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New Product Diffusion Models in Marketing: A Review and Directions for Research

Since the publication of the Bass model in 1969, research on the modeling of the diffusion of innovations has resulted in a body of literature consisting of several dozen articles, books, and assorted other publications. Attempts have been made to reexamine the structural and conceptual assumptions and estimation issues underlying the diffusion models of new product acceptance. The authors evaluate these developments for the past two decades. They conclude with a research agenda to make diffusion models theoretically more sound and practically more effective and realistic.

THE diffusion of an innovation traditionally has been defined as the process by which that innovation "is communicated through certain channels over time among the members of a social system" (Rogers 1983, p. 5). As such, the diffusion process consists of four key elements: innovation, communication channels, time, and the social system.

As a theory of communications, diffusion theory's main focus is on communication channels, which are the means by which information about an innovation is transmitted to or within the social system. These means consist of both the mass media and interpersonal communications. Members of a social system have different propensities for relying on mass media or interpersonal channels when seeking information about an innovation. Interpersonal communications,

including nonverbal observations, are important influences in determining the speed and shape of the diffusion process in a social system.

Since its introduction to marketing in the 1960s (Arndt 1967; Bass 1969; Frank, Massy, and Morrison 1964; King 1963; Robertson 1967; Silk 1966), innovation diffusion theory has sparked considerable research among consumer behavior, marketing management, and management and marketing science scholars. Researchers in consumer behavior have been concerned with evaluating the applicability of hypotheses developed in the general diffusion area to consumer research (Gatignon and Robertson 1985). The marketing management literature has focused on the implications of these hypotheses for targeting new product prospects and for developing marketing strategies aimed at potential adopters (see, e.g., Engel, Blackwell, and Miniard 1986, Chap. 20; Kotler and Zaltman 1976; McKenna 1985, Chap. 4). Researchers in management and marketing science have contributed to the development of diffusion theory by suggesting analytical models for describing and forecasting the diffusion of an innovation in a social system. More recently, this literature also has been concerned with developing normative guidelines for how an innovation should be diffused in a social system.

We focus on the contributions of management and

marketing science literature to the cumulative understanding of the dynamics of innovation diffusion. The main impetus underlying these contributions is a new product growth model suggested by Bass (1969). The Bass model and its revised forms have been used for forecasting innovation diffusion in retail service, industrial technology, agricultural, educational, pharmaceutical, and consumer durable goods markets (Akinola 1986; Bass 1969; Dodds 1973; Kalish and Lilien 1986a; Lancaster and Wright 1983; Lawton and Lawton 1979; Nevers 1972; Tigert and Farivar 1981). Representative companies that have used the model include Eastman Kodak, RCA, IBM, Sears, and AT&T (Bass 1986).

Since publication of the Bass model, research on the modeling of the diffusion of innovations in marketing has resulted in an extensive literature. Contributions of this literature through the 1970s were reviewed by Mahajan and Muller (1979). However, in the ensuing decade a plethora of studies has contributed to our understanding of the structural, estimation, and conceptual assumptions underlying diffusion models. Though some of these recent developments have been documented by Mahajan and Peterson (1985) and Mahajan and Wind (1986a), we now extend these efforts by presenting a critical evaluation of the cumulative developments since the Bass (1969) and Mahajan and Muller (1979) articles. Table 1 is a summary of these developments over the last two decades across five subareas: basic diffusion models, parameter estimation considerations, flexible diffusion models, refinements and extensions, and use of diffusion models.

The Basic First-Purchase Diffusion Models

Mahajan and Muller (1979) have stated that the objective of a diffusion model is to present the level of spread of an innovation among a given set of prospective adopters over time. The purpose of the diffusion model is to depict the successive increases in the number of adopters and predict the continued development of a diffusion process already in progress. In the product innovation context, diffusion models focus on the development of a life cycle curve and serve the purpose of forecasting first-purchase sales of innovations. That is, in the first-purchase diffusion models one assumes that, in the product planning horizon being considered, there are no repeat buyers and purchase volume per buyer is one unit. The number of adopters defines the unit sales for the product. Diffusion models, by definition, are concerned with representing the growth of a product category.

The best-known first-purchase diffusion models of new product diffusion in marketing are those of Bass

(1969), Fourt and Woodlock (1960), and Mansfield (1961). These early models attempted to describe the penetration and saturation aspects of the diffusion process. After briefly reviewing the original formulations of these models, we review the recent developments that further evaluate their basic structure.[1]

The Bass Model

The main impetus underlying diffusion research in marketing is the Bass model. Subsuming the models proposed by Fourt and Woodlock (1960) and Mansfield (1961), the Bass model assumes that potential adopters of an innovation are influenced by two means of communication—mass media and word of mouth. In its development, it further assumes that the adopters of an innovation comprise two groups. One group is influenced only by the mass-media communication (external influence) and the other group is influenced only by the word-of-mouth communication (internal influence). Bass termed the first group "Innovators" and the second group "Imitators." Unlike the Bass model, the model proposed by Fourt and Woodlock (1960) assumes that the diffusion process is driven primarily by the mass-media communication or the external influence. Similarly, the model proposed by Mansfield (1961) assumes this process is driven by word of mouth.

Figure 1 is a plot of the conceptual and analytical structure underlying the Bass model. As noted in Figure 1A, the Bass model conceptually assumes that "Innovators" or buyers who adopt exclusively because of the mass-media communication or the external influence are present at any stage of the diffusion process. Figure 1B shows the analytical structure underlying the Bass model. As depicted, the noncumulative adopter distribution peaks at time T*, which is the point of inflection of the S-shaped cumulative adoption curve. Furthermore, the adopter distribution assumes that an initial pm (a constant) level of adopters buy the product at the beginning of the diffusion process. Once initiated, the adoption process is symmetric with respect to time around the peak time T* up to 2T*. That is, the shape of the adoption curve from time T* to 2T* is the mirror image of the shape of the adoption curve from the beginning of the dif

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[1]Related to the Mansfield model is the imitation model suggested by Fisher and Pry (1971) and the Gompertz curve. For applications of the Gompertz curve and its comparison with the Mansfield model, see Hendry (1972), Dixon (1980), and Ziemer (1988). Several other growth models also have been proposed in the marketing, economics, and technological substitution literatures to depict the growth phenomenon (e.g., the Weibull distribution). As some of these models either do not explicitly consider the diffusion effect in their formulation or combine other models, they are not included in our review. For applications of such models to new product growth situations, see DeKluyver (1982), Sharif and Islam (1980), Meade (1984), Lee and Lu (1987), and Skiadas (1985, 1986).

TABLE 1
Emergence of Diffusion Modeling Literature in Marketing

Research Areas	Time Period		
	1960s	1970s	1980s
Basic diffusion models	Formulation of relationship between imitators and innovators over time Saturation effect		Unbundling of adopters Definition of innovators/imitators Development of diffusion models from individual-level adoption decisions
Parameter estimation considerations	Estimation when data are available: Ordinary least squares estimation procedure		Estimation when no prior data are available: Algebraic estimation procedures Product/market attribute-based analogical estimation procedures Estimation when data are available: Time-invariant parameter estimation procedures (maximum likelihood, nonlinear least squares) Time-varying parameter estimation procedures (Bayesian estimation, feedback filters)
Flexible diffusion models			Systematic (or random) variation in diffusion model parameters over time Flexible diffusion patterns in terms of timing and magnitude of peak of adoption curve
Refinements and extensions		Dynamic diffusion models: market saturation changes over time Multi-innovation diffusion models: other innovations influence diffusion of an innovation Space/time diffusion models: diffusion of an innovation occurs simultaneously in space and time Multistage diffusion models: adopters pass through a series of stages in the innovation-decision process	Multigeneration models: timing and adoption of different generations of an innovation Multistage diffusion models: effect of negative word of mouth in the innovation decision process Diffusion models with marketing mix variables: effect of price (linkage with experience curves), advertising, personal selling, distribution, and timing of new product introduction on diffusion patterns Product/market attribute-based diffusion models: effect of social system characteristics and perceived product attributes on diffusion patterns Controlled diffusion models: effect of supply restrictions on diffusion patterns Multiadoption diffusion models: incorporation of repeat sales and replacment sales in diffusion patterns Competitive diffusion models: effect of competitive actions in terms of pricing, advertising, and number of brands on diffusion patterns
Use of diffusion models	Forecasting	Forecasting: problems in use of diffusion models for forecasting	Forecasting: problems in the use of diffusion models for forecasting Descriptive: testing of hypotheses related to diffusion of innovations across countries, effect of product/market attributes on diffusion patterns, relationship between innovation diffusion and market structure factors such as the experience curve phenomenon and proliferation of number of brands Normative: derivation of optimal pricing, advertising, and timing strategies in monopoly and oligopoly markets

fusion process up to time T* (Mahajan, Muller, and Srivastava 1990).

The Bass model derives from a hazard function (the probability that an adoption will occur at time t given that it has not yet occurred). Thus, $f(t)/[1 - F(t)] = p + qF(t)$ is the basic premise underlying the Bass model. The density function of time to adoption is given by $f(t)$ and the cumulative fraction of adopters at time t is given by $F(t)$. This basic premise states that the conditional probability of adoption at time t (the fraction of the population that will adopt at time t) is increasing in the fraction of the population that has already adopted. Therefore, the basic premise states that part of the adoption influence depends on imitation or "learning" and part of it does not. The parameter q reflects that influence and the parameter p re-

