known results about the problem ($\tilde{\mathbf{P}}$). We say $(\tilde{U},\tilde{V})(\xi)$ is a monotone pair if $\tilde{U}(\xi)$ is increasing and $\tilde{V}(\xi)$ is decreasing. In [5], Kan-on derived the following fact that for any $(a,b,c,d)\in\tilde{\mathcal{P}}$, there exists a monotone pair $(\tilde{U},\tilde{V})(\xi;a,b,c,d)$ and $\tilde{s}=\tilde{s}(a,b,c,d)$ satisfy (1.5)-(1.7). Moreover, $\tilde{s}=\tilde{s}(a,b,c,d)$ is unique and $(\tilde{U},\tilde{V})(\xi;a,b,c,d)$ is also unique up to translation. On the other hand, for any $d>0$ and for any positive numbers b,c with $bc>1$, there exists a unique positive number $\bar{a}=\bar{a}(b,c,d)\in(1/c,b)$ such that $\tilde{s}(\bar{a},b,c,d)=0$.

As for the monotone dependence on parameters about $\tilde{s}(a,b,c,d)$, we have

$$\frac{\partial}{\partial a}\tilde{s}(a,b,c,d)>0,\ \frac{\partial}{\partial b}\tilde{s}(a,b,c,d)<0,\ \frac{\partial}{\partial c}\tilde{s}(a,b,c,d)>0, \tag{2.1}$$

if $(a,b,c,d)\in\tilde{\mathcal{P}}$. From this, we also have the following property about $s=s(a,h,k,d)$:

$$\frac{\partial}{\partial k}s(a,h,k,d)>0>\frac{\partial}{\partial h}s(a,h,k,d), \tag{2.2}$$

if $(a,h,k,d)\in\mathcal{P}$. But, we do not know the monotone dependence on the parameter a for $s(a,h,k,d)$.

Now, we focus on the case when $s(a,h,k,d)=0$. The following lemma can be proved by the uniqueness of wave speed and a suitable change of variables.

Lemma 2.1. *There hold:*

(1) $s(1,h,h,1)=0$ *for all* $h>1$,
(2) *If* $s(a,h,k,d)=0$ *for some* $(a,h,k,d)\in\mathcal{P}$, *then* $s(d,k,h,a)=0$ *and* $s(la,h,k,ld)=0$ *for all* $l>0$.

In particular, $s(d,h,h,d)=0$ *for all* $d>0,h>1$.

Proof. (1) Let $(s,U(\xi),V(\xi))$ be a solution of ($\mathbf{P}$) with $a=d=1$ and $k=h$ for some $h>1$. Then, by setting $(U_1,V_1)(\xi):=(V,U)(-\xi)$, the functions U_1 and V_1 satisfy the following system

$$\begin{cases} 0=U_1''+(-s)U_1'+U_1(1-U_1-hV_1),\\ 0=V_1''+(-s)V_1'+V_1(1-V_1-hU_1),\end{cases}$$

with

$$(U_1,V_1)(-\infty)=(0,1),\ (U_1,V_1)(+\infty)=(1,0).$$

By the uniqueness of wave speed, we have $s(1,h,h,1)=0$ for all $h>1$.

(2) Suppose $s(a,h,k,d)=0$ for some $(a,h,k,d)\in\mathcal{P}$. That is to say, there exist functions U_2 and V_2 satisfy the following system

$$\begin{cases} 0=U_2''+U_2(1-U_2-kV_2),\\ 0=dV_2''+aV_2(1-V_2-hU_2),\end{cases}$$

with

$$(U_2, V_2)(-\infty) = (0, 1),\ (U_2, V_2)(+\infty) = (1, 0).$$

By defining $(U_3, V_3)(\xi) := (V_2, U_2)(-\sqrt{d/a}\ \xi)$, the functions U_3 and V_3 satisfy the following system

$$\begin{cases} 0 = U_3'' + U_3(1 - U_3 - hV_3), \\ 0 = aV_3'' + dV_3(1 - V_3 - kU_3), \end{cases}$$

with

$$(U_3, V_3)(-\infty) = (0, 1),\ (U_3, V_3)(+\infty) = (1, 0).$$

Again, by the uniqueness of wave speed, we have $s(d, k, h, a) = 0$.

