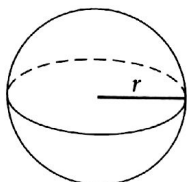


- a. Show that the derivative of the area formula is $2\pi r$, the formula for the circumference of a circle.
- b. Give an explanation for this in terms of rates of change.

36. GENERAL: Geometry The formula for the volume of a sphere is $V = \frac{4}{3}\pi r^3$, where r is the radius of the sphere and π is a constant.



- a. Show that the derivative of the volume formula is $4\pi r^2$, the formula for the surface area of a sphere.
- b. Give an explanation for this in terms of rates of change.

2.4 The Product and Quotient Rules

37–46. Find the derivative of each function.

37. $f(x) = 2x(5x^3 + 3)$

38. $f(x) = x^2(3x^3 - 1)$

✓ 39. $f(x) = (x^2 + 5)(x^2 - 5)$

40. $f(x) = (x^2 + 3)(x^2 - 3)$

41. $y = (x^4 + x^2 + 1)(x^5 - x^3 + x)$

42. $y = (x^5 + x^3 + x)(x^4 - x^2 + 1)$

43. $y = \frac{x - 1}{x + 1}$

44. $y = \frac{x + 1}{x - 1}$

45. $y = \frac{x^5 + 1}{x^5 - 1}$

46. $y = \frac{x^6 - 1}{x^6 + 1}$

✓ 47. Find the derivative of $f(x) = \frac{2x + 1}{x}$ in three different ways, and check that the answers agree:

- a. By the Quotient Rule
- b. By writing the function in the form $f(x) = (2x + 1)(x^{-1})$ and using the Product Rule.
- c. By thinking of another way, which is the easiest of all.

✓ 48. **BUSINESS: Sales** The manager of an electronics store estimates that the number of cassette tapes that a store will sell at a price of x dollars is

$$S(x) = \frac{2250}{x + 9}$$

Find the rate of change of this quantity when the price is \$6 per tape, and interpret your answer.

49. **BUSINESS: Marginal Average Profit** A company profit function is $P(x) = 6x - 200$ dollars, where x is the number of units.

- a. Find the average profit function.
- b. Find the marginal average profit function.
- c. Evaluate the marginal average profit function at $x = 10$ and interpret your answer.

50. **BUSINESS: Marginal Average Cost** A company can produce a mini optical computer mouse at a cost of \$7.50 each while fixed costs are \$50. Therefore, the company's cost function is $C(x) = 7.5x + 50$.

- a. Find the average cost function $AC(x) = \frac{C(x)}{x}$.
- b. Find the marginal average cost function $MAC(x)$.
- c. Evaluate $MAC(x)$ at $x = 50$ and interpret your answer.

Source: Green Pearle International

2.5 Higher-Order Derivatives

51–54. Find the *second* derivative of each function.

51. $f(x) = 12\sqrt{x^3} - 9\sqrt[3]{x}$

52. $f(x) = 18\sqrt[3]{x^2} - 4\sqrt{x^3}$

53. $f(x) = \frac{1}{3x^2}$

54. $f(x) = \frac{1}{2x^3}$

55–60. Evaluate each expression.

55. If $f(x) = \frac{2}{x^3}$, find $f''(-1)$.

56. If $f(x) = \frac{3}{x^4}$, find $f''(-1)$.

57. $\left. \frac{d^2}{dx^2} x^6 \right|_{x=-2}$