FIGURE 1
The Bass New Product Diffusion Model

A. Adoptions Due to External and Internal Influences in the Bass Model
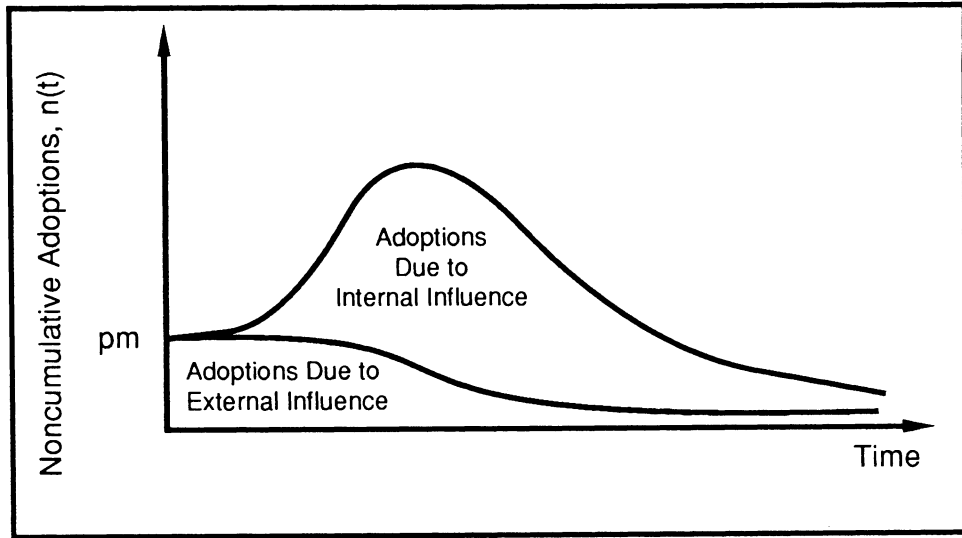
B. Analyical Structure of the Bass Model
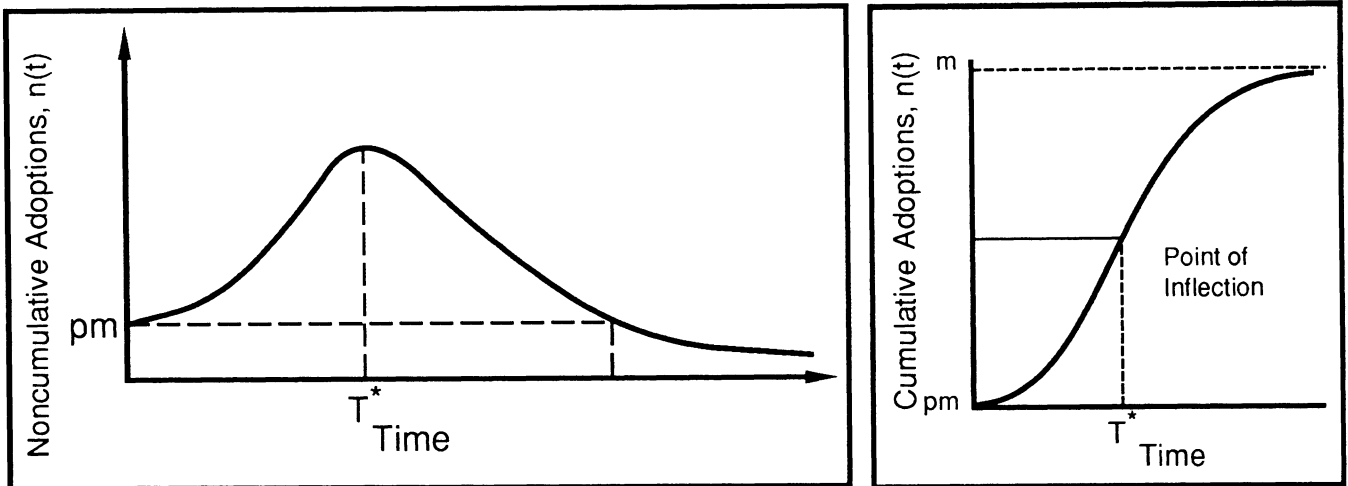
flects an influence that is independent of previous adoption. If q is zero, f(t) will follow the negative exponential distribution. If m is the potential number of ultimate adopters, the number of adopters at time t will be mf(t) = n(t) and the cumulative number of adopters at time t will be mF(t) = N(t). The basic premise of the Bass model can be manipulated, along with the definitions just provided, to yield

$$n(t) = \frac{dN(t)}{dt} = p[m - N(t)] + \frac{q}{m} N(t)[m - N(t)]. \quad (1)$$

The first term, p[m − N(t)], in equation 1 represents adoptions due to buyers who are not influenced in the timing of their adoption by the number of people who already have bought the product. Bass (1969) referred to p as the "coefficient of innovation." The second term in equation 1, q/m N(t)[m − N(t)], represents adoptions due to buyers who are influenced by the number of previous buyers. Bass (1969) referred to q as the "coefficient of imitation." Note in equation 1 that at time t = 0, n(0) = pm.

Equation 1 is a first-order differential equation. It

can be integrated to yield the S-shaped cumulative adopter distribution, N(t). Once N(t) is known, further differentiation yields expressions for the noncumulative number of adopters, n(t), and the time (T*) and magnitude (n(t*) and N(t*)) of the peak of the adoption curve.[2]

Given the basic structure of the Bass diffusion model, three questions can be raised:

- How does the Bass model compare with the classical normal distribution model proposed by Rogers (1983)?
- Is the Bass model complete in capturing the communication structure between the two assumed distinct groups of innovators and imitators?
- How can the Bass model, which captures diffusion at the aggregate level, be linked to the adoption decisions at the individual level?

Recent developments that address these three questions are discussed next.

Unbundling of Adopters

Rogers (1983, p. 244) has articulated that because of the interpersonal interaction, the adoption curve should have a normal distribution. In fact, using two basic statistical parameters of the normal distribution—mean and standard deviation—Rogers has proposed an adopter categorization scheme dividing adopters into five categories of Innovators, Early Adopters, Early Majority, Late Majority, and Laggards.

To establish the linkage between the Bass model and the classical normal distribution model, Mahajan, Muller, and Srivastava (1990) compared the two approaches. In their comparison, they highlight two points. First, they argue that adopters termed "Innovators" in the Bass model should not be called innovators because they are not necessarily the first adopters of an innovation, as defined by Rogers. Following Lekvall and Wahlbin (1973), they suggest that because the Bass model captures the spread of an innovation due to the mass media and interpersonal communication channels, the Bass model coefficients p and q should be referred to as the coefficient of external influence and the coefficient of internal influence, respectively. (We use these labels in the rest of this article.) They also provide an explicit expression to estimate the total number of adoptions due to ex-

ternal influence at any time in the diffusion process:

$$N_1(t) = m \frac{p}{q} \ln \left[\frac{1 + \frac{q}{p}}{1 + \frac{q}{p} e^{-(p+q)t}} \right],$$

where $N_1(t)$ represents adoptions due to external influence. Hence, adoptions due to internal influence are $N_2(t) = N(t) - N_1(t)$.

Second, Mahajan, Muller, and Srivastava (1990) suggest that because one standard deviation away from the mean of the normal distribution represents its points of inflection (the analytical logic underlying the categorization scheme proposed by Rogers), the same analytical logic can be used to develop adopter categories for the Bass model. This scheme also yields five adopter categories with the number of buyers (pm) who initiate the Bass model being defined as innovators. Examining the diffusion of personal computers, Mahajan, Muller, and Srivastava (1990) show how the adopter categories based on the Bass model can be used to study differences among their profiles.

Innovators Versus Imitators

Irrespective of the term "Innovators" used to label buyers who adopt because of external influence in the Bass model, a question can be raised as to whether the Bass model really captures the communication structure between the two assumed groups of adopters called "Innovators" and "Imitators." Emphasizing this argument, Tanny and Derzko (1988) suggest that the communication structure assumed in the Bass model is not complete. They propose an extension of the Bass model wherein (1) potential adopters are divided into two distinct groups of Potential Innovators (say m_1) and Potential Imitators (say m_2), (2) both Potential Innovators and Potential Imitators are influenced by the mass-media communication, and (3) *only* Potential Imitators are influenced by word of mouth due to Innovators and Imitators. To appreciate the linkage between the Bass model and its extension proposed by Tanny and Derzko (1988), consider the following rate equations they proposed.

$$\text{Innovators:} \quad \frac{dN_1(t)}{dt} = p_1[m_1 - N_1(t)] \quad (2)$$

$$\text{Imitators:} \quad \frac{dN_2(t)}{dt} = p_2[m_2 - N_2(t)] + q_2[N_1(t) + N_2(t)][m_2 - N_2(t)] \quad (3)$$

If we assume that $p_1 = p_2 = p$ (i.e., the coefficient of external influence is the same for both groups), the total adoptions can be represented by summing the two

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[2] These expressions are given by:

$$N(t) = m \left[\frac{1 - e^{-(p+q)t}}{1 + \frac{q}{p} e^{-(p+q)t}} \right], \quad n(t) = m \left[\frac{p(p + q)^2 e^{-(p+q)t}}{(p + q e^{-(p+q)t})^2} \right]$$

$$n(T^*) = \frac{1}{4q} (p + q)^2, \quad N(T^*) = m \left(\frac{1}{2} - \frac{p}{2q} \right)$$

$$T^* = -\frac{1}{(p + q)} \ln \left(\frac{p}{q} \right).$$

rate equations (and noting that $m_1 + m_2 = m$ and $N(t) = N_1(t) + N_2(t)$).

$$\frac{dN(t)}{dt} = p[m_1 + m_2 - N_1(t) - N_2(t)]$$
$$+ q_2 N(t)[m_2 - N_2(t)]$$
$$= p[m - N(t)] + q_2 N(t)[m_2 - N_2(t)] \quad (4)$$

Note that equation 4 is identical to the Bass model, equation 1, except for the fact that equation 4 considers the word-of-mouth influence on the potential adopters who are Potential Imitators rather than on all of the potential adopters as is done in the Bass model. In their empirical work on some of the consumer durable products analyzed by Bass (1969), Tanny and Derzko (1988) did not find satisfactory results for their proposed extension (the model either reduced to the Bass model or it failed to provide estimates for the additional model coefficients). These empirical results are not surprising because as the diffusion process progresses, the population of potential adopters mostly comprises Potential Imitators, justifying the parsimonious model suggested by Bass.

Diffusion Models From Individual Adoption Decisions

A key aspect of the Bass model is that it addresses the market in the aggregate. The typical variable measured is the number of adopters who purchase the product by a certain time t. The emphasis is on the total market response rather than an individual customer. This approach is convenient in practical terms but it raises the following issue: Can the diffusion model be built by aggregating demand from consumers who behave in a neoclassical microeconomic way? That is, assume that potential adopters are smart and are not just carriers of information. They therefore maximize some objective function such as expected utility or benefit from the product, taking into account the uncertainty associated with their understanding of its attributes, its price, pressure from other adopters to adopt it, and their own budget. Because the decision to adopt the innovation is individual-specific, all potential adopters do not have the same probability of adopting the product in a given time period. Is it possible to develop the adoption curve at the aggregate market level, given the heterogeneity among potential adopters in terms of their probability of adopting the product at any time t? Development of a model that answers this question can potentially assist in ascertaining the effect of marketing mix and other variables on demand for the product via their effect on individual consumers.