At the same time, the functions U_2 and V_2 also satisfy the following system

$$\begin{cases} 0 = U_2'' + U_2(1 - U_2 - kV_2), \\ 0 = (ld)V_2'' + (la)V_2(1 - V_2 - hU_2) \end{cases}$$

for any positive number l. By the uniqueness, we also have $s(la, h, k, ld) = 0$ for all $l > 0$.

Finally, it follows from (1) and (2) that $s(d, h, h, d) = 0$ for all $d > 0, h > 1$. Therefore, the lemma follows. □

Next, we study the following problem $(\mathbf{P_0})$ when $s = 0$:

$$U'' + U(1 - U - kV) = 0 < U', \tag{2.3}$$

$$V'' + rV(1 - V - hU) = 0 > V', \tag{2.4}$$

with

$$(U, V)(-\infty) = (0, 1),\ (U, V)(+\infty) = (1, 0),$$

where $r \neq 1$, $h > 1$, $k > 1$. Note that $r = a/d$.

Proposition 2.2. *If (U, V) is a solution of $(\mathbf{P_0})$, then*

$$\int_{-\infty}^{\infty} U^2V' = -\frac{1}{3k},\ \int_{-\infty}^{\infty} U'V^2 = \frac{1}{3h}, \tag{2.5}$$

$$\left(\frac{k}{3} - r\right)\int_{-\infty}^{\infty} U'V^3 + (1 - rh)\int_{-\infty}^{\infty} UU'V^2 - \int_{-\infty}^{\infty} U'V'V' = \frac{1 - 2r}{6h}, \tag{2.6}$$

$$\frac{2r}{3}\int_{-\infty}^{\infty} U'V^3 + 2rh\int_{-\infty}^{\infty} UU'V^2 - \int_{-\infty}^{\infty} U'V'V' = \frac{r}{3h}. \tag{2.7}$$

Proof. First, multiplying (2.3) by U' and integrating it over $(-\infty, +\infty)$, we get

$$\int_{-\infty}^{\infty} U'U'' + \int_{-\infty}^{\infty} U(1 - U)U' = k\int_{-\infty}^{\infty} UU'V.$$

Then, by the boundary condition and the integration by parts, we can easily obtain

$$\int_{-\infty}^{\infty} U^2V' = -\frac{1}{3k}.$$

The case for the value of $\int_{-\infty}^{\infty} U'V^2$ is similar. This proves (2.5).

Next, multiplying (2.3) by VV' and integrating this equation over $(-\infty,\infty)$, we have

$$\int_{-\infty}^{\infty} U''VV' + \int_{-\infty}^{\infty} UVV' - \int_{-\infty}^{\infty} U^2VV' - k\int_{-\infty}^{\infty} UV^2V' = 0.$$

By (2.4) and the integration by parts, we get

$$\begin{aligned} &-\int_{-\infty}^{\infty} U'V'V' + r\int_{-\infty}^{\infty} U'V^2(1-V-hU) \\ &+\int_{-\infty}^{\infty} UVV' + \int_{-\infty}^{\infty} UU'V^2 + \frac{k}{3}\int_{-\infty}^{\infty} U'V^3 = 0. \end{aligned}$$

Hence (2.6) follows from (2.5) and a direct computation.

Finally, multiplying (2.4) by UV' and integrating this equation over $(-\infty,\infty)$, it follows from (2.5) and the integration by parts that (2.7) holds. This completes the proof. □

3. Proofs of Main Theorems

In this section, we give the proofs of the main theorems stated in Section 1. The main idea is to apply the information of standing wave with the help of the monotone dependence on parameters.

3.1. **Proof of Theorem 1.** The theorem follows from Lemma 2.1 and (2.2). □

3.2. **Proof of Theorem 2.** First, recall from Lemma 2.1(2) that $s(d,h,h,d) = 0$ for all $h > 1$, $d > 0$. It follows from (1.8) that $\tilde{s}(d,dh,h/d,d) = 0$ for all $h > 1$, $d > 0$. Now, using (2.1) in $\tilde{\mathcal{P}}$, we have

$$\tilde{s}(a,d\hat{h},\hat{h}/d,d) > 0,$$

if $a > d > 0$ and $\hat{h} > a/d$. This is equivalent to

$$s(a,h,k,d) > 0, \quad h = d\hat{h}/a, \quad k = a\hat{h}/d,$$

if $a > d > 0$ and $\hat{h} > a/d$, due to (1.8).

