"高分子加工原理"　課程

Upper-Convected Maxwell (UCM) rheological model

$$\lambda \frac{\delta \underset{\approx}{\tau}}{\delta t} = 2G\lambda \underset{\approx}{d} - \underset{\approx}{\tau} \quad (1)$$

where

$$\frac{\delta \underset{\approx}{\tau}}{\delta t} = \frac{\partial \underset{\approx}{\tau}}{\partial t} + (\underset{\sim}{v} \cdot \underset{\sim}{\nabla}) \underset{\approx}{\tau} - \underset{\sim}{\nabla} \underset{\sim}{v}^{T} \cdot \underset{\approx}{\tau} - \underset{\approx}{\tau} \cdot \underset{\sim}{\nabla} \underset{\sim}{v} \quad (2)$$

$\underline{V} = (v_1, v_2, v_3)$，　$2\underline{\underline{d}} = \nabla \underline{V} + (\nabla \underline{V})^T$,　assume that G, λ *are constant.*

Given a flow field,

$$v_1 = v_1(x_1) = \dot{\varepsilon} \cdot x_1 \quad , \quad (3)$$

$$v_2(x_2) = -\frac{1}{2} \dot{\varepsilon} \cdot x_2 \quad , \quad (4)$$

$$v_3(x_3) = -\frac{1}{2} \dot{\varepsilon} \cdot x_3 \quad . \quad (5)$$

where $\dot{\varepsilon} = const.$　$\underset{\approx}{\tau} = \begin{pmatrix} \tau_{11} & \tau_{12} & \tau_{13} \\ \tau_{21} & \tau_{22} & \tau_{23} \\ \tau_{31} & \tau_{32} & \tau_{33} \end{pmatrix}$

(A)　寫出以上 Tensor 方程式(1)之展開式 [將(1)式 展開成 9 條純量（scalar）方程式]。

(B)　求 $\nabla \cdot \underline{v} = ?$

給定公式如下：

$$\{\mathbf{v} \cdot \nabla \boldsymbol{\tau}\}_{xx} = (\mathbf{v} \cdot \nabla) \tau_{xx}$$
$$\{\mathbf{v} \cdot \nabla \boldsymbol{\tau}\}_{xy} = (\mathbf{v} \cdot \nabla) \tau_{xy}$$
$$\{\mathbf{v} \cdot \nabla \boldsymbol{\tau}\}_{xz} = (\mathbf{v} \cdot \nabla) \tau_{xz}$$
$$\{\mathbf{v} \cdot \nabla \boldsymbol{\tau}\}_{yx} = (\mathbf{v} \cdot \nabla) \tau_{yx}$$
$$\{\mathbf{v} \cdot \nabla \boldsymbol{\tau}\}_{yy} = (\mathbf{v} \cdot \nabla) \tau_{yy}$$
$$\{\mathbf{v} \cdot \nabla \boldsymbol{\tau}\}_{yz} = (\mathbf{v} \cdot \nabla) \tau_{yz}$$
$$\{\mathbf{v} \cdot \nabla \boldsymbol{\tau}\}_{zx} = (\mathbf{v} \cdot \nabla) \tau_{zx}$$
$$\{\mathbf{v} \cdot \nabla \boldsymbol{\tau}\}_{zy} = (\mathbf{v} \cdot \nabla) \tau_{zy}$$
$$\{\mathbf{v} \cdot \nabla \boldsymbol{\tau}\}_{zz} = (\mathbf{v} \cdot \nabla) \tau_{zz}$$