In recent years, attempts have been made by Hiebert (1974), Stoneman (1981), Feder and O'Mara (1982), Jensen (1982), Oren and Schwartz (1988), Chatterjee and Eliashberg (1989), and Lattin and Roberts (1989) to develop diffusion models by specifying adoption decisions at the individual level. In these models one assumes that, at any time t, a potential adopter's utility for an innovation is based on his uncertain perception of the innovation's performance, value, or benefits. The potential adopter's uncertain perceptions about the innovation, however, change over time as he learns more about the innovation from external sources (e.g., advertising) or internal sources (e.g., word of mouth). Therefore, because of this learning, whenever his utility for the innovation becomes greater than the status quo (he is better off with the innovation), he adopts the innovation. Aggregation across the various potential adopters yields the cumulative adoption curve.

Table 2 contrasts the various individual-level diffusion models on several dimensions. Of all the models compared in Table 2, only three provide explicit functions for aggregate diffusion models. Depending on the assumptions made about the distribution of parameters that measure heterogeneity across individuals, the model by Chatterjee and Eliashberg (1989) yields several basic diffusion models. If risk aversion across potential adopters is assumed to follow a negative exponential distribution, the model by Oren and Schwartz (1988) reduces to the Mansfield model. If the perceived differences in the potential benefits of the product across potential adopters are assumed to follow a uniform distribution, Lattin and Roberts (1989) suggest the following model.

$$N(t) = a + bN(t - 1) - \frac{d}{c + N(t - 1)} \quad (5)$$

where a, b, c, and d are constants. Using the data on several consumer durable products, they indicate that their model provides a better fit to the data than the Bass model. Their model contains four parameters, however (vs. three in the Bass model) and, unlike the Bass model, it does not provide N(t) as an explicit function of time, which limits its long-term forecasting ability.

Parameter Estimation Considerations

The use of the Bass model for forecasting the diffusion of an innovation requires the estimation of three parameters: the coefficient of external influence (p), the coefficient of internal influence (q), and the market potential (m). Though the estimate for the market potential of a new product can be derived from the diffusion time-series data, recent applications of diffusion models have obtained better forecasting results by using exogenous sources of information (such as

	Hiebert (1974)	Stoneman (1981)	Feder and O'Mara (1982)	Jensen (1982)	Oren and Schwartz (1988)	Chatterjee and Eliashberg (1989)	Lattin and Roberts (1989)
Nature of Innovation Studied	High yielding seed varieties (agricultural)	New technology (industrial)	New technology (agricultural)	Exogenously developed innovation (industrial)	Any new product that is potential substitute for current product	Durable goods	Durable goods
Perceptions							
Type of uncertain attribute/benefit	Net income	Return	Profit	Success rate	Performance	Benefit	
Perceptual uncertainty model (distributional assumption)	No specific model: uncertainty due to imperfect information about yield response to inputs (e.g., fertilizer)	Normal distribution	Normal distribution	Discrete: binary (innovation is profitable or unprofitable; uncertainty captured via subjective probability of innovation being profitable)	Beta distribution	Normal distribution	Normal distribution
Preference Structure							
Attribute(s) incorporated in utility function	Net income	Returns from new and old technologies, adjustment costs	Profit	Expected return	Success rate	Performance, price	Benefit
Assumption about attitude toward risk	No specific assumption: different attitudes considered	Risk averse	Risk neutral	Risk neutral	Risk averse	Risk averse	Risk averse
Adoption Decision Rule	Maximize expected utility (partial adoption of innovation possible)	Maximize utility to determine proportion of output produced on new technology	Expected profit from new technology exceeds profit from current technology	Expected return from adoption is greater than expected value of continuing waiting for additional information	Expected utility for new product exceeds expected utility for current product	Expected utility for new product exceeds expected utility for status quo	Expected utility for new product exceeds expected utility for status quo
Learning							
Learning model	Not explicit: learning reduces uncertainty	Bayesian	Bayesian	Bayesian	Bayesian	Bayesian	Bayesian
Source of information		Previous experience with new technology	Internal (previous adopters)	External	Internal (previous adopters)	Both internal and external	Internal (previous adopters)
Aggregation							
Heterogeneity criterion on which aggregation is done across potential adopters	No aggregation	Not applicable: considers only intrafirm diffusion	Mean of initial perceptions about profitability	Initial subjective probability of innovation being profitable	Risk aversion parameter (note: model assumes constant flow of consumers so that aggregation yields market share rather than cumulative penetration)	Initial perceptions (both expectation and degree of uncertainty); perceived reliability of information; risk aversion parameter; price/performance tradeoff	Difference in mean of perceptions about benefit from status quo

market surveys, secondary sources, management judgments, or other analytical models) for estimating m (see, e.g., Heeler and Hustad 1980; Mesak and Mikhail 1988; Oliver 1987; Souder and Quaddus 1982; Teotia and Raju 1986).

In the 1980s, several estimation procedures were proposed to estimate the Bass model parameters (see Table 1). Meta-analyzing the results of 15 such diffusion studies, Sultan, Farley, and Lehmann (1990) report average values of .03 and .38 for the coefficients of external influence and internal influence, respectively. Their analyses further suggest that the values of these coefficients are influenced by the type of estimation procedure used to estimate them. For a practitioner, the main question is which of the several estimation procedures should be used and why. The answer to this question depends partially on the amount of data available to estimate these parameters. We review estimation procedures that are designed to develop estimates both in the absence of and in the presence of time-series diffusion data. (A brief analytical description of these procedures is given in an appendix in the unabridged version of this article.)

No Prior Data Available

If no data are available, parameter estimates can be obtained by using either management judgments or the diffusion history of analogous products.

One procedure that exclusively uses management judgments to estimate the diffusion parameters is an algebraic estimation procedure suggested by Mahajan and Sharma (1986). The implementation of this procedure requires information from managers on three items: (1) the market size (m), (2) the time of the peak of the noncumulative adoption curve, and (3) the adoption level at the peak time (n*). That is, the key information required by the estimation procedure is the peak of the noncumulative adoption curve. Knowing this information, one can estimate the coefficients of external influence and internal influence. Though the algebraic estimation procedure has been implemented in actual applications by some firms (e.g., Institute for the Future), Bass (1986) has questioned its desirability, suggesting that one of the key outputs of the diffusion model is the prediction of the timing and magnitude of the peak. Therefore, if one can guess these items, there is no need to estimate model parameters.

An alternative algebraic estimation procedure has been suggested by Lawrence and Lawton (1981). This procedure also involves obtaining information from managers on three items: (1) the potential market size (m), (2) the number of adoptions in the first time period, and (3) an estimate of the sum of the coefficients of external influence and internal influence, that is, the p + q value. Though managers may be able to

guess the adoption level for the first time period, how does one guess the p + q value? A record of the parameter values of earlier new products may provide a basis, by analogy, for guessing p + q. From an analysis of the diffusion patterns of several products, Lawrence and Lawton (1981), for example, recommend using a value of .66 for industrial product innovations and a value of .50 for consumer product innovations (for an application of this procedure to consumer durable products, see DeKluyver 1982). Such a recommendation may be too general, however, and does not consider indosyncratic characteristics of a particular diffusion situation. Thomas (1985) therefore has recommended that, for a new product under consideration, the parameters can be estimated by taking a weighted sum of the parameters of analogous products where weights are determined by establishing the similarity/dissimilarity relationships between the new product and the various analogous products on five bases of comparison: environmental situation, market structure, buyer behavior, marketing mix strategy, and characteristics of innovation itself. In fact, to consider idiosyncratic characteristics of a new product in a particular social system, recent analogical approaches estimate its coefficients of external influence and internal influence from regression models that express a historical empirical relationship between these coefficients and product or market attributes of several current products. Once this relationship has been established, the values for the coefficients of a new product can be estimated by knowing its characteristics. Four such approaches for the Bass model have been suggested by Srivastava et al. (1985), Gatignon, Eliashberg, and Robertson (1989), Sultan, Farley, and Lehmann (1990), and Montgomery and Srinivasan (1989).

In studying parameter estimates of the Bass model, Lawrence and Lawton (1981) found that p + q ranged from .3 to .7 over several innovations. They note that first year sales, S_1, can be expressed as $m(1 - e^{-(p+q)})/[1 + (q/p)e^{-(p+q)}]$ and hence q/p can be expressed as $[m(1 - e^{-(p+q)}) - S_1]/S_1e^{-(p+q)}$. It is possible to use judgment in guessing m and S_1. In strategic terms, probably the most critical forecast deriving from the Bass model is the time of peak of adoptions, T*. This value is given by $[1/(p + q)]Ln(q/p)$. Because p + q varies over a relatively narrow range and has a mode around .5, for consumer products, guesses of p + q, m, and S_1 may provide good estimates of T*. Lawrence and Lawton (1981, p. 535) report good results with this method.

Availability of Data

Because the Bass model contains three parameters (p, q, and m), adoption data for a minimum of three time periods are required to estimate these parameters. Re-

cent empirical studies, however, have documented that estimates of these parameters, and hence the adoption forecasts, are sensitive to the number of datapoints used to estimate them (see, e.g., Hyman 1988; Tigert and Farivar 1981). In fact, these studies suggest that stable and robust parameter estimates for the Bass model are obtained only if the data under consideration include the peak of the noncumulative adoption curve (Heeler and Hustad 1980; Srinivasan and Mason 1986). Because of these concerns, attempts have been made in recent years to develop estimation procedures that update parameter estimates as additional data become available after the initiation of the diffusion process. These procedures include Bayesian estimation procedures and adaptive filtering approaches that provide time-varying parameter estimates.

Time-invariant estimation procedures. One of the first procedures suggested to estimate the diffusion parameters is the ordinary least squares (OLS) procedure proposed by Bass. The OLS procedure involves estimation of the parameters by taking the discrete or regression analog of the differential equation formulation of the Bass model (i.e., equation 1). In fact, rearrangement of equation 1 yields:

$$N(t + 1) - N(t) = pm + (q - p)N(t) - \frac{q}{m} N^2(t)$$

$$n(t + 1) = \alpha_1 + \alpha_2 N(t) + \alpha_3 N^2(t) \qquad (6)$$

where $\alpha_1 = pm$, $\alpha_2 = q - p$, and $\alpha_3 = -q/m$. That is, regression analysis is used to estimate α_1, α_2, and α_3 in equation 6. Once α's are known, p, q, and m can be estimated. If one has reason to believe that all datapoints in the diffusion time series should not have an equal weighting in the least squares procedure, discounted least squares can be used for estimating α's (for an application of discounted least squares to the discrete analog of the Bass model, see Young and Ord 1985).