Next, for $a > d > 0$ and $h > 1$, we choose $\hat{h} = ah/d$ so that $\hat{h} > a/d$. Then $s(a,h,k,d) > 0$ when $k = a\hat{h}/d = (a/d)^2h$. It follows from (2.2) that $s(a,h,k,d) > 0$, if $a > d > 0$ and

$$h > 1, \quad k \geq (a/d)^2h.$$

Similarly, we can prove that $s(a,h,k,d) < 0$, if $0 < a < d$, $k > 1$ and $h \geq (d/a)^2k$. This proves Theorem 2. □

3.3. **Proof of Theorem 3.** When $a = d$, the conclusion follows from Theorem 1. Without loss of generality, we may assume $a > d > 0$. Suppose the conclusion is false. By Lemma 2.1(2), there exist $a_1 > d_1 > 0,\ h_1 > 1,\ k_1 > 1,\ l_1 > 0$ such that $s(a_1, h_1, k_1, d_1) > 0 > s(l_1a_1, h_1, k_1, l_1d_1)$. By (2.2), we have $s(a_1, h, k, d_1) > 0$ if $h = h_1,\ k \geq k_1$ or $1 < h \leq h_1,\ k = k_1$. Similarly, we also have $s(l_1a_1, h, k, l_1d_1) < 0$ if $h = h_1,\ 1 < k \leq k_1$ or $h \geq h_1,\ k = k_1$. Combining these facts with Lemma 2.1(2) and the continuous dependence on h and k, we have

$$s(a_1, h, k, d_1) > 0 > s(l_1a_1, h, k, l_1d_1), \tag{3.1}$$

if $(h, k) \in \{h = h_1, k > 1\} \cup \{h > 1, k = k_1\}$. Indeed, this can be proved by a contradiction argument. Otherwise, there exists $k \in (1, k_1)$ such that $s(a_1, h_1, k, d_1) = 0$. Then, by Lemma 2.1(2), we have $s(l_1a_1, h_1, k, l_1d_1) = 0$, a contradiction. The other cases are similar. Hence (3.1) follows.

Next, we choose two positive numbers $\tilde{h}$ and k_2 with $\tilde{h} > h_1$ and $k_2 > k_1$ such that $d^2k_2 \geq a^2\tilde{h}$. By Theorem 2, we have $s(l_1a_1, \tilde{h}, k_2, l_1d_1) > 0$. But, by (3.1), we have $s(l_1a_1, h_1, k_2, l_1d_1) < 0$. Since $s(a, h, k, d)$ is continuous on h, there exists $h_2 \in (h_1, \tilde{h})$ such that $s(l_1a_1, h_2, k_2, l_1d_1) = 0$. This implies that $s(a_1, h_2, k_2, d_1) = 0$, by Lemma 2.1(2). On the other hand, (3.1) implies that $s(a_1, h_2, k_1, d_1) > 0$. It follows from (2.2) that $s(a_1, h_2, k_2, d_1) > 0$, a contradiction. So the theorem follows. □

3.4. **Proof of Theorem 4.** First, we define $m := (a/d) - 1 > 0$. Then, multiplying (1.2) by U^mV' and (1.3) by $(1/d)U^mU'$, respectively, and integrating it over $(-\infty, \infty)$, we obtain