The OLS procedure, however, has three shortcomings (Schmittlein and Mahajan 1982):

- Because of the likely multicollinearity between independent variables in equation 6, that is, N(t) and N^2(t), the procedure may yield parameter estimates that are unstable or have wrong signs.
- The procedure does not directly provide standard errors for the estimated parameters p, q, and m (and, hence, statistical significance of these estimates cannot be assessed).
- There is a time-interval bias because discrete time-series data are used for estimating a continuous model (i.e., the solution of the differential equation specification of the Bass model).

To overcome these shortcomings, Schmittlein and Mahajan (1982) have suggested a maximum likelihood estimation procedure to estimate the parameters directly from the solution of the differential equation specification of the Bass model. This procedure also has limitations, however. For example, Srinivasan and Mason (1986) point out that because the maximum likelihood procedure considers only sampling errors and ignores all other errors such as the effects of excluded marketing variables that influence the diffusion process, it underestimates the standard errors of the estimated parameters, resulting in possible wrong inferences about the statistical significance of the parameters. To overcome this shortcoming, they suggest a formulation by means of which estimates of p, q, and m can be obtained by using any appropriate nonlinear regression package (a similar formulation has been suggested by Jain and Rao 1989). This formulation also uses the solution to the differential equation specification of the Bass model for parameter estimation.

From the preceding descriptions, it is clear that both the maximum likelihood and the nonlinear estimation procedures offer better choices than the OLS procedure. An empirical comparison of these estimation procedures (along with the algebraic estimation procedure suggested by Mahajan and Sharma 1986) by Mahajan, Mason, and Srinivasan (1986) suggests an overall superiority of the nonlinear estimation procedure, but the maximum likelihood procedure performs equally well when survey-type diffusion data are used to estimate the parameters because of the dominance of sampling errors (Mahajan, Mason, and Srinivasan 1986; Srinivasan and Mason 1986).

Parameter estimation for diffusion models is primarily of historical interest; by the time sufficient observations have developed for reliable estimation, it is too late to use the estimates for forecasting purposes. The estimates can be used for model testing and for comparison across products. Considered in such a context, the methods often yield estimates that do not differ greatly.

Time-varying estimation procedures. These procedures are designed to update parameter estimates as new data become available.[3] The updating of param-

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[3] The idea that coefficients of a market response model should change over time is not new in marketing. In fact, several theoretical approaches that assist in developing market response models when model coefficients have a time-varying behavior have been applied and documented in the marketing literature (see, e.g., Mahajan, Bretschneider, and Bradford 1980; Wildt and Winer 1978). Two such approaches also have been examined in the context of diffusion models: the systematic parameter variation methods and the random coefficient methods. The systematic parameter variations assume *a priori* the time path of the model coefficients. These methods have generated a new set of diffusion models termed "flexible diffusion models" (Mahajan and Peterson 1985) that are reviewed here.

In the random coefficient methods, the random parameters are assumed to constitute a sample from a common multivariate distribution with an estimated mean and variance-covariance structure. Following Karmeshu and Pathria (1980a, b), Eliashberg, Tapiero, and Wind (1987)

eters is achieved either with the Bayes procedure or feedback filters.

Such procedures have been applied in various diffusion settings by Sultan, Farley, and Lehmann (1990), Lenk and Rao (1989), and Bretschneider and Mahajan (1980). All of these procedures have two elements in common: (1) they require an initial estimate of the diffusion model parameters before the diffusion data become available and (2) they specify an updating formula to upgrade the initial estimates as additional diffusion data become available.

In the Bayesian estimation procedure advocated by Sultan, Farley, and Lehmann (1990), statistical results of their meta-analysis study are used to develop initial estimates for the coefficients of external influence and internal influence for a new product. For each of these two coefficients, the procedure updates the initial estimates by taking a weighted sum of its two values, the initial estimate and the estimate developed from the actual data (by using any procedure such as the nonlinear estimation procedure of Srinivasan and Mason 1986). The weights in the updating formula are expressed as a function of the variation in the parameter estimates from the actual data so that as these time-varying estimates stabilize, the weight for the initial estimate based on the meta-analysis goes to zero. A Bayesian estimation procedure also has been reported by Lenk and Rao (1989). Their procedure explicitly considers the between-product and within-product variations in establishing initial estimates for the new product.

An alternative approach to updating the diffusion model parameters for the Bass model has been demonstrated by Bretschneider and Mahajan (1980). It estimates the time-varying values of the Bass model parameters by updating the regression coefficients in the discrete analog of the Bass model, equation 6. The updating formula is based on a feedback filter suggested by Carbone and Longini (1977). This feedback filter estimates an adjustment to the current values of parameters at time t based on the error between the actual and the predicted values of the noncumulative number of adopters at time t. Though the procedure provides time-varying estimates for the diffusion model coefficients, it has the same shortcomings as the or-

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explored the applicability of these methods to the Bass diffusion model. They consider the coefficients of innovation (p) and imitation (q) in the Bass model as stochastic and hence time-varying by assuming that $\bar{p} = p + \epsilon_p(t)$ and $\bar{q} = q + \epsilon_q(t)$, where p and q denote constant means and ϵ_p and ϵ_q denote normally distributed error terms surrounding those means such that their means are zero and the variances are constant. Their empirical results suggest that their stochastic formulation of the Bass model does as well as the deterministic version of the Bass model. There are other types of stochastic diffusion models, but they are not included in our review. Reviews of such models are given by Bartholomew (1982), Eliashberg and Chatterjee (1986), and Boker (1987).

dinary least squares procedure.[4]

Flexible Diffusion Models

The basic structure of a diffusion model can be characterized in terms of two mathematical properties, point of inflection and symmetry. The point of inflection on a diffusion curve occurs when the maximum rate of diffusion is reached. If the diffusion pattern after the point of inflection is the mirror image of the diffusion pattern before the point of inflection, the diffusion curve is characterized as being symmetric. For example, as depicted in Figure 1B, the adopter distribution for the Bass model peaks at time T*, which is the point of inflection of the S-shaped cumulative adoption curve, and is symmetric with respect to time around the peak time T* up to time 2T*. Furthermore, the Bass model assumes that the maximum penetration rate cannot occur after the product has captured 50% of the market potential. In practice as well as in theory, the maximum rate of diffusion of an innovation should be able to occur at any time during the diffusion process. Additionally, diffusion patterns can be expected to be nonsymmetric as well as symmetric.

Easingwood, Mahajan, and Muller (1983) have suggested that flexibility can be achieved in the diffusion models by recognizing an important underlying assumption. In most of the diffusion models, the impact of the word of mouth on potential adopters is assumed to remain constant throughout the diffusion span. This assumption is tenuous because, for most innovations, the word of mouth is likely to increase, decrease, or remain constant over time (Hernes 1976). Easingwood, Mahajan, and Muller (1983) suggest that the time-varying nature of the word-of-mouth effect can be incorporated in the Bass model by specifying the coefficient of internal influence as systematically varying over time as a function of penetration level. That is,

$$w(t) = q\left[\frac{N(t)}{m}\right]^\alpha \tag{7}$$

where α is a constant and w(t) is the time-varying coefficient of external influence. Substitution of equation 7 into the Bass model, equation 1, yields the *non-uniform-influence* (NUI) model suggested by those authors (in terms of the cumulative fraction of adopters):

$$\frac{dF(t)}{dt} = [p + qF^\delta(t)][1 - F(t)] \tag{8}$$

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[4]For an application of this approach to the diffusion of robotics in the State of New York, see Bretschneider and Bozeman (1986). Other feedback filters also can be used to estimate time-varying diffusion parameters. For example, the use of the Kalman filter to estimate the time-varying coefficients for the Mansfield model has been reported by Meade (1985).

where $F(t) = N(t)/m$ and $\alpha + 1 = \delta$. When $p = 0$ (i.e., coefficient of external influence is zero), equation 8 or 9 yields a flexible extension of the Mansfield model termed *nonsymmetric responding logistic* (NSRL) by Easingwood, Mahajan, and Muller (1981). An interesting alternative interpretation of the NSRL model in terms of experience curve and price elasticity is provided by Sharp (1984).

In addition to the NUI and NSRL models, Table 3 reports characteristics of nine other diffusion models. The following observations are warranted from this table.

- In addition to the NUI and NSRL models, only two models offer complete flexibility in capturing diffusion patterns (i.e., point of inflection can occur from 0% to 100% penetration and the diffusion patterns can be symmetric or nonsymmetric). These are the models proposed by Von Bertalanffy (1957) (an identical model has been proposed by Nelder 1962) and Bewley and Fiebig (1988).

- Like the NUI and NSRL models proposed by Easingwood, Mahajan, and Muller (1981, 1983), the model by Von Bertalanffy expresses the coefficient of internal influence as systematically changing over time as a function of penetration level, that is,

$$w(t) = \frac{q(1 - F^\phi)}{(1 - F)} \qquad (9)$$

where ϕ is a constant. Unlike the NUI and NSRL models, the differential equation used to specify the diffusion process by the Von Bertalanffy model has a closed-form solution enabling one to represent cumulative adoption as an explicit function of time. This model, however, assumes that the word-of-mouth effect decreases over time. The NUI and NSRL models can accommodate the word-of-mouth effect that increases, decreases, or remains constant over time.

- In comparison with the models suggested by Easingwood, Mahajan, and Muller (1981, 1983) and Von Bertalanffy (1957), the FLOG (*flexible logistic growth*) model suggested by Bewley and Fiebig (1988) expresses the systematic variation in the coefficient of internal influence as a function of time, that is,

$$w(t) = q[(1 + kt)^{1/k}]^{\mu-k} \qquad (10)$$

where k and μ are constants. The FLOG model offers a closed-form solution and, like the NUI and NSRL models, can accommodate the time-varying word-of-mouth effect.

Though some evidence suggests that, in comparison with the basic diffusion models (such as the Bass model), the flexible models provide a better fit to diffusion data (see Easingwood 1987, 1988; Lattin and Roberts 1989; McGowan 1986; Rao 1985), this advantage is obtained by incorporating additional parameters. Hence these models are more difficult to use in the absence of diffusion time-series data (using the historical data on existing products, however, Easingwood 1989 has demonstrated how the NUI model can be used to develop analogical parameter estimates for a new product).

Refinements and Extensions of the Bass Diffusion Model

Several assumptions underlie the Bass model. Most are simplifying assumptions that provide a parsimonious analytical representation of the diffusion process. However, recognition of these assumptions is important to properly understand and interpret the dynamics of innovation diffusion captured by the Bass model. Table 1 lists several of these assumptions that have been of concern to diffusion modelers in the 1970s and 1980s. Nine of these assumptions warrant attention.

Market potential of the new product remains constant over time. The Bass model assumes that the market potential (m) of a new product is determined at the time of introduction and remains unchanged over its entire life (Kalish 1985; Mahajan and Peterson 1978; Sharif and Ramanathan 1981). Theoretically, there is no rationale for a static potential adopter population. Instead, a potential adopter population continuously in flux is to be expected.

Extensions of the Bass model that address this assumption have attempted to relax it by specifying the market potential as a function of relevant exogenous and endogenous variables—controllable as well as uncontrollable—that affect the market potential. Examining the diffusion of a durable product, Kalish (1985), for example, specified the dynamics of the market potential as a function of price of the product and reduction of uncertainty associated with the product with its increased adoption. Assuming full product awareness in the population, he specified

$$m(t) = m_0 \exp\left[-dP(t)\left(\frac{a + 1}{a + \frac{N(t)}{m_0}}\right)\right] \qquad (11)$$

where a and d are constants, m_0 is the size of the market potential at the time of product introduction, $P(t)$ is the product price, and the term $[(a + 1)/(a + N(t)/m_0)]$ represents the effect of market penetration in increasing the size of market potential due to the word-of-mouth effect. Other applications have represented the market potential as a function of growth in the number of households (Mahajan and Peterson 1978), population growth (Sharif and Ramanathan 1981), product profitability (Lackman 1978), price (Chow 1967; Jain and Rao 1989; Kamakura and Balasubramanian 1988), growth in the number of retailers making the product available to potential customers (Jones and Ritz 1987), and income distribution, price, and product uncertainty (Horsky 1990).

TABLE 3
Flexible Diffusion Models

Model	Model Equation $(dF/dt =)$	Model Solution $(F =)$	Point of Inflection (F^*)	Symmetry[a]	Coefficient of Internal Influence	Illustrated Reported Applications
1. Bass (1969)[b]	$(p + qF)(1 - F)$	$\dfrac{1 - e^{-(p+q)t}}{1 + \dfrac{q}{p} e^{-(p+q)t}}$	.0–.5	NS	Constant	Consumer durable goods; retail service, agricultural, education, and industrial innovations; electronics, photographic products, industrial processes
2. Gompertz curve[c] (see Hendry 1972; Dixon 1980)	$qF \ln\left(\dfrac{1}{F}\right)$	$e^{-e^{-(c+qt)}}$	.37	NS	Constant	Consumer durable goods, agricultural innovations
3. Mansfield (1961)	$qF(1 - F)$	$\dfrac{1}{1 + e^{-(c+qt)}}$	.5	S	Constant	Industrial, high technology, and administrative innovations
4. Floyd (1962)	$qF(1 - F)^2$	*	.33	NS	Decreasing to zero	Industrial innovations
5. Sharif and Kabir (1976)[d]	$\dfrac{qF(1 - F)^2}{1 - F(1 - \sigma)}$	*	.33–.5	S or NS	Constant or decreasing to zero	Industrial innovations
6. Jeuland (1981)[e]	$(p + qF)(1 - F)^{1+\gamma}$	*	.0–.5	S or NS	Constant or decreasing to zero	Consumer durable goods
7. Nonuniform influence (NUI) (Easingwood, Mahajan, and Muller 1983)	$(p + qF^\delta)(1 - F)$	*	.0–1.0	S or NS	Increasing, decreasing, or constant	Consumer durable, retail service, and education innovations
Nonsymmetric responding logistic (NSRL, p = 0 in NUI) (Easingwood, Mahajan, and Muller 1981)	$qF^\delta(1 - F)$	*	.0–1.0	S or NS	Increasing, decreasing, or constant	Medical innovations
8. Nelder[f,g] (1962; see McGowan 1986)	$qF(1 - F^\phi)$	$\dfrac{1}{[1 + \phi e^{-(c+qt)}]^{1/\phi}}$	.0–1.0	S or NS	Decreasing to a constant	Agricultural innovations
Von Bertalanffy[h] (1957; see Richards 1959)	$\dfrac{q}{1 - \theta} F^\theta (1 - F^{1-\theta})$	$[1 - e^{-(c+qt)}]^{1/1-\theta}$	.0–1.0	S or NS	Decreasing to a constant	
9. Stanford Research[i] Institute (e.g., Teotia and Raju 1986)	$\dfrac{q}{t} F(1 - F)$	$\dfrac{1}{1 + \left(\dfrac{T^*}{t}\right)^q}$	.0–.5	NS	Decreasing to zero	Energy-efficient innovations
10. Flexible logistic[j] growth (FLOG; (Bewley and Fiebig 1988)	$q[(1 + kt)^{1/k}]^{\mu-k}$	$\dfrac{1}{1 + e^{-[c+qt(\mu,k)]}}$	.0–1.0	S or NS	Increasing, decreasing, or constant	Telecommunication innovations

[a]S = symmetric, NS = nonsymmetric.
[b]The model is symmetric around the peak time T* up to 2T*.
[c]c is a constant.
[d]$0 \le \sigma \le 1$.
[e]$\gamma \ge 0$.
[f]The model suggested by Nelder (1962) is identical to the model originally suggested by Von Bertalanffy (1957). The equivalence between the two can be shown by substituting $\phi = \theta - 1$ in the Von Bertalanffy model.
[g]c is a constant; model reduces to Mansfield model for $\Phi = 1$ and the Gompertz curve as Φ approaches zero.
[h]c is a constant; $\theta \ge 0$; model reduces to Mansfield model when $\theta = 2$ and the Gompertz curve as θ approaches 1.
[i]The model is not invariant to the choice of time scale. A linear transformation of time t is required to make it time-scale independent. T* is time of 50% penetration. See Bewley and Fiebig (1988).
[j]μ and k are constants and $t(\mu,k) = \{[(1 + kt)^{(1/k)}]^\mu - 1\}/\mu$, $\mu \ne 0$, $k \ne 0$, $t(\mu,k) = (1/k) \log (1 + kt)$, $\mu = 0$, $k \ne 0$, $t(\mu,k) = (e^{\mu t} - 1)/\mu$, $\mu \ne 0$, $k = 0$, $t(\mu,k) = t$, $\mu = 0$, $k = 0$.

Diffusion of an innovation is independent of all other innovations. The Bass model assumes that the adoption of an innovation does not complement, substitute for, detract from, or enhance the adoption of any other innovation (and vice versa) (Peterson and Mahajan 1978). In reality, however, an innovation is not introduced into a vacuum nor does it exist in isolation. Other innovations are present in the marketplace and may have an influence (positive or negative) on its diffusion. Consideration of simultaneous diffusion of multiple innovations is especially critical if the diffusion of one innovation is contingent upon the diffusion of another innovation (e.g., compact disc software and compact disc hardware) or if the diffusion of one innovation complements the diffusion of another innovation (e.g., washers and dryers).

Following the contingent diffusion model suggested by Peterson and Mahajan (1978), Bayus (1987), for example, conducted an empirical study examining the diffusion dependence between compact disc software and compact disc hardware. In the contingent diffusion model, the market potential of the dependent product is contingent upon the diffusion of the primary product. That is, in the Bass model representation of its growth, equation 1, its market potential is specified as $(N_1(t) - N_2(t))$ where $N_1(t)$ is the cumulative number of adopters of the primary product (e.g., compact disc hardware) and $N_2(t)$ is the cumulative number of adopters of the contingent product (e.g., compact disc software).

Nature of an innovation does not change over time. Manufacturers of high technology products usually achieve diffusion in the marketplace by offering successive generations of an innovation. Each generation is positioned to be better than its predecessors on relevant product attributes. Assessment of market penetration therefore is critical for successive generations of a high technology product. In addition to creating its own demand, each generation of the product cannibalizes the diffusion of its predecessors. The important application of diffusion models for assessing technological substitution has been demonstrated by Norton and Bass (1987) for the growth of two basic types of integrated circuits, memory and logic circuits. If τ_2 represents the time of the introduction of the second generation, Norton and Bass suggest that the word-of-mouth effect within each generation and substitution effects across successive generations can be represented by the following extension of the Bass model.

$$S_1(t) = m_1 F_1(t) - m_1 F_1(t) F_2(t - \tau_2) \qquad (12)$$

$$S_2(t) = m_2 F_2(t - \tau_2) + m_1 F_2(t) F_2(t - \tau_2) \qquad (13)$$

where equation 12 is the diffusion equation for the first generation product and equation 13 represents the second generation product, S_1 and S_2 are their shipments at time t, and $F_1(t)$ and $F_2(t)$ are fractions of adoptions for each generation and are given by the Bass model (solution of equation 1). In equations 12 and 13, the term $m_1 F_1(t) F_2(t - \tau_2)$ represents the cannibalization or substitution effect.

The geographic boundaries of the social system do not change over the diffusion process. Despite the fact that the diffusion of an innovation occurs simultaneously in space and time, research on these two dimensions of diffusion seldom has been integrated in a marketing context. For example, the new product rollout is clearly a popular option used by many firms to diffuse their products from market to market over time (in both the national and the international markets). Such a new-product launch strategy enables a firm to capitalize on word-of-mouth communication, referred to as the "neighborhood effect" (Brown 1981; Gore and Lavaraj 1987), across markets. Simultaneous assessment of market penetration within a market and across markets therefore is necessary.

One application that addresses diffusion from a joint space and time perspective has been reported by Mahajan and Peterson (1979). In examining the adoption of tractors in 25 states in the central agricultural production region of the United States for the period 1920–1964, they extend the Bass model by assuming that (1) the innovation is introduced initially in one market and (2) the relative number of total adoptions is greater in markets that are closest to the market of innovation origination (i.e., the neighborhood effect diminishes with increased distance from the market of innovation origination, decreasing the size of market potential across markets).

The diffusion process is binary. The Bass model assumes that potential adopters of an innovation either adopt or do not adopt the innovation. As a consequence of this assumption, the Bass model does not take into account stages in the adoption process (e.g., awareness, knowledge, etc.). Some of the attempts to extend the two-stage models to incorporate the multistage (or polynomial) nature of the diffusion process include models by Midgley (1976), Dodson and Muller (1978), Sharif and Ramanathan (1982), Mahajan, Muller, and Kerin (1984), and Kalish (1985). Most of these extensions tend to characterize stages in which positive, negative, or neutral information is communicated about the product. The implementation of these models is rather cumbersome as they require detailed information about the customer flow across the various stages. In empirical applications, the developers of these models therefore either collapse the various stages (Kalish 1985 assumes full product awareness), attempt to derive the population in various stages by decomposing the time-series diffusion data (Midgley

1976; Sharif and Ramanathan 1982) with too many parameters to be estimated with the limited available data (Silver 1984), or trace the innovation diffusion with the panel data (Mahajan, Muller, and Kerin 1984; Mahajan, Muller, and Sharma 1984).