$$\begin{aligned}
0 \;=\;& \int_{-\infty}^{\infty} (U'V')'U^m + \left(1 + \frac{1}{d}\right) s \int_{-\infty}^{\infty} U^mU'V' + \left(\int_{-\infty}^{\infty} U^{m+1}V' + \frac{a}{d}\int_{-\infty}^{\infty} U^mU'V\right) \\
&+ \left(-\int_{-\infty}^{\infty} U^{m+2}V' - \frac{ah}{d}\int_{-\infty}^{\infty} U^{m+1}U'V\right) + \left(-k\int_{-\infty}^{\infty} U^{m+1}VV' - \frac{a}{d}\int_{-\infty}^{\infty} U^mU'V^2\right) \\
=\;& -m\int_{-\infty}^{\infty} U^{m-1}U'U'V' + \left(1 + \frac{1}{d}\right) s\int_{-\infty}^{\infty} U^mU'V' + \left(m + 2 - \frac{ah}{d}\right)\int_{-\infty}^{\infty} U^{m+1}U'V \\
&+ \left(\frac{(m+1)k}{2} - \frac{a}{d}\right)\int_{-\infty}^{\infty} U^mU'V^2 \\
=\;& -m\int_{-\infty}^{\infty} U^{m-1}U'U'V' + \left(1 + \frac{1}{d}\right) s\int_{-\infty}^{\infty} U^mU'V' + \left(\frac{a + d - ah}{d}\right)\int_{-\infty}^{\infty} U^{m+1}U'V \\
&+ \frac{a(k-2)}{2d}\int_{-\infty}^{\infty} U^mU'V^2.
\end{aligned}$$

Using the fact $U' > 0 > V'$, the theorem follows. □

3.5. **Proof of Theorem 5.** Suppose that (U, V) is a solution of $(\mathbf{P_0})$ and that $r = a/d > 1$. Then, by (2.6) and (2.7), we have

$$\int_{-\infty}^{\infty} UU'V^2 = \frac{4r-1}{6h(3rh-1)} + \frac{k-5r}{3(3rh-1)} \int_{-\infty}^{\infty} U'V^3. \tag{3.2}$$

Using $0 < V < 1$ and the integration by parts, it follows from (2.5) that

$$\int_{-\infty}^{\infty} UU'V^2 < \int_{-\infty}^{\infty} UU'V = -\frac{1}{2}\int_{-\infty}^{\infty} U^2V' = \frac{1}{6k}. \tag{3.3}$$

If we assume that $k \geq 5r$, then, by (3.2) and (3.3), we obtain

$$(3rh-1)h > (4r-1)k.$$

Hence we see that

$$s(a, h, k, d) \neq 0 \text{ if } \quad a > d > 0,\ h > 1,\ k \geq 5a/d,\ (3ah-d)h \leq (4a-d)k. \tag{3.4}$$

Suppose that $s(a_1, h_1, k_1, d_1) < 0$ for some (a_1, h_1, k_1, d_1) such that

$$a_1 > d_1 > 0,\ h_1 > 1,\ k_1 \geq 5a_1/d_1,\ (3a_1h_1 - d_1)h_1 \leq (4a_1 - d_1)k_1.$$

Then, by Theorem 4, we can choose a positive number h_2 with $1 < h_2 < \min\{h_1, 1 + d_1/a_1\}$ such that $s(a_1, h_2, k_1, d_1) > 0$. Since $s(a, h, k, d)$ is continuous on h, there exists a positive number $\tilde{h} \in (h_2, h_1)$ such that $s(a_1, \tilde{h}, k_1, d_1) = 0$. This contradicts (3.4), since $(3a_1\tilde{h} - d_1)\tilde{h} \leq (4a_1 - d_1)k_1$. Hence the theorem follows. $\square$

3.6. **Proof of Theorem 6.** When $h = 1/(3r)$ and $k = 5r$, it follows from (2.6) and (2.7) that $r = 1/4$. Let $d > 0$ be given. Since $r = a/d$, we have

$$s(a, \frac{d}{3a}, \frac{5a}{d}, d) \neq 0 \quad \text{if} \quad a \in (d/5, d/3) \setminus \{d/4\}.$$

By the relation (1.8), we obtain

$$\tilde{s}(a, \frac{d}{3}, \frac{5}{d}, d) \neq 0 \quad \text{if} \quad a \in (d/5, d/3) \setminus \{d/4\}.$$

However, for any positive numbers b, c with $bc > 1$, there exists a unique positive number $\bar{a} = \bar{a}(b, c, d) \in (1/c, b)$ such that $\tilde{s}(\bar{a}, b, c, d) = 0$. So $\tilde{s}(d/4, d/3, 5/d, d) = 0$ for all $d > 0$. In other words, $s(d/4, 4/3, 5/4, d) = 0$ for all $d > 0$. Hence the theorem follows from the monotone dependence on parameters (2.2). $\square$

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