Diffusion of an innovation is not influenced by marketing strategies. Since the pioneering work of Robinson and Lakhani (1975) that incorporated the impact of price in the Bass model, several efforts have been made to study systematically the impact of marketing mix variables such as price, advertising, promotion and personal selling, and distribution on product growth (efforts related to price and advertising are reviewed extensively by Kalish and Sen 1986). As the Bass model contains three parameters (coefficients of external influence and internal influence, and the market potential), the impact of marketing mix variables has been incorporated into the Bass model by representing these parameters as a function of relevant variables. Attempts have been made to represent the market potential as a function of price (e.g., Kalish 1983, 1985) and distribution growth (Jones and Ritz 1987). Other attempts to incorporate marketing mix variables have been concerned with representing the coefficients of external influence and internal influence as a function of diffusion-influencing variables. Though analytically very elegant, most of these modeling efforts lack empirical validation (Mahajan and Wind 1986b). However, they can be useful in establishing working hypotheses to examine the likely impact of marketing mix variables on innovation diffusion. As these hypotheses are presented in the next section, we briefly comment here on studies that have provided some empirical support for their extensions.

Two empirical studies by Horsky and Simon (1983) and Simon and Sebastian (1987), respectively, have examined the impact of advertising on innovation diffusion. Studying the diffusion of a new banking service, Horsky and Simon argue that because advertising provides information to innovators, the coefficient of external influence in the Bass model should be represented as a function of advertising expenditures (with diminishing returns). Their empirical results provided a good fit to their diffusion data, supporting their argument. Studying the diffusion of new telephones in West Germany, Simon and Sebastian suggest that, though advertising may influence innovators (and hence the coefficient of external influence) in the early stage of the product life cycle, it is more likely to influence the coefficient of imitation in the intermediate life cycle stage of a new product because the objective of the advertising content in the intermediate stage is to influence potential customers through evaluation by customers and social pressure. Furthermore, the advertising effect is cumulative over time. They report

a good fit to their diffusion data, supporting their arguments about incorporation of the cumulative effect of advertising into the coefficient of imitation.

The question of the inclusion of price in the Bass model intrigued diffusion analysts in the 1970s and 1980s. Examining the diffusion of a (unmentioned) durable product, Kalish (1985) suggested that price affects the market potential of a product (see equation 11). However, recent empirical studies by Kamakura and Balasubramanian (1988) and Jain and Rao (1989), employing data on several consumer durable products, show that price affects the rate of diffusion (via the coefficients of external influence and internal influence) rather than the market potential.

Product and market characteristics do not influence diffusion patterns. The Bass model does not consider explicitly the impact of product and market characteristics on diffusion patterns. Empirical studies reported in the innovation diffusion literature, however, have found that product and market characteristics have a substantial impact on innovation diffusion patterns (Rogers 1983; Tornatzky and Klein 1982). Three empirical studies (Gatignon, Eliashberg, and Robertson 1989; Kalish and Lilien 1986a; Srivastava et al. 1985) have attempted to incorporate product and market characteristics into the Bass model by expressing the coefficients of external influence and/or internal influence as a function of these characteristics. Whereas Srivastava et al. (1985) and Kalish and Lilien (1986a) examine the impact of product characteristics on diffusion patterns, Gatignon, Eliashberg, and Robertson (1989) study the impact of market characteristics on the diffusion of a product across markets. Only Kalish and Lilien (1986a), however, explicitly consider the changing consumer perceptions of the product characteristics as the product is accepted over time. They define the coefficient of imitation as changing over time due to changes in the product characteristics.

There are no supply restrictions. The Bass model is a demand model. If the demand for a product cannot be met because of supply restrictions, such as the unavailability of the product due to limitations on production capacity or difficulties in setting up distribution systems, the excess unmet demand is likely to generate a waiting line of potential adopters (Simon and Sebastian 1987). In such a situation, the adopter distribution is the same as the supply distribution and applying the Bass model to these adoption data is inappropriate. Therefore the Bass model must be extended to integrate the demand-side dynamics with the supply-side restrictions.

A model that captures innovation diffusion dynamics in the presence of supply restrictions has been suggested by Jain, Mahajan, and Muller (1989). Their

model conceptualizes diffusion as a three-stage process: potential adopters → waiting adopters → adopters. They have demonstrated the application of their model for the diffusion of new telephones in Israel.

There is only one adoption by an adopting unit. The objective of a diffusion model is to represent the level or spread of an innovation among a given set of prospective adopters. For a great many product innovations, the increase in the number of adopters may consist of first-time buyers as well as repeat buyers. The Bass model, however, captures only the first-time buyers.

In recent years, five empirical studies have been reported that capture the repeat/replacement dynamics of innovation diffusion. Two of these studies, by Lilien, Rao, and Kalish (1981) and Mahajan, Wind, and Sharma (1983), include repeat purchase in the Bass model to examine diffusion of ethical drugs. Two other studies, by Olson and Choi (1985) and Kamakura and Balasubramanian (1987), include product replacements in the Bass model to assess long-term sales for consumer durable products. Norton and Bass (1987) assume that adopters continue to buy and that the average repeat buying rate over the population of adopters is constant.

Uses of Diffusion Models

Innovation diffusion models traditionally have been used in the context of sales forecasting. However, as pointed out by Mahajan and Wind (1986b) and Kalish and Lilien (1986a), sales forecasting is only one of the objectives of diffusion models. In addition to forecasting, perhaps the most useful applications of diffusion models are for *descriptive* and *normative* purposes. Because diffusion models are an analytical approach to describing the spread of a diffusion phenomenon, they can be used in an explanatory mode to test specific diffusion-based hypotheses. Further, because diffusion models are designed to capture the product life cycle of a new product, they can be used for normative purposes as the basis of how a product should be marketed.

Descriptive Uses

Table 4 is a listing of nine illustrative studies in which the diffusion modeling framework has been used to test hypotheses. Srivastava et al. (1985) and Rao and Yamada (1988) use diffusion models to test hypotheses related to the impact of perceived product attributes on diffusion patterns. Kobrin (1985), Takada and Jain (1988), and Gatignon, Eliashberg, and Robertson (1989) use diffusion models to test hypotheses related to innovation diffusion across countries. Bass (1980), Olshavsky (1980), and Modis and Debecker

(1988) use diffusion models to test hypotheses related to the life-cycle dynamics of a new product. Finally, Mahajan, Sharma, and Bettis (1988) evaluate the hypothesis that any S-shaped curve may not be a result of the imitation process. The preceding studies clearly demonstrate how the diffusion models can be used to evaluate hypotheses related to the dynamics of innovation diffusion.

Normative Uses

Though diffusion models are concerned with representing the growth of a product category, that growth can be influenced by the individual or by collective actions of competitors that have long-term effects on the growth or decline of the market. Alternatively, even if there is only one firm in the industry, it must consider the life cycle dynamics over time to determine optimal marketing mix strategy for maximizing its profitability. That is, it must find out what trajectory (pattern or strategy) of the relevant marketing mix variables it should follow to maximize its discounted profits over the planning period given the constraint that the life cycle of the product follows a certain growth pattern. It therefore solves the following dynamic optimization problem.

$$\text{Maximize } \pi = \text{Total discounted profits} \quad (14)$$
$$\text{over the planning period}$$

$$\text{Subject to: A given life cycle growth pattern} \quad (15)$$

The dynamic optimization formulation outlined in expressions 14 and 15 is the general framework that has been used by several authors in the 1980s to develop optimal marketing mix strategies, especially for price and advertising. Most of these studies use the Bass model, and its extensions incorporating marketing mix variables, in expression 15 to represent the life cycle dynamics over time. They usually consider a single marketing mix variable, such as price, to isolate its effects on product growth. Before we comment on these studies, a further elaboration on expressions 14 and 15 is warranted.

Note that the determination of trajectory of the marketing mix variable(s) that maximizes expression 14 depends on the specification of the growth model used to specify the life cycle growth pattern in expression 15. Therefore, though most of the studies use the Bass model to capture the word-of-mouth effect in expression 15, different optimal strategies can be obtained depending on how the relevant marketing mix variables are incorporated in the Bass model. To highlight this point, we consider here derived optimal pricing strategies for new durable goods.

When launching a new product, a firm usually can choose between two distinct pricing strategies, market

TABLE 4
Illustrative Descriptive Applications of Diffusion Models

Study By	Hypothesis Tested	Diffusion Model Used	Remarks
Bass (1980)	As a result of learning and accumulated experience, declining patterns of costs and prices result for technological innovations	Bass	Reports results for six durable products. The hypothesis is generally confirmed for most of these products. Similar results are provided by DeKluyver (1982).
Olshavsky (1980)	Product life cycles of consumer durable goods are shortening because of rapidly accelerating technological developments	Mansfield	Study uses data from 25 consumer durable products. Hypothesis is tested by examining relationship between coefficient of imitation and time of introduction of an innovation. Findings confirm hypothesis.
Kobrin (1985)	The pattern of oil production nationalization across countries is a "social interaction" phenomenon	Bass	Study examines pattern of number of countries per year that nationalized oil production from 1960 to 1979. Supplementing quantitative results with detailed qualitative analyses, study confirms hypothesis.
Srivastava et al. (1985)	Potential adopters' perceptions of innovation attributes explain the diffusion pattern of a product	Bass	Study examines diffusion of 14 investment alternatives. To explain diffusion patterns across investment alternatives, coefficient of imitation is expressed as a function of perceived product attributes. Two attributes of perceived information cost and perceived likelihood of loss of principal/ negative return explain those differences. Findings confirm hypothesis.
Mahajan, Sharma, and Bettis (1988)	Adoption of the M-form organizational structure by the U.S. firms resulted from an imitation behavior	Bass	Study examines adoption of M-form organization structure among 127 U.S. firms from 1950 to 1974. Findings question validity of the hypothesis.
Modis and Debecker (1988)	There is a relationship between the number of new computer models and the number of new computer manufacturers.	Mansfield	Study uses data on number of new models introduced and number of new manufacturers that emerged in computer market from beginning of 1958 to end of 1984. Study is also done for personal computers. By examining relationship between growth patterns of number of new computer models and number of new computer manufacturers, the authors conclude that, on average, a new computer manufacturer emerges for every five new models that appear on the market. For the personal computers market, this relationship is around one for every six.
Rao and Yamada (1988)	Potential adopters' perceptions of innovation attributes explain the diffusion pattern of a product	Lilien, Rao, and Kalish (1981)	Study examines diffusion of 21 ethical drugs using repeat-purchase diffusion model suggested by Lilien, Rao, and Kalish. Model coefficients are expressed as a function of six perceived attributes of ethical drugs. Findings confirm hypothesis.
Takada and Jain (1988)	Cultural differences among countries will lead to different diffusion patterns	Bass	Study examines diffusion of eight consumer durable products in Japan, Korea, and United States. By testing differences between coefficients of innovation and imitation across the three countries, the authors conclude that among the three countries analyzed, a product is adopted in Korea at a much faster rate than in either the U.S. or in Japan. No significant differences are found between the diffusion patterns in Japan and U.S.
Gatignon, Eliashberg, and Robertson (1989)	Three dimensions explain the differences in the diffusion patterns across countries: level of cosmopolitanism of a country, mobility, and the role of women in the society	Bass	The study examines the diffusion of six consumer durable products in 14 European countries. Coefficients of imitation and innovation are expressed as a function of variables measuring the three hypothesized dimensions, and their impact on the two coefficients is determined simultaneously across products and across countries. Findings confirm hypothesis.

skimming and market penetration. A market-skimming strategy uses a high price initially to "skim" the market when the market is still developing. The market penetration strategy, in contrast, uses a low price initially to capture a large market share.

Introduction of the impact of price in the Bass model framework generally has resulted in two types of normative pricing strategies. One derived pricing strategy

posits that price will increase at introduction, peak, and decrease later (Dolan and Jeuland 1981; Jeuland and Dolan 1982; Kalish 1983; Robinson and Lakhani 1975). Kalish and Sen's (1986, p. 94) intuitive explanation for this pricing strategy is that if early adopters have a strong positive effect on late adopters, a low introductory ("subsidized") price should encourage them to adopt the product. Consequently, once a product is established, price can be raised because the contribution to sales due to additional adopters decreases over time. Studies deriving this pricing strategy generally assume that price does not affect the population of potential adopters and produces a *multiplicative* effect on the rate of diffusion. That is, from equation 1,

$$\frac{dN(t)}{dt} = \left[p + \frac{q}{m} N(t) \right](m - N(t))g(P) \qquad (16)$$

where g(P) is the price response function for the dynamic price P at time t. Equation 16 assumes that price affects the rate of diffusion.

The second derived pricing strategy posits that price is more likely to decrease over time, supporting the market-skimming strategy (Kalish 1983). In deriving this optimal strategy, some researchers have assumed that price affects the market potential. That is:

$$\frac{dN(t)}{dt} = \left[p + \frac{q}{m} N(t) \right][m(P) - N(t)]. \qquad (17)$$

The preceding analyses illustrate that we must be cautious about the normative policies derived from the diffusion-based dynamic optimization framework because the derived policies could be simply an artifact of the underlying assumptions made for analytical convenience. Despite this observation, the diffusion modeling framework has provided an excellent opportunity to develop a "theory" of life cycle analysis for empirical validation.

Table 5 is a summary of some of the major results from various studies for optimal strategies for three variables: pricing, advertising, and product introduction time. We summarize these results for two industry settings, monopoly and oligopoly. The major results reported for each study reflect the issue raised in the study.

Conclusions and Discussion

From our review of the emerging literature on innovation diffusion modeling in marketing, we can highlight research issues that must be addressed to make these models theoretically more sound and practically more effective and realistic. We discuss such research possibilities related to the five subareas of recent developments.

Basic Diffusion Models

Though several assumptions underlying the Bass model have been of concern in the 1980s (Mahajan and Wind 1986a), we believe five issues warrant further investigation.

Adoptions due to internal influence. One of the key factors of the Bass model is that it explicitly considers the influence of internal (word of mouth) as well as external sources of communication on innovation diffusion. As depicted in Figure 1A, the Bass model assumes that adopters whose purchase decisions are influenced by external sources of information are present at any stage of the diffusion process. Such adopters, however, should not be labeled "innovators" because innovators, by definition, are characterized as the first adopters of an innovation (Mahajan, Muller, and Srivastava 1990). The questions now are: What are the characteristics of adopters who, despite a large product penetration in the marketplace, are predominantly influenced by external sources? How do they differ from innovators and other adopter categories on those characteristics? Because, within a certain time period in the diffusion process, the Bass model implies the presence of adopters due to both internal influence and external influence, how do those two groups differ from each other?

In a recent empirical study, Feick and Price (1987) suggest that in any social system there are individuals who assimilate and disseminate information on products (and therefore influence others) and tend to rely on external sources of information. They label these individuals "market mavens." On the basis of their empirical results, however, they conclude that "the concepts of the market maven and the innovative consumer are distinct" (1987, p. 90). Their findings raise research questions about the linkage in the Bass model between market mavens and adopters who buy as a result of external influence.

Multiple adoptions. The Bass model has been developed to represent the conversion of potential adopters to adopters. It explicitly assumes that each potential adopter buys only one unit of the product. However, certain innovations are bought in multiple units by potential adopters (e.g., multiple units of scanners by a supermarket and multiple units of personal computers by a firm). For these innovations, the sales data must be linked with the number of adopters by using a function that explicitly takes into consideration the multiple-unit-adoption behavior of the potential adopters (see Norton and Bass 1987).

Effect of consumer expectations. For certain innovations (e.g., computers), consumer expectations about the innovation's future characteristics (e.g., price) influence purchase intentions (see, e.g., Holak,

TABLE 5
Optimal Marketing Mix Strategies for Innovation Diffusion

Issue and Marketing Mix Variable	Major Assumptions/Comments	Major Normative Results	Illustrative References
Industry Setting: Monopoly			
Price			
How should a monopolist price a new product over its life cycle?	1. Price interacts with diffusion (rate of adoption) 2. Demand saturation effect causes decline in price over time and diffusion effect causes price to increase over time; experience effect (learning by doing) causes a decline in price over time	For a long planning horizon, if imitation effect is dominating, price first increases and then decreases	Robinson and Lakhani (1975), Dolan and Jeuland (1981), Kalish (1983), Clarke, Darrough, and Heineke (1982)
	Price has a multiplicative effect on diffusion and interacts with experience curve	Price declines over time	Kalish (1983), Bass and Bultez (1982)
	Price affects market potential	Price declines over time	Kalish (1983)
How should a monopolist price over time a new product that can be copied?	Monopolist produces a new product that can be copied. Market potential is affected by price	If product is not protected against copying, price is initially high and then decreases as copying increases	Nascimento and Vanhonacker (1988)
How should a monopolist price a repeat-purchased product over its life cycle?	Price affects market potential	1. Price increases monotonically over time 2. With experience effect, price may first increase (strong imitation effect) then decrease (strong experience effect)	Feichtinger (1982), Jorgensen (1983, 1986), Kalish (1983), Jeuland and Dolan (1982)
How should a monopolist price over time a new product or service whose consumption value increases with the expansion of the "network" of adopters referred to as a network externality (e.g., electronic mail)?	Price and cumulative adoption affect market potential	Price increases over time	Dhebar and Oren (1985)
Advertising			
How should a monopolist advertise a new product over time?	Advertising affects coefficients of innovation and/or imitation; three types of response functions can be used to represent this effect: linear, concave (diminishing returns), S-shaped (increasing and then diminishing returns)	1. Linear response function implies a blitz followed by a constant maintenance level 2. If advertising affects only innovators, concave response function implies a policy whereby advertising decreases over time, gradually approaching the maintenance level; if advertising affects imitators, concave response function implies a policy whereby advertising increases over time 3. S-shaped response function implies a high intensity blitz level followed by a pulsing policy	Horsky and Simon (1983), Dockner and Jorgensen (1988a), Mahajan and Muller (1986)
Timing			
When should a monopolist introduce a product if both positive and negative word of mouth affect diffusion process? How should product be advertised over time?	Members in a social system pass through three stages in innovation decision process: unaware, potential customers, and adopters; both potential customers and adopters circulate positive as well as negative word of mouth	Optimal timing calls for advertising before product is introduced and withdrawal of product after end of advertising period	Mahajan, Muller, and Kerin (1984)
When should a monopolist introduce a second generation product? Should firm shelve it or introduce it as soon as it is available?	No pricing or advertising effect	In most cases, optimal timing decision is "now or never"; if optimal introduction time exists, it is early in life cycle of first product	Wilson and Norton (1989)

Issue and Marketing Mix Variable	Major Assumptions/Comments	Major Normative Results	Illustrative References
Industry Setting: Oligopoly			
Price			
How will firms in an oligopoly price their products over their life cycle?	Price has a multiplicative effect on diffusion; demand saturation causes decline in price; diffusion effect causes price increase; experience effect causes a decline in price	Monopoly results extend to oligopoly case: if imitation effect is strong, price increases initially, and if planning horizon is long, it decreases toward end of planning horizon	Thompson and Teng (1984), Clarke and Dolan (1984), Dockner and Jorgensen (1988b)
How does an industry set a price of a new product class over time?	Market price is a function of quantities set by oligopolists	Same as above	Rao and Bass (1985)
Advertising			
How would firms in an oligopoly advertise their products over time?	Advertising affects innovators or imitators: linear or concave advertising response	1. In many cases, advertising starts with a high level that decreases to a constant maintenance policy 2. Emphasis on final market shares causes an increase in advertising toward end of planning horizon 3. In some cases, advertising may increase; moreover, in some cases advertising for one competitor may increase while that of the second competitor may decrease, or both may increase	Deal (1979), Teng and Thompson (1983), Thompson and Teng (1984), Erickson (1985)
Timing			
Does order of entry affect long-term market share? How would anticipation of entry affect investment decision of a monopolist?	Multiplicative price effect on diffusion	1. Order of entry has no long-term effects on final market shares 2. Monopolist who does not foresee entry overcapitalizes in contrast to foresighted monopolist who anticipates entry	Fershtman, Mahajan, and Muller (1990)
How would anticipation of entry affect pricing decision of a monopolist?	Multiplicative price effect on diffusion	Foresighted monopolist who anticipates entry reduces price in contrast to a surprised monopolist who does not foresee entry	Eliashberg and Jeuland (1986)

Lehmann, and Sultan 1987; Winer 1985). For such innovations, in addition to influencing the nature of the adoption curve, consumer expectations can influence the optimal marketing mix strategy used by a firm. For example, incorporating consumer expectations related to price in the Bass model, Narasimhan (1989) suggests that the optimal pricing strategy for a monopolist cycles over time and within each cycle the price increases at introduction, peaks, and decreases later. Given the importance of consumer expectations in understanding diffusion dynamics, we expect future research to incorporate them into the Bass model.

Exploration of recent developments in hazard models. The different diffusion models can be viewed as making different assumptions about the "hazard rate" for nonadopters as a function of time (the hazard rate being the likelihood that an individual who has remained a nonadopter through time t will become an

adopter in the next instant of time). The Bass model specifies this rate as a linear function of previous adopters. Since the publication of the Bass model, however, much work developing and applying hazard models has appeared in the statistics, biometrics, and econometrics literatures (e.g., Cox and Oakes 1984; Kalbfleisch and Prentice 1980; for possible marketing applications of hazard models, see Helsen and Schmittlein 1989; for interpretation of diffusion models as hazard models, see Lavaraj and Gore 1990). The key development in hazard models over the last decade has been in the area of understanding covariate effects on the hazard rate (and consequently on duration times). This development is particularly important because attempts to incorporate marketing mix variables (and other covariate effects) in diffusion models to date have been very limited in scope and *ad hoc* in their choice of model specifications for those effects. Exploration of recent developments in the

hazard modeling framework may provide a unifying theme for understanding of covariate/marketing mix effects in diffusion models.

Understanding of diffusion processes at the micro (individual) level. Diffusion models based on individual-level adoption decisions offer an opportunity to study the actual pattern of social communication and its impact on product perceptions, preferences, and ultimate adoption. The empirical evidence provided by Chatterjee and Eliashberg (1989) on the development of aggregate diffusion models from individual-level adoption decisions, though limited, is encouraging. Further empirical work on such models may assist in developing the aggregate diffusion models prior to launch.

Parameter Estimation Considerations

In comparison with the other subareas we review, parameter estimation considerations for the Bass model probably received the most attention in the 1980s. These developments are timely and encouraging, but further empirical work on the validation of meta-analysis procedures (Montgomery and Srinivasan 1989; Sultan, Farley, and Lehmann 1990), Bayesian estimation procedures (Lenk and Rao 1989; Sultan, Farley, and Lehmann 1990), and procedures that capitalize on the information provided by managers and potential adopters (e.g., Randles 1983; Souder and Quaddus 1982) is important. An emerging body of literature in the forecasting area suggests that combining parameter estimates from different estimation procedures can yield better forecasting results (see Mahajan and Wind 1988). Empirical studies that explore the feasibility of such findings for diffusion models are desirable (Lawrence and Geurts 1984).

Flexible Diffusion Models

Flexible diffusion models have the advantage of capturing penetration patterns that are symmetric as well as nonsymmetric with no restrictions on the point of inflection. However, among all the models reviewed in Table 3, only the models by Von Bertalanffy (1957) (or Nelder 1962) and Bewley and Fiebig (1988) offer closed-form solutions to the differential equations used to specify the diffusion dynamics (i.e., express the number of adopters as an explicit function of time, which is desirable for long-term forecasting). Furthermore, these models have a flexibility advantage by requiring estimation of additional numbers of parameters. However, two important questions remain: How much additional long-term forecasting accuracy is provided by the flexible models, in comparison with the basic diffusion models such as the Bass model, when controlled for the number of parameters? Given the parameter estimation considerations discussed here,

how can parameters in these models be calibrated prior to launch for long-term forecasting? Further empirical work related to these questions is desirable.

Refinements and Extensions

We briefly discuss 10 possibilities for further refinement and extension of the Bass model.

- A decade ago, Mahajan and Muller (1979) concluded that it was not clear how marketing mix variables should be incorporated into the Bass model. The few empirical studies reported in the 1980s still do not provide conclusive guidelines on this question. Despite the arguments made in favor of including price in the market potential, empirical studies on consumer durable products by Kamakura and Balasubramanian (1988) and Jain and Rao (1989) suggest that price affects the rate of diffusion (by influencing the coefficients of external influence and internal influence). Similarly, in relation to the inclusion of advertising in the Bass model, the two reported empirical studies suggest different alternatives. Horsky and Simon (1983) recommend that it be included in the coefficient of external influence whereas Simon and Sebastian (1987) report better results by including it in the coefficient of internal influence. Interestingly, though both of these studies examine the effect of advertising on the diffusion of a service (a banking service by Horsky and Simon and a telephone service by Simon and Sebastian), they were conducted in two different markets (U.S. and West Germany) and under different market conditions (there was a supply problem with the availability of telephones in West Germany). Whether these differences had an impact on the reported results is an empirical question. Given the importance of including marketing mix variables in understanding diffusion dynamics, we expect more empirical work including other marketing mix variables such as distribution.

- Several of the empirical studies reported in Table 5 have incorporated product attributes in the Bass model. A natural extension of these studies is to develop procedures to determine optimal product design to obtain the desirable penetration rate.

- For high technology products, the time interval between successive generations of technologies has been decreasing. Norton and Bass (1987) have shown how diffusion of successive generations interacts within the context of the Bass model. Forecasting possibilities stemming from this work appear to be promising. Extensions involving pricing of generations of technology would be desirable and feasible.

- When should a firm introduce a second generation product? Though the analytical results of Wilson and Norton (1989) suggest the answer is "now or never," they exclude the impact of other variables such as price. Further theoretical and empirical work addressing this question would be welcome.

- For high technology products, the product offering of a firm generally includes both hardware and software, such as Nintendo hardware (keypad) and Nintendo software (video games) for children. Because of the contingency inherent in the relationship, it is important to develop diffusion models that examine the diffusion of the entire bundle of product offerings. In addition to forecasting,

normative questions may relate to its optimal pricing and distribution. For example, how should a monopolist (e.g., Nintendo) manufacture and distribute its hardware and software? Should it keep a monopoly on both of them? Should it keep a monopoly on hardware and create an oligopoly for software to increase demand for the hardware?

- How do the number of competitors and the rivalry among them influence the growth of a product category? Does the growth affect the entry/exit patterns of competitors? Answers to these questions are within the domain of the diffusion modeling framework and provide a linkage with the strategic planning literature. Theoretical and empirical work on these questions will enhance the utility of diffusion models.

- Supply restrictions influence diffusion patterns. For certain types of products (e.g., prescription drugs), it may be desirable to retard the diffusion process by controlling their supply and distribution. Further empirical and theoretical work on this linkage would enable managers to control the life cycle of a product by managing the supply.

- Market interventions (e.g., patent violations) are externalities that can influence the growth pattern of a new product. Though the use of intervention analysis is well established in the time-series analysis literature, no attempt seems to have been made to conduct intervention analysis with the diffusion models (Mahajan, Sharma, and Wind 1985). Theoretical and empirical work in this area could assist in assessing the impact (e.g., assessing patent violation damages in a legal case) of market interventions on the product life cycle.

- Though integration of the time and spatial dimensions has been of interest to geographers, their integration is equally important in marketing to evaluate alternative product distribution strategies across markets. Such extensions of the Bass model could assist in evaluating the impact on the growth of a new product of how and where the product is made available.

- The diffusion literature has emphasized consistently the importance of negative word of mouth on the growth of a new product (Mahajan, Muller, and Kerin 1984). The multistage extensions of the Bass model offer an avenue for considering its impact on the growth pattern. These extensions lack empirical validation, however. Data collection and estimation procedures should be developed to make these extensions practicable.

- Not all new products are accepted by consumers at the time of their introduction. Some products are much slower than others in being accepted by potential adopters. That is, they differ in terms of how long it takes them to "take off." The "take-off" phenomenon is not considered explicitly in the Bass model. The Bass model assumes the presence of a certain number of consumers before "take off" (i.e., pm). Extensions of the Bass model that explicitly consider this phenomenon will be useful in explaining and predicting the take-off behavior of a new product.

Use of Diffusion Models

One of the critical uses of diffusion models has been for forecasting the first-purchase sales volume curve. In recent years, questions have been raised about the forecasting accuracy of diffusion models (Bernhardt

and MacKenzie 1972; Heeler and Hustad 1980). We sympathize with such concerns and believe that further empirical work is needed to identify conditions under which diffusion models work or do not work. For example, recent work by Jain, Mahajan, and Muller (1989) suggests that the use of the Bass model is inappropriate in international settings where the supply of the product is restricted. Furthermore, as the diffusion models capture the dynamics of innovation diffusion for first-time buyers, it is not clear that the same diffusion dynamics are applicable to replacement sales. Therefore the use of diffusion models for such adoption data may be inappropriate (see, e.g., Bayus 1988; Bayus, Hong, and Labe 1989). Finally, diffusion models are imitation models. Any S-shaped curve, however, may not be a result of the imitation process, and alternative time-series models may be more appropriate for such data (Mahajan, Sharma, and Bettis 1988). Even in the presence of the imitation effect, it may be necessary to examine various diffusion models systematically to identify the one that best describes the data (Rust and Schmittlein 1985). There is also a growing body of literature on "chaos theory" suggesting that for certain parameter values, diffusion models generate persistent chaotic behavior within predictable boundaries (Gordon and Greenspan 1988). Understanding of such phenomena may be essential to decipher the impact of changes that affect the diffusion dynamics.

The use of diffusion models to test diffusion-based hypotheses is very encouraging. The empirical studies documented in Table 4 clearly attest to their potential in such applications. We expect additional empirical work employing a diffusion modeling framework to test hypotheses related to life cycle dynamics (e.g.: How does the number of competitors change over the life cycle of a product? How does the number of brands available in a market influence the growth of a product? How does the rivalry among competitors in an industry affect the life cycle of a product? Etc.)

The use of diffusion models to derive normative results for the dynamics of innovation diffusion received considerable attention in 1980s. However, as summarized in Table 5, these results are simply working hypotheses. Furthermore, the nature of these results is contingent on the assumptions made in their analytical derivation. For most of these studies, the analytical elegance surpasses the empirical validation of the derived results. Empirical evidence is needed to find out if and when the firms use the derived normative strategies.

Finally, it is important to acknowledge that several firms have used diffusion models for forecasting the demand of a new product. By sharing their experiences, industry users can contribute to the further validation of diffusion models